Academic Year: 2019 – 20 **Semester: II**

Class : III B.Tech **Subject: Heat Transfer**

Learning Material

UNIT-I

Introduction and One Dimensional Steady State Conduction Heat Transfer

Syllabus:

Introduction: Modes and mechanisms of heat transfer, Basic laws of heat transfer, General discussion about applications of heat transfer. Difference between heat transfer and thermodynamics.

Conduction Heat Transfer: Fourier Law of Heat conduction, General heat conduction equation in Cartesian, Cylindrical and Spherical coordinates.

Simplification and forms of the field equation, steady, unsteady heat transfer, Initial and boundary conditions

One Dimensional Steady State Conduction Heat Transfer: Homogeneous slabs, hollow cylinders and spheres, Electrical Analogy, Thermal Contact Resistance, Thermal Shape Factor, Critical radius of insulation, Variable Thermal conductivity, systems with heat generation.

1.1 Introduction:

From the study of thermodynamics, you have learned that energy can be transferred by interactions of a system with its surroundings. These interactions are called work and heat. However, thermodynamics deals with the end states of the process during which an interaction occurs and provides no information concerning the nature of the interaction or the time rate at which it occurs.

For example, imagine a water bottle is kept in refrigerator for cooling purpose. Using the laws of thermodynamics, we can find only the amount of heat required to be removed from the water to achieve the specified cold temperature. But we can't find the time it will take to reach that temperature. Also we can't find the temperature of water at different intervals of time in the process of cooling. Thus Thermodynamics deals only with the transfer of heat between two equilibrium states, where as heat transfer deals with rate of energy transfer. In thermodynamics we represent heat in Joules or kJ. But in heat transfer we measure rate of heat transfer by introducing time effect and the unit is Watt or kilowatt.

Importance of Heat Transfer:

- It is the science that deals with the mechanism of energy transfer and the rate of energy transfer due to a difference in temperature.
- It enables an ***engineer*** to design an equipment in which the process occurs.
- It provides the fundamental information needed to estimate the size and hence the cost of an equipment necessary to transfer a specified amount of heat in a given time.
- A ***mechanical engineer*** is concerned with developing and designing process equipment for thermal and nuclear power plants, and heating and cooling systems for comfort conditions.
- A ***chemical engineer*** concerns with heat transfer processes in numerous chemical reactions.
- A ***metallurgical engineer*** has to monitor rates of temperature change during heat transfer processes in order to achieve the desired properties in a material.

- An ***electronics and computer engineer*** has to ensure that heat is dissipated efficiently from transistors and chips, so that their temperature do not exceed safe limits.
- An ***electrical engineer*** has to ensure that the designed electrical appliances (motors, generators, fans etc.) do not overheat during operation.

1.2 Modes of Heat transfer:

The study of heat transfer requires the knowledge of three modes of heat transfer - conduction, convection and radiation.

1.2.1 Conduction

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. Conduction can take place in solids, liquids, or gases.

In gases and liquids, conduction is due to the *collisions* and *diffusion* of the molecules during their random motion. In solids, it is due to the combination of *vibrations* of the molecules in a lattice and the energy transport by *free electron.*

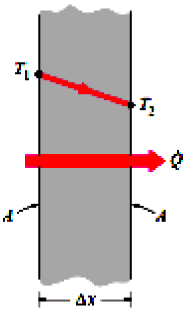


fig.1

The *rate* of heat conduction through a medium depends on the *geometry* of the medium, its *thickness,* and the *material* of the medium, as well as the *temperature difference* across the medium.

Fourier's Law: The rate of heat transfer by conduction is proportional to the temperature gradient in the direction of heat flow and the area normal to the heat flow direction. Which is called **Fourier's law of heat conduction** after J. Fourier. Mathematically, it is expressed as

$$\dot{Q}_{cond} = -kA\frac{dT}{dx} \qquad (1.1)$$

Where, the constant of proportionality k is the **thermal conductivity** of the material, which is a measure of the ability of a material to conduct heat and its unit is W/m^{0}C or W/m-K. $\frac{dT}{dx}$ is the **temperature gradient,** which is the slope of the temperature curve ona *T*-*x* diagram (the rate of change of *T* with *x*) at a location *x.* A is the area normal to heat flow direction.

Negative sign is introduced in the equation (1), as the heat is conducted in the direction of decreasing temperature, and the temperature gradient becomes negative when temperature decreases with increasing *x.*

Thermal Conductivity

The thermal conductivity *k* is a measure of a material's ability to conduct heat. For example, *k* =0.608 W/m^{0}C for water and *k* = 80.2 W/m^{0}C for iron at room temperature, which indicates that iron conducts heat more than 100 times faster than water can. Thus we say that water is a poor heat conductor relative to iron, although water is an excellent medium to store thermal energy.

The **thermal conductivity** of a material can be defined as *the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.*

A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or *insulator.* The thermal conductivities of some common materials at room temperature are given in Table 1–1.

➢ Note that materials such as copper and silver that are good electric conductor are also good heat conductors, and have high values of thermal conductivity.

TABLE 1–1

The thermal conductivities of some materials at room temperature

Material	k, W/m · °C*
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.613
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026

*Multiply by 0.5778 to convert to Btu/h · ft · °F.

- Materials such as rubber, wood, and Styrofoam are poor conductors of heat and have low conductivity values.
- Pure crystals and metals have the highest thermal conductivities, and gases and insulating materials the lowest.

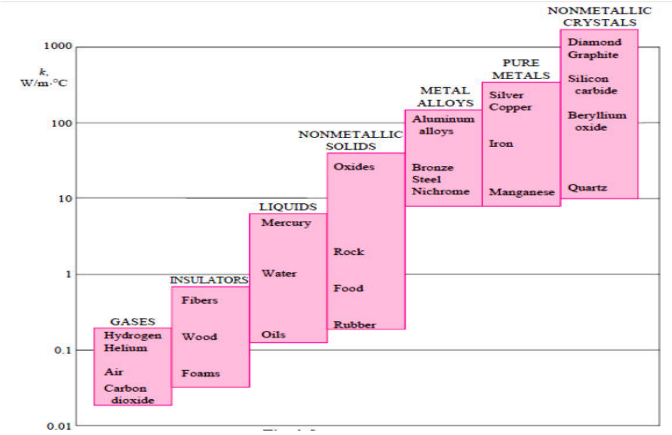


Fig. 1.2 Thermal conductivity of different materials

Heat conduction in gases:

- The kinetic theory of gases predicts and the experiments confirm that the thermal conductivity of gases is proportional to the square root of the thermal dynamic temperature T, and inversely proportional to the square root of the molar mass M.

- Therefore, the thermal conductivity of a gas increases with increasing temperature and decreasing molar mass. So it is not surprising that the thermal conductivity of helium (M=4) is much higher than those of air (M=29) and argon (M=40).

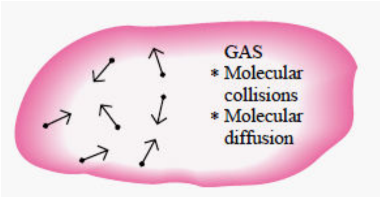


Fig. 1.3a Conduction heat transfer in gases

Heat conduction in liquids:

- The mechanism of heat conduction in a *liquid* is complicated by the fact that the molecules are more closely spaced, and they exert a stronger intermolecular force field.
- The thermal conductivities of liquids usually lie between those of solids and gases.
- The thermal conductivity of a substance is normally highest in the solid phase and lowest in the gas phase.
- Unlike gases, the thermal conductivities of most liquids decrease with increasing temperature, with water being a notable exception.
- Like gases, the conductivity of liquids decreases with increasing molar mass.
- Liquid metals such as mercury and sodium have high thermal conductivities and are very suitable for use in applications where a high heat transfer rate to a liquid is desired, as in nuclear power plants.

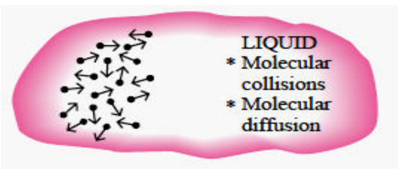


Fig. 1.3b Conduction heat transfer in liquids

Heat conduction in Solids:

- In *solids,* heat conduction is due to two effects: the *lattice vibrational waves* induced by the vibrational motions of the molecules positioned at relatively fixed positions in a periodic manner called a lattice, and the energy transported via the *free flow of electrons* in the solid (see Fig. 1.3b).
- The thermal conductivity of a solid is obtained by adding the lattice and electronic components.
- The relatively high thermal conductivities of pure metals are primarily due to the electronic component.

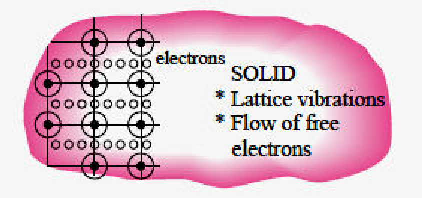


Fig. 1.3c Conduction heat transfer in Solids

- The lattice component of thermal conductivity strongly depends on the way the molecules are arranged. For example, diamond, which is a highly ordered crystalline solid, has the highest known thermal conductivity at room temperature.
- Pure metals have high thermal conductivities.
- The thermal conductivity of an alloy of two metals is usually much lower than that of either metal, as shown in Table 1–2. This is due to structural heterogeneity, which brings about electron scattering. Even small amounts in a pure metal of "foreign" molecules that are good conductors themselves seriously disrupt the flow of heat in that metal.
- The thermal conductivities of materials vary with temperature (Table 1–3). The thermal conductivities of certain solids exhibit dramatic increases at temperatures near absolute zero, when these solids become *superconductors*. For example, the conductivity of copper reaches a maximum value of about 20,000 W/m °C at 20 K, which is about 50 times the conductivity at room temperature.

TABLE 1–2

The thermal conductivity of an alloy is usually much lower than the thermal conductivity of either metal of which it is composed

Pure metal or alloy	*k*, W/m · °C, at 300 K
Copper	401
Nickel	91
Constantan (55% Cu, 45% Ni)	23
Copper	401
Aluminum	237
Commercial bronze (90% Cu, 10% Al)	52

TABLE 1–3

Thermal conductivities of materials vary with temperature

T, K	Copper	Aluminum
100	482	302
200	413	237
300	401	237
400	393	240
600	379	231
800	366	218

1.2.2 Convection:

Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of *conduction* and *fluid motion.*

- The faster the fluid motion, the greater the convection heat transfer.

- In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction.
- The presence of bulk motion of the fluid enhances the heat transfer between the solid surface and the fluid.

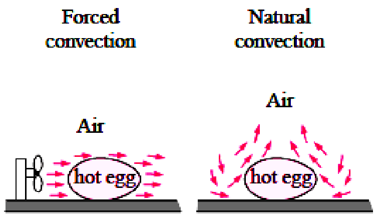


Fig. 1.4 Convection Mechanism

Forced convection: Convection is called **forced convection** if the fluid is forced to flow over the surface or through a tube/duct by external means such as a fan, pump, or the wind.
Natural convection: In contrast, convection is called **natural** (or **free**) **convection** if the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid.
Newton's law of cooling: The rate of convection heat transfer is observed to be proportional to the temperature difference, and is conveniently expressed by as

$$\dot{Q}_{conv} = hA_s\left(T_s - T_\infty\right) \qquad (1.2)$$

where h is the convection heat transfer coefficient in W/m^2-^0C, $A_s$is the surface area exposed to the fluid, T_s is the surface temperature, and T_∞ is the temperature of the fluid sufficiently far from the surface.

- The convection heat transfer coefficient *h* is not a property of the fluid. It is an experimentally determined parameter whose value depends on all the variables influencing convection such as the surface geometry, the nature of fluid motion, the properties of the fluid, and the bulk fluid velocity.

1.2.3 Radiation

Radiation is the energy emitted by matter in the form of *electromagnetic waves* (or *photons*) as a result of the changes in the electronic configurations of the atoms or molecules.

- Unlike conduction and convection, the transfer of energy by radiation does not require the presence of an *intervening medium.*
- In fact, energy transfer by radiation is fastest (at the speed of light) and it suffers no attenuation in a vacuum. This is how the energy of the sun reaches the earth.
- In heat transfer studies we are interested in *thermal radiation,* which is the form of radiation emitted by bodies because of their temperature.
- All bodies at a temperature above absolute zero emit thermal radiation.
- Radiation is a *volumetric phenomenon,* and all solids, liquids, and gases emit, absorb, or transmit radiation to varying degrees.
- However, radiation is usually considered to be a *surface phenomenon* for solids that are opaque to thermal radiation such as metals, wood, and rocks since the radiation emitted by the interior regions of such material can never reach the surface, and the radiation incident on such bodies is usually absorbed within a few microns from the surface.

Stefan–Boltzmann law: The rate of heat flux emitted from the surface of a black body is directly proportional to the fourth power of its absolute temperature. Mathematically,

$$\dot{Q}_{max} = \sigma A_s T_s^4 \tag{1.3}$$

where $\sigma = 5.667 \times 10^{-8} W/m^2K^4$ is called Stefan Boltzmann constant.

- The idealized surface that emits radiation at this maximum rate is called a **blackbody,** and the radiation emitted by a blackbody is called **blackbody radiation.**

The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature, and is expressed as

$$\left(\dot{Q}_{max}\right)_{real} = \sigma \in A_s T_s^4 \tag{1.4}$$

where $\in$ is the emissivity of the surface.

1.3 Conduction Heat Transfer

The Fourier Law of heat conduction states that the rate of heat flux in a given direction is directly proportional to temperature gradient. Thus heat flux in x-direction we have

$$q_x = -k_x \frac{\partial T}{\partial x}$$

Conduction in solids can occur in all the possible three directions $x, y \; or \; z$. Such treatment considering heat transfer in all directions is known as 3- dimensional conduction analysis but often it is treated as one or two dimensional.

The Fourier law can be expressed for other directions as $q_y = -k_y \frac{\partial T}{\partial y}$ & $q_z = -k_z \frac{\partial T}{\partial z}$ along y and z directions respectively.

where $k_x, \; k_y, \; and \; k_z$ represent the thermal conductivities of the substances in $x, y \; and \; z \; directions$ respectively.

- In the case of wood, asbestos, laminated boards and carbon fiber composites, the thermal conductivity depends on the direction of heat flow.
- In wood or asbestos, the thermal conductivity parallel to the grains differ from that perpendicular to these grains. Such a material, which exhibits different k values in different directions, is **anisotropic** material.
- However, for metals and alloys the directional variation in thermal conductivity is negligible and we assume that they are isotropic, i.e. same property in all directions.

1.3.1 General conduction Equation in Cartesian System

Let us consider a volume element as shown in fig. 1.5. having dimensions $dx, dy \; and \; dz$. We assume that the element has an internal heat generation of $q_G \, W/m^3$ due to chemical reaction or electrical resistance heating or nuclear reaction etc.

We further consider the conduction heat transfer in three directions x, y, z as $Q_x, Q_y, and \; Q_z$ entering the element and $Q_{x+dx}, Q_{y+dy}, Q_{z+dz}$ as leaving.

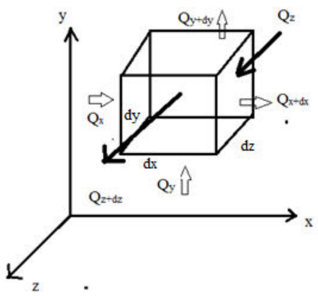


Fig. 1.5 Heat conduction

From the Fourier Law of Heat conduction,

$$Q_x = -k_x[dydz]\frac{\partial T}{\partial x}$$

$$Q_y = -k_y[dxdz]\frac{\partial T}{\partial y} \text{ and , } Q_z = -k_z[dxdy]\frac{\partial T}{\partial z} \tag{1.5}$$

Further, the heat leaving in $x \; direction$ at $x + dx$ is mathematically expressed as

$$Q_{x+dx} = Q_x + \frac{\partial}{\partial x}[Q_x]dx$$

Similarly, in y and z directions

$$Q_{y+dy} = Q_y + \frac{\partial}{\partial y}[Q_y]dy \text{ and } Q_{z+dz} = Q_z + \frac{\partial}{\partial z}[Q_z]dz \quad (1.6)$$

The amount of total heat generated in the elemental volume= $q_g \times vol = q_g \times dx \times dy \times dz$.

If we assume the density of the element ρ, then the mass of element, $m = \rho dxdydz$.

Now applying ***Law of conservation of energy*** as

Heat energy entering + Heat generated - Heat energy leaving = Rate of energy stored in the element. The stored energy is responsible for increasing the internal heat capacity of the element given by $m.c.\frac{\partial T}{\partial t}$ where c is the specific heat of the material in $J/kg.K$ and t represents time.

Now substituting the respective quantities in the energy balance equation:

$$Q_x + Q_y + Q_z + q_g(dx \times dy \times dz) - Q_{x+dx} - Q_{y+dy} - Q_{z+dz} = \rho cdxdydz \quad (1.7)$$

$$k_x dxdydz\frac{\partial^2 T}{\partial x^2} + k_y[dxdydz]\frac{\partial^2 T}{\partial y^2} + k_z[dxdydz]\frac{\partial^2 T}{\partial z^2} + q_g(dx \times dy \times dz) = \rho cdxdydz\frac{\partial T}{\partial t} \quad (1.8)$$

On simplification, we get: $$k_x\frac{\partial^2 T}{\partial x^2} + k_y\frac{\partial^2 T}{\partial y^2} + k_z\frac{\partial^2 T}{\partial z^2} + q_g = \rho c\frac{\partial T}{\partial t} \quad (1.9)$$

Assuming isotropic behavior: $$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{\rho c}{k}\frac{\partial T}{\partial t} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (1.10)$$

$\frac{k}{\rho c}$ is known as thermal diffusivity and its units are m^2/s. Thermal diffusivity is a property of the material and indicates the rate at which the heat energy is distributed in the material. Metals have large diffusivity compared to non-metals.

1.3.2 General Conduction Equation in Cylindrical Coordinates

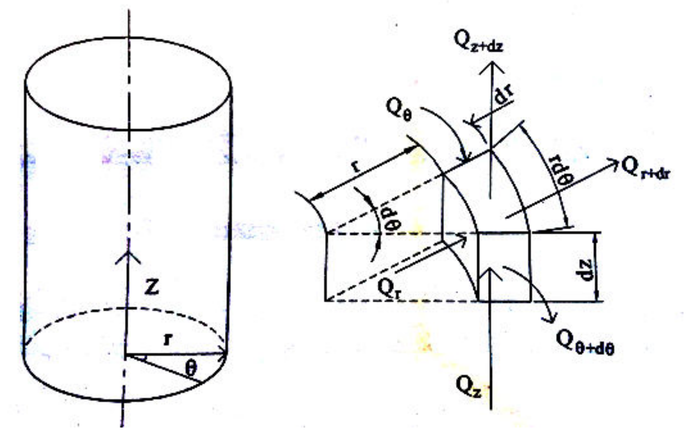


Fig. 1.6 Element in spherical coordinate system

Consider an element cut from a cylinder by taking radial and longitudinal sections as shown in the fig. 1.6.

The volume of the element considered = $rd\theta drdz$.

Heat entering at radius r as per Fourier's Law $Q_r = -k(rd\theta dz)\frac{\partial T}{\partial r}$ (1.11)

Heat leaving at $(r + dr)$ in radial direction is $Q_{r+dr} = Q_r + \frac{\partial}{\partial r}[Q_r]dr$ (1.12)

Net heat entering in r direction = $Q_r - Q_{r+dr} = k\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] rd\theta drdz$ (1.13)

Heat entering in θ $direction$ is $Q_\theta = -k(drdz)\frac{\partial T}{r\partial\theta}$ (1.14)

Heat leaving At $\theta + d\theta$ *in tangential direction is* $Q_{\theta+d\theta} = Q_\theta + \frac{\partial[Q_\theta]}{r\partial\theta} rd\theta$ (1.15)

Net heat entering in θ *direction* is $Q_\theta - Q_{\theta+d\theta} = k\left[\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}\right] rd\theta drdz$ (1.16)

Heat entering in z direction is $Q_z = -k[rd\theta dr]\frac{\partial T}{\partial z}$ (1.16)

At $z + dz$ *in axial direction is* $Q_{z+dz} = Q_z + \frac{\partial}{\partial z}[Q_z]dz$ (1.17)

Net heat leaving z direction $= Q_z - Q_{z+dz} = k\left[\frac{\partial^2 T}{\partial z^2}\right] rd\theta drdz$ (1.18)

Total heat generated in the control volume = $q_G(rd\theta drdz)$ (1.19)

Applying the energy balance

Heat entering into the element + internal heat generation - heat leaving the element = Rate of change of energy in the element.

Now substituting the respective quantities in the energy balance equation and on simplification, we get:

$$\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] + \left[\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}\right] + \left[\frac{\partial^2 T}{\partial z^2}\right] + \frac{\dot{q}_G}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (1.20)$$

1.3.3 Generalized heat conduction equation in Spherical coordinate system:

Consider an element in a sphere of isotropic material through which heat is conducted along with generation of heat. The element is isolated and presented in fig. 1.7. Let the coordinates in spherical system be (r, ϕ, ψ).

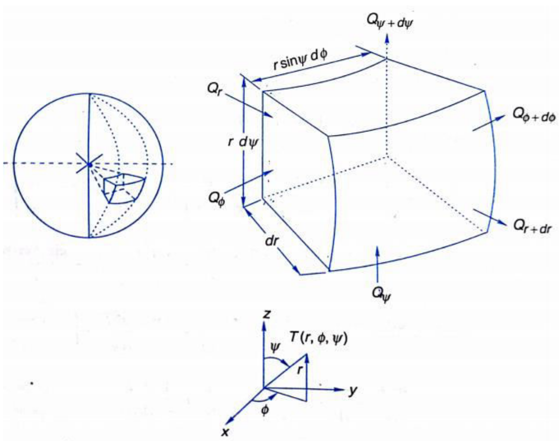


Fig.1.7 Element from a sphere

The rate of heat flowing into the element in r-direction is:

$$Q_r = -k\frac{\partial T}{\partial r}(rd\psi)(r\sin\psi)d\phi = -k\frac{\partial T}{\partial r}r^2\sin\psi d\psi d\phi \quad (1.21)$$

The rate of heat flow out of the element in r-direction is:

$$Q_{r+dr} = Q_r + \frac{\partial Q_r}{\partial r}dr = Q_r + \frac{\partial T}{\partial r}\left(-k\frac{\partial T}{\partial r}r^2\sin\psi d\psi d\phi\right)dr \quad (1.22)$$

The net rate of heat entering the element in r-direction is:

$$Q_r - Q_{r+dr} = \frac{\partial T}{\partial r}\left(k\frac{\partial T}{\partial r}r^2\sin\psi d\psi d\phi\right)dr = k\sin\psi drd\psi d\phi\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}r^2\right) = \frac{kr^2\sin\psi drd\psi d\phi}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}r^2\right) \quad (1.23)$$

The rate of heat flowing into the element in ψ -direction is:

$$Q_{\psi} = -k\frac{\partial T}{r\partial\psi}\left(r\sin\psi d\phi\right)dr \tag{1.24}$$

The rate of heat flow out of the element in ψ -direction is given by:

$$Q_{\psi+d\psi} = Q_{\psi} + \frac{\partial}{r\partial\psi}\left(-k\frac{\partial T}{r\partial\psi}\left(r\sin\psi d\phi\right)dr\right)rd\psi \tag{1.25}$$

The net rate of heat entering the element in ψ -direction is:

$$Q_{\psi} - Q_{\psi+d\psi} = \frac{\partial}{r\sin\psi\partial\psi}\left(-k\frac{\partial T}{r\partial\psi}\left(r\sin\psi d\phi\right)dr\right)rd\psi\sin\psi = \frac{r^2 kdrd\phi d\psi\sin\psi}{r^2\sin\psi}\frac{\partial}{\partial\psi}\left(\frac{\partial T}{\partial\psi}\left(\sin\psi\right)\right) \tag{1.26}$$

The rate of heat flowing into the element in ϕ -direction is $Q_{\phi} = -k\left(rdrd\psi\right)\frac{\partial T}{\partial\phi}$ (1.27)

The rate of heat flow out of the element in ϕ -direction is given by:

$$Q_{\phi+d\phi} = Q_{\phi} + \frac{\partial}{r\sin\psi\partial\phi}\left(-k\left(rdrd\psi\right)\frac{\partial T}{r\sin\psi\partial\phi}\right)r\sin\psi d\phi \tag{1.28}$$

The net rate of heat entering the element in ψ -direction is:

$$Q_{\phi} - Q_{\phi+d\phi} = \frac{\partial}{r\sin\psi\partial\phi}\left(k\left(rdrd\psi\right)\frac{\partial T}{r\sin\psi\partial\phi}\right)r\sin\psi d\phi = \frac{k\left(r^2 drd\psi d\phi\sin\psi\right)}{r^2\sin^2\psi}\frac{\partial^2 T}{\partial\phi^2} \tag{1.29}$$

Let $\ddot{q}$ is the internal heat generated per unit time per unit volume of the element.

Total internal heat generated per unit time = $\ddot{q}\left(r^2\sin\psi drd\phi d\psi\right)$ (1.30)

Change in internal energy per unit time = $\rho c_p\left(r^2\sin\psi drd\phi d\psi\right)\frac{\partial T}{\partial t}$ (1.31)

From the first law of thermodynamics, the energy balance is written as:

Net heat conducted into the element per unit time + Net heat conducted into the element per unit time =

Increase in internal energy per unit time + Work done by the element per unit time

$$\frac{kr^2\sin\psi drd\psi d\phi}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}r^2\right) + \frac{r^2 kdrd\phi d\psi\sin\psi}{r^2\sin\psi}\frac{\partial}{\partial\psi}\left(\frac{\partial T}{\partial\psi}\left(\sin\psi\right)\right) + \frac{k\left(r^2 drd\psi d\phi\sin\psi\right)}{r^2\sin^2\psi}\frac{\partial^2 T}{\partial\phi^2}$$

$$\ddot{q}\left(r^2\sin\psi drd\phi d\psi\right) = \rho c_p\left(r^2\sin\psi drd\phi d\psi\right)\frac{\partial T}{\partial t} \tag{1.32}$$

On simplification, we get

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}r^2\right) + \frac{1}{r^2\sin\psi}\frac{\partial}{\partial\psi}\left(\frac{\partial T}{\partial\psi}\left(\sin\psi\right)\right) + \frac{1}{r^2\sin^2\psi}\frac{\partial^2 T}{\partial\phi^2} + \frac{\ddot{q}}{k} = \frac{\rho c_p}{k}\frac{\partial T}{\partial t} \tag{1.33}$$

Let thermal diffusivity, $\alpha = \frac{k}{\rho c_p}$

Finally, three dimensional unsteady state heat conduction equation in spherical coordinate system with internal heat generation is obtained as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}r^2\right)+\frac{1}{r^2\sin\psi}\frac{\partial}{\partial\psi}\left(\frac{\partial T}{\partial\psi}(\sin\psi)\right)+\frac{1}{r^2\sin^2\psi}\frac{\partial^2 T}{\partial\phi^2}+\frac{\ddot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (1.34)$$

1.4 Forms of Heat Conduction Equations:

Case (i): For a homogeneous and isotropic material with constant thermal conductivity, generalized heat conduction equation with internal heat generation in 3-D form is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\ddot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (1.35)$$

This equation is called as ***Diffusion equation.***

Case (ii): Generalized heat conduction equation without internal heat generation in 3-D form is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (1.36)$$

This is called ***Fourier's equation***

Case (iii): Steady heat conduction equation with internal heat generation in 3-D form is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\ddot{q}}{k}=0 \quad (1.37)$$

This equation is called ***Poisson's equation***.

Case (iv): Steady state heat conduction equation without internal heat generation is

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0 \quad (1.38)$$

This equation is called ***Laplace equation.***

Note: Generalized steady state heat conduction equation in one-dimension without internal heat generation in simplified form is

$$\frac{1}{r^n}\left(\frac{\partial}{\partial r}\left(r^n\frac{\partial T}{\partial r}\right)\right)=0 \quad (1.39)$$

where, n = 0 for Cartesian coordinate system

n = 1 for Cylindrical coordinate system

n = 2 for Spherical coordinate system

1.5 Objectives of Conduction Analysis

To estimate the variation of temperature with space coordinates and time. The temperature field is T(x,y,z,t). The temperature field depends on:

- Initial condition
- Boundary condition
- Material properties
- Geometry of the body

Temperature field is useful to:

- Compute heat flux at any location
- Compute thermal stresses, expansion, and deflection due to temperature etc.,
- Design insulation thickness
- Simulate heat treatment of metals

Conduction equation is second order in spatial coordinate and first order in time. Hence, to solve unsteady state problem,

Problem	No. of Boundary Conditions needed	No. of initial conditions
1-D	Two in x-direction	One initial condition
2-D	Two each in x and y directions	

3-D	Two each in x, y & z-directions	

1.6 Initial and Boundary Conditions

The initial conditions describe the temperature distribution in a medium at the initial moment of time. These are needed only for the time dependent (transient) problems. It can be expressed as:

$$at\ t=0,\ T=T(x,y,z) \quad (1.40)$$

Boundary conditions specify the temperature or the heat flow at the surface of the body. They can be specified in many ways:

1. **Boundary Conditions of First Kind: (Prescribed Surface Temperature)**
   It is also called as Dirichlet's condition.
   For each moment of time, the temperature distribution at the surface (represented in fig. 1.8) is specified as

   $$T_s = T(x,y,z,t)$$

   Example: For a slab, boundary conditions of first kind are specified as

   At $x=0,\ T(\mathrm{x},y,t)=0$

   At $y=0,\ T(\mathrm{x},y,t)=0$

   At $x=a,\ T(\mathrm{x},y,t)=f_1(y)$

   At $y=b,\ T(\mathrm{x},y,t)=f_2(x)$

   (1.41)

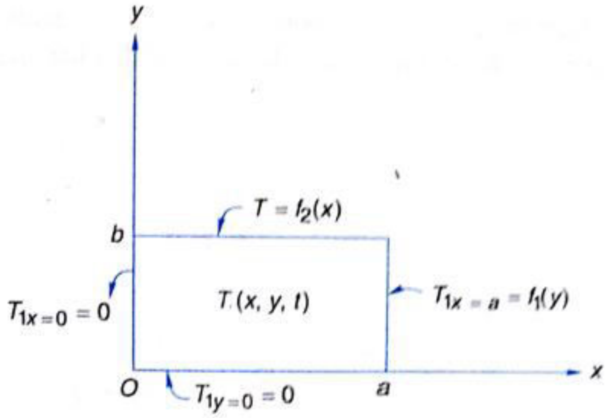


Fig. 1.8 Boundary condition of 1st Kind

   Example: Phase change process occurring at a surface whose temperature is constant.

2. **Boundary Conditions of Second Kind: (Prescribed Heat Flux)**
   It is also called as Neumann condition.
   Heat flux is specified at the boundary surface (as shown in fig. 1.9) and it is expressed as

   At $x=0,\ -k\dfrac{\partial T(x,t)}{\partial x}=q_0$

   At $x=0,\ \dfrac{\partial T(x,t)}{\partial x}=\dfrac{q_0}{k} \quad (1.42)$

   At the plane of symmetry, insulated or adiabatic boundary is described as $x=0,\ \dfrac{\partial T}{\partial x}=0$. (1.43)

   Example: Bonding a thin film electric heater to the surface.

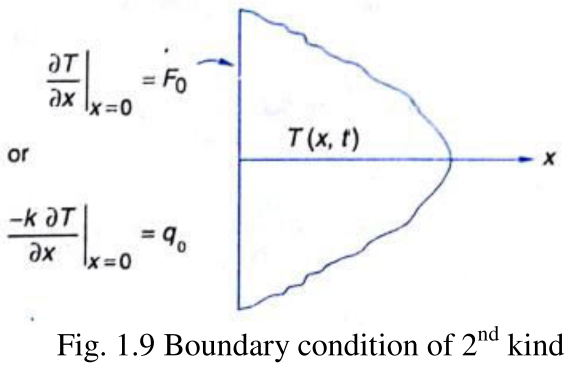


Fig. 1.9 Boundary condition of 2nd kind

3. **Boundary Conditions of Third Kind: (Convective Condition)**
   This condition is obtained when the boundary surface is subjected to a convective heat transfer into a medium at the ambient temperature as shown in fig. 1.10.One example is heat conducted into the surface by conduction is equal to the heat leaving the surface by convection.
   Applying the energy balance equation at the left and right surfaces of the wall:
   Heat entering the surface by conduction = Heat leaving the surface by convection

   At left face, i.e. at x=0, $\quad h_1\left(T_1-T|_{x=0}\right)=-k\left.\dfrac{\partial T}{\partial x}\right|_{x=0}$

At right face, i.e. at x=L, $-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h_2\left(T|_{x=L} - T_2\right)$ (1.44)

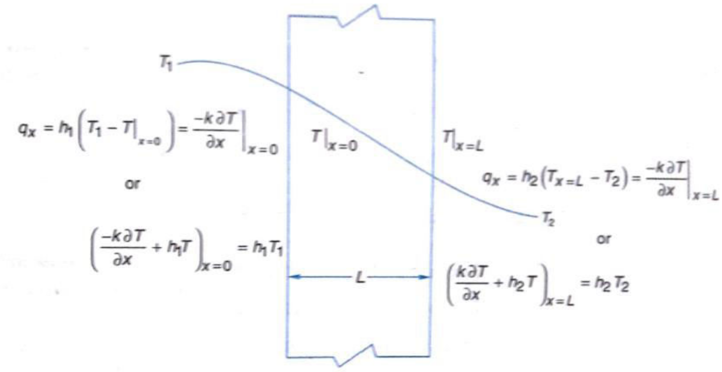


Fig. 1.10 Boundary condition of 3rd Kind

4. **Boundary Conditions of Fourth Kind:**
It describes the process of conduction between two bodies in perfect contact as shown in fig. 1.12. Since the surfaces in contact are at the same temperature, the heat fluxes through them are equal.

$$-k_1\left(\frac{\partial T_1}{\partial n}\right)_i = -k_2\left(\frac{\partial T_2}{\partial n}\right)_i \quad (1.45)$$

From this condition, it can be concluded that the ratio of slopes of tangents to the temperature curves at the points of contact of two bodies is a constant.

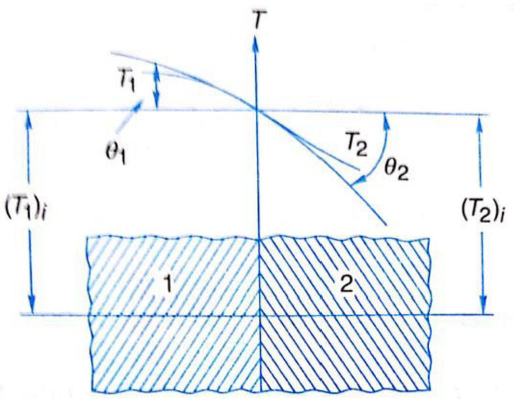


Fig. 1.11 Boundary condition of 4th Kind

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{k_2}{k_1} = constant \quad (1.46)$$

1.7 Heat Conduction through Plane Walls

Consider a plane wall of thickness 'L' through which heat 'Q' is conducted as shown in fig. 1.13.Let the heat is flowing from left face to right face which are at temperatures T_1 and T_2 respectively. It is a case of one-dimensional heat conduction problem. In this case, under steady state conditions without heat generation, the conduction equation becomes $\frac{\partial^2 T}{\partial x^2} = 0$.

(1.45)

On integrating this equation once, w.r.t 'x', we get $\frac{\partial T}{\partial x} = C_1$ (1.46)

On again integrating w.r.t 'x', we get final solution as $T = C_1 x + C_2$ (1.47)

Considering the boundary conditions:

At left face, x=0, $T=T_1 \Rightarrow T_1 = C_2$

and at right face, x=L, $T=T_2 \Rightarrow T_2 = C_1 L + T_1 \Rightarrow C_1 = \frac{T_2 - T_1}{L}$ (1.48)

Temperature distribution through the wall of thickness 'L' is obtained as

$$T = C_1 x + C_2 = \left(\frac{T_2 - T_1}{L}\right)x + T_1 = T_1 - \left(\frac{T_1 - T_2}{L}\right)x \quad (1.49)$$

This equation tells that the temperature decreases linearly from hot surface to cold surface.

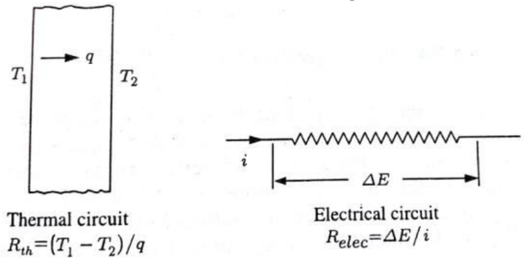
The temperature gradient along x-direction is $\frac{dT}{dx} = \frac{T_2 - T_1}{L}$ (1.50)

From the Fourier law, Heat conducted through the wall, $Q = -kA\frac{dT}{dx} = kA\left(\frac{T_1 - T_2}{L}\right)$ (1.51)

Electrical Analogy

Flow of heat through a conductor is analogous to flow of current through an electrical conductor since both are due to flow of electrons. Hence thermal resistance is defined similar to electrical resistance.

Corresponding to the current 'i' and the potential difference 'ΔE' in an electrical circuit, the analogous quantities in a thermal circuit are the heat flow rate 'q' and the temperature difference (T_1-T_2).



The thermal resistance can therefore be defined as $R_{th} = \left(\frac{T_1 - T_2}{q}\right)$ (1.52)

In a plane wall with surface temperatures T_1 and T_2 and thickness 'L', heat flow rate is written as:

$$Q = KA\left(\frac{T_1 - T_2}{L}\right) \Rightarrow R_{th} = \left(\frac{L}{kA}\right) = \frac{T_1 - T_2}{Q} \quad (1.53)$$

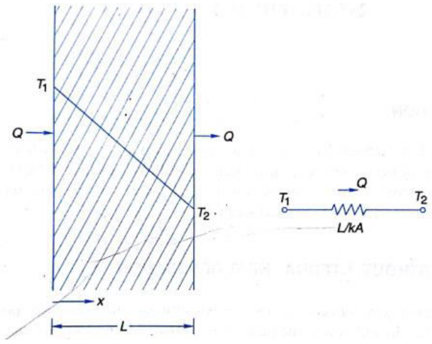


Fig. 1.12 Plane wall

1.8 Heat Conduction through Cylinder

Consider a hollow cylinder of inner radius 'r_i' and outer radius 'r_0' of length 'l' with uniform surface temperatures T_i and T_0 respectively, as shown in fig. 1.14. Assuming that the heat is flowing one-dimensionally along the radius, the steady state heat conduction equation without heat generation is given as

$$\frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right) = 0 \quad (1.54)$$

Integrating this equation w.r.t 'r' gives $r\frac{\partial T}{\partial r} = C_1 \Rightarrow \frac{\partial T}{\partial r} = \frac{C_1}{r}$ (1.55)

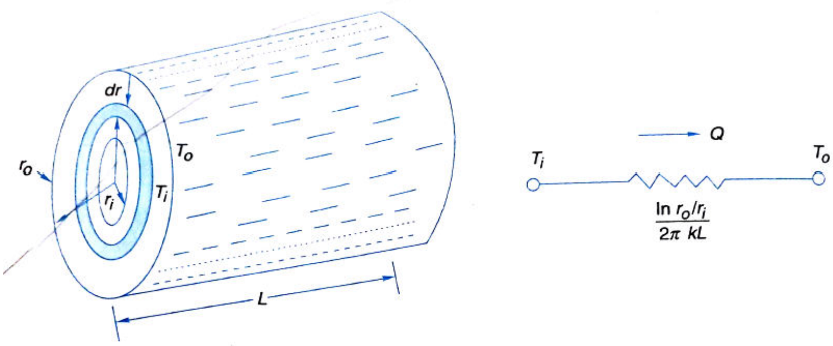


Fig. 1.13 Hollow cylinder

On again integrating the above equation, we get $T = C_1 \ln(r) + C_2$ (1.56)

Boundary conditions:

at inner surface, $r = r_i, T = T_i \Rightarrow Ti = C_1 \ln(r_i) + C_2$ (1.57)

at outer surface, $r = r_0, T = T_0 \Rightarrow T0 = C_1 \ln(r_0) + C_2$ (1.58)

On simplifying these equations, we get, $C_1 = \dfrac{T_i - T_0}{\ln\left(r_i / r_0 \right)}$ (1.59)

Also, $C_2 = T_i - \left(\dfrac{T_i - T_0}{\ln\left(r_i / r_0 \right)} \right) \ln(r_i)$ (1.60)

Temperature distribution in a hollow cylinder is obtained as $T = C_1 \ln(r) + C_2$

$$\Rightarrow T = \frac{T_i - T_0}{\ln\left(r_i / r_0 \right)} \ln(r) + T_i - \left(\frac{T_i - T_0}{\ln\left(r_i / r_0 \right)} \right) \ln(r_i)$$

$$\Rightarrow T = T_i + \left(\frac{T_i - T_0}{\ln\left(r_i / r_0 \right)} \right) \ln\left(r / r_i \right) \quad (1.61)$$

Heat transferred from inner to outer cylinder surface,

$$Q = -kA \left[\frac{\partial T}{\partial r} \right]_{r=ri} = -k(2\pi r_i l) \frac{1}{ri} \left(\frac{T_i - T_0}{\ln\left(r_i / r_0 \right)} \right) = (2\pi kl) \left(\frac{T_i - T_0}{\ln\left(r_0 / r_i \right)} \right) \quad (1.62)$$

Thermal resistance of a hollow cylinder is written as:

$$Q = 2\pi kL \frac{(T_i - T_0)}{\ln\left(\frac{r_0}{r_i} \right)} \Rightarrow R_{th} = \left(\frac{1}{2\pi kL} \ln\left(\frac{r_0}{r_i} \right) \right) = \frac{T_1 - T_2}{Q}$$

$$R_{th} = \frac{1}{2\pi kL} \ln\left(\frac{r_0}{r_i} \right) \quad (1.63)$$

1.9 Heat Conduction through Sphere

Consider a hollow sphere of inner radius ‘r_i’ and outer radius ‘r_0’ with uniform surface temperatures T_i and T_0 respectively, as shown in fig. 1.15. Assuming that the heat is flowing one-dimensionally along the radius, the steady state heat conduction equation without heat generation is given as

$$\frac{1}{r^2} \left(\frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \right) = 0 \quad (1.64)$$

Integrating this equation w.r.t ‘r’ gives $r^2 \dfrac{\partial T}{\partial r} = C_1 \Rightarrow \dfrac{\partial T}{\partial r} = \dfrac{C_1}{r^2}$ (1.65)

On again integrating the above equation, we get $T = C_1\left(\frac{-1}{r}\right) + C_2$ (1.66)

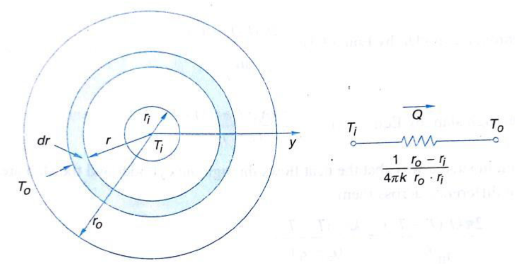


Fig. 1.14 Hollow sphere

Boundary conditions:

at inner surface, $r = r_i, T = T_i \Rightarrow Ti = C_1\left(\frac{-1}{r_i}\right) + C_2$

at outer surface, $r = r_0, T = T_0 \Rightarrow T_0 = C_1\left(\frac{-1}{r_0}\right) + C_2$ (1.67)

On simplifying these equations, we get, $C_1 = \frac{T_i - T_0}{\left(\frac{1}{r_0} - \frac{1}{r_i}\right)}$ (1.68)

Also, $C_2 = T_i - \left(\frac{T_i - T_0}{\frac{1}{r_0} - \frac{1}{r_i}}\right)\left(\frac{1}{r_i}\right)$ (1.69)

Temperature distribution in a hollow sphere is obtained as

$$T = \frac{T_i - T_0}{\left(\frac{1}{r_0} - \frac{1}{r_i}\right)}\left(\frac{-1}{r}\right) + T_i - \left(\frac{T_i - T_0}{\frac{1}{r_0} - \frac{1}{r_i}}\right)\left(\frac{1}{r_i}\right)$$

(1.70)

$$\Rightarrow T = T_i - \left(\frac{T_i - T_0}{\frac{1}{r_0} - \frac{1}{r_i}}\right)\left(\frac{1}{r} - \frac{1}{r_i}\right)$$

Rate of heat transferred through the hollow sphere,

$$Q = -kA\left[\frac{\partial T}{\partial r}\right]_{r=ri} = -k\left(4\pi r_i^2\right)\left|\frac{T_i - T_0}{\left(\frac{1}{r_0} - \frac{1}{r_i}\right)}\left(\frac{1}{r^2}\right)\right|_{r=r_i} = \frac{4\pi k r_i r_0}{r_0 - r_i}\left(T_i - T_0\right) \quad (1.71)$$

Thermal resistance of a hollow cylinder is written as:

$$Q = 4\pi k r_0 r_i \frac{(T_i - T_0)}{r_0 - r_i} \Rightarrow R_{th} = \left(\frac{r_0 - r_i}{4\pi k r_0 r_i}\right) = \frac{T_1 - T_2}{Q}$$

$$R_{th} = \left(\frac{r_0 - r_i}{4\pi k r_0 r_i} \right) \quad (1.72)$$

Note:

Thermal Resistance in Convection is written as $Q = hA\left(T_s - T_\infty\right) \Rightarrow Rth = \frac{1}{hA} = \frac{T_s - T_\infty}{Q}$ (1.73)

1.10 Composite Systems:

To reduce the heat loss to surroundings, insulation is provided for the surfaces of tubes, refrigerators, cold storage tanks, and hot water tanks, in furnaces etc., this makes two or more materials of different thermal conductivity arranged in series or parallel. These will come under the problem of composite systems, which can be solved by applying the concept of thermal resistance.

It is assumed that in all the parallel layers, the temperature is continuous i.e. all are in perfect thermal contact or the resistance due to interface contact is negligible.

(a) Composite Plane Wall:

Consider a multilayered plane wall as shown in the figure. It is assumed that the interior and exterior surfaces are subjected to convective heat transfer to fluids at mean temperatures T_a and T_b with heat transfer coefficients h_a and h_b respectively.

The diagram has the configuration of all the layers connected in series in which the rate of heat flow is constant in each layer.

$$Q = h_a A\left(T_a - T_1\right) = K_1 A\left(\frac{T_1 - T_2}{L_1}\right) = K_2 A\left(\frac{T_2 - T_3}{L_2}\right) = K_3 A\left(\frac{T_3 - T_4}{L_3}\right) = h_b A\left(T_4 - T_a\right)$$

(1.74)

Writing as

$$T_a - T_1 + T_1 - T_2 + T_2 - T_3 + T_3 - T_4 + T_4 - T_b = Q\left(\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A}\right)$$

$$T_a - T_b = Q\left(R_1 + R_2 + R_3 + R_4 + R_5\right) \Rightarrow Q = \frac{T_a - T_b}{\left(R_1 + R_2 + R_3 + R_4 + R_5\right)} = \frac{\left(\Delta T\right)_{overall}}{R_{total}} \quad (1.75)$$

(b) Composite Cylinder:

Consider a multilayered cylinder as shown in the figure. It is assumed that the interior and exterior surfaces are subjected to convective heat transfer to fluids at mean temperatures T_a and T_b with heat transfer coefficients h_a and h_b respectively.

The diagram has the configuration of all the layers connected in series in which the rate of heat flow is constant in each layer.

The rate of heat transfer is given as:

$$Q = \frac{T_a - T_b}{\left(R_1 + R_2 + R_3 + R_4 + R_5\right)} = \frac{\left(\Delta T\right)_{overall}}{R_{total}} \quad (1.76)$$

(c) Composite Sphere:

Consider a multilayered sphere as shown in the figure. It is assumed that the interior and exterior surfaces are subjected to convective heat transfer to fluids at mean temperatures T_a and T_b with heat transfer coefficients h_a and h_b respectively.

The diagram has the configuration of all the layers connected in series in which the rate of heat flow is constant in each layer.

The rate of heat transfer is given as:

$$Q=\frac{T_a-T_b}{\left(R_1+R_2+R_3+R_4+R_5\right)}=\frac{\left(\Delta T\right)_{overall}}{R_{total}} \tag{1.77}$$

1.11 Systems with variable thermal conductivity:

- The thermal conductivity of a material, in general, varies with temperature. However, this variation is mild for many materials in the range of practical interest and can be disregarded. In such cases we can use an average value for the thermal conductivity and treat it as constant.
- When the variation of thermal conductivity with temperature in a specified temperature interval is large, however, it may be necessary to account for this variation to minimize the error.
- Under normal circumstances, for limited ranges of temperature, it is sufficiently accurate to use the linear expansion for k, i.e. $k=k_0(1+\beta T)$; where k_0 is the value of thermal conductivity at T=0 K and β is the value of temperature coefficient of thermal conductivity.

Case (a): Plane Slab

Consider a plane wall in the region $0 \le x \le L$ having boundary surfaces at x=0 and x=L kept at uniform temperatures T_1 and T_2 respectively. The problem can be formulated as

$$\frac{d}{dx}\left[\left(k\left(T\right)\frac{dT}{dx}\right)\right]=0 \qquad \text{for } 0 \le x \le L \tag{1.78}$$

Boundary conditions are:

at x=0, T= T_1 and at x=L,T= T_2. (1.79)

$$\frac{d}{dx}\left[\left(k_0\left(1+\beta T\right)\frac{dT}{dx}\right)\right]=0 \tag{1.80}$$

The integration of above equation w.r.t x gives, $k_0\left[\left(1+\beta T\right)\frac{dT}{dx}\right]=C_1$ (1.81)

Integrating again we get, $k_0\left[T+\beta\frac{T^2}{2}\right]=C_1x+C_2$ (1.82)

Using the Boundary Condition at x=0; we get, $C_2=k_0\left[T_1+\beta\frac{T_1^2}{2}\right]$ (1.83)

Using the second boundary condition at x=L; $C_1L=k_0\left[T_2+\beta\frac{T_2^2}{2}\right]-k_0\left[T_1+\beta\frac{T_1^2}{2}\right]$ (1.84)

$$C_1=\left(\frac{k_0}{L}\right)\left[T_2-T_1+\beta\frac{T_2^2-T_1^2}{2}\right]=\left(\frac{k_0}{L}\right)\left(T_2-T_1\right)\left[1+\beta\frac{T_2+T_1}{2}\right]=\left(\frac{k_0}{L}\right)\left(T_2-T_1\right)\left[1+\beta T_m\right] \tag{1.85}$$

Where T_m is arithmetic mean of the boundary surface temperatures i.e., $(T_1+T_2)/2$

The substitution of C_1&C_2 in Eq. 1.82 gives

$$k_0\left[T+\beta\frac{T^2}{2}\right]=\left[k_0\left(T_2-T_1\right)(1+T_m)\right]+k_0\left[T_1+\beta\frac{T_1^2}{2}\right]$$

$$\frac{\beta}{2}T^2+T+\frac{x}{L}\left(T_1-T_2\right)(1+\beta T_m)-T_1\left(1+\frac{\beta}{2}T_1\right)=0$$

This is quadratic equation in T and its solution is given by

$$T=\frac{-1}{\beta}\pm\sqrt{\frac{1}{\beta^2}+\frac{2}{\beta}T_1\left(1+\frac{\beta}{2}T_1\right)-\frac{x}{L}\left(T_1-T_2\right)\left(1+\beta Tm\right)} \quad (1.86)$$

The heat flow rate Q is given by $Q=-KA\left(\frac{dT}{dx}\right)=K_mA\left(\frac{T_1-T_2}{L}\right)$ (1.87)

where $k_m = k_0\left(1+\beta T_m\right)= k_0\left(1+\beta\left(T_1+T_2\right)/2\right)$ (1.88)

Case (b): Hollow cylinder

Consider a hollow cylinder with boundary surfaces at r_i and r_0 kept at uniform temperatures T_i and T_o respectively. The thermal conductivity of material of cylinder is temperature dependant, and is given by

$\frac{d}{dr}[rk(T)\frac{dT}{dr}]=0 \rightarrow \frac{d}{dr}[r.k_0(1+\beta T)\frac{dT}{dr}]=0$ (1.89)

B.C are $T=T_i$ at $r=r_i$ and $T=T_o$ at $r=r_o$ (1.90)

Integrating the above equation we get, $k_0(1+\beta T)\frac{dT}{dr}=\frac{C_1}{r}$ (1.91)

Again Integrating on both sides we get, $k_0[T+\frac{\beta}{2}T^2]=C_1\ln(r)+C_2$ (1.92)

Using Boundary conditions at $r=r_i$ and $r=r_o$ we get

$k_0[T_i+\frac{\beta}{2}T_i^2]=C_1\ln(r_i)+C_2$ (1.93)

$k_0[T_o+\frac{\beta}{2}T_o^2]=C_1\ln(r_o)+C_2$ (1.94)

Solving above two equations for C_1&C_2

$C_1\ln\left(\frac{r_i}{r_o}\right)= k_o[(T_i-T_o)+\frac{\beta}{2}(T_i^2-T_o^2)] = k_o(T_i-T_o)\{1+\frac{\beta}{2}(T_i+T_o)\} = k_o(T_i-T_o)(1+\beta T_m)$ (1.95)

Where $T_m=(T_1+T_2)/2$

$\therefore C_1=\frac{ko(Ti-To)(1+\beta Tm)}{\ln\left(\frac{r_i}{r_o}\right)}$ (1.96)

And substituting the value of C_1 in eq-2 we get $C_2=k_0[T_i+\frac{\beta}{2}T_i^2]-\frac{ko(Ti-To)(1+\beta Tm)}{\ln\left(\frac{r_i}{r_o}\right)}$ (1.97)

Substituting the values of $C_1 \& C_2$ in eq-1 we get

$$k_0[T+\frac{\beta}{2}T^2] = \frac{ko(Ti-To)(1+\beta Tm)}{\ln(\frac{r_i}{r_o})} \ln(r) + k_0[T_i+\frac{\beta}{2}T_i^2] - \frac{ko(Ti-To)(1+\beta Tm)}{\ln(\frac{r_i}{r_o})} \quad (1.98)$$

The above equation can be simplified to

$$\frac{\beta}{2}T^2+T-\frac{(Ti+To)(1+\beta Tm)}{\ln(\frac{r_i}{r_o})}\ln\left(\frac{r}{r_i}\right)-T_i(1+\frac{\beta}{2}T_i) = 0s$$

$$\therefore T=\frac{-1\pm[1+4\frac{\beta}{2}\left\{(Ti-To)(1+\beta Tm)\left(\frac{\ln(\frac{r}{r_i})}{\ln(\frac{r_i}{r_o})}\right)+Ti\left(1+\frac{\beta}{2}Ti\right)\right\}]^{1/2}}{\beta} \quad (1.99)$$

The heat flow rate is given by $Q=-k(T)A\frac{dT}{dr}$

$$Q=\frac{-2\pi k_m L(Ti-To)}{\ln(\frac{r_o}{r_i})} \quad (1.100)$$

Where $k_m=k_0(1+\beta T_m)=k_0(1+\beta(T_1+T_2)/2)$

Case (c): Hollow sphere

Consider a hollow sphere with boundary surfaces at $r=r_i$ and r_o kept at uniform temperatures T_i and T_o respectively.

For 1-D flow of heat in radial direction,

$$Q=-k(T)A\frac{dT}{dr}$$

$$=-k_0(1+\beta T)4\pi r^2\frac{dT}{dr} \quad (1.101)$$

Separating the variables and integrating we get

$$\int\frac{dr}{r^2}=\frac{-4\pi k0}{Q}\int(1+\beta T)dT$$

$$\frac{-1}{r}=\frac{-4\pi k0}{Q}[T+\frac{\beta}{2}T^2]+C_1 \quad (1.102)$$

Constant $C_1$is evaluated by using the boundary conditions $T=T_i$ at $r=r_i$

$$C_1=\frac{-1}{r_1}+\frac{4\pi k0}{Q}[T_i+\frac{\beta}{2}T_i^2] \quad (1.103)$$

Substitution of C_1in the equation gives

$$\frac{4\pi k0}{Q}[T_i+\frac{\beta}{2}T_i^2]-\frac{1}{r_1}-\frac{4\pi k0}{Q}[T+\frac{\beta}{2}T^2]+\frac{1}{r}=0 \quad (1.104)$$

$$\frac{\beta}{2}T^2+T-[T_i+\frac{\beta}{2}T_i^2]+\frac{Q}{4\pi k0}\left(\frac{1}{r_i}-\frac{1}{r}\right)=0 \quad (1.105)$$

$$\therefore T=\frac{\frac{-\beta}{2}\pm[1+\frac{4\beta}{2}\{\left(Ti+\frac{\beta}{2}Ti2\right)-\frac{Q}{4\pi k0}\left(\frac{1}{r_i}-\frac{1}{r}\right)\}]^{1/2}}{\beta} \quad (1.106)$$

$$T=-\frac{1}{2}\pm\frac{1}{\beta}[(1+\beta T_i)^2-\frac{Q}{4\pi k0}\left(\frac{1}{r_i}-\frac{1}{r}\right)]^{1/2} \quad (1.107)$$

1.13Plane wall with internal heat generation:

Let us consider a slab of thickness L, in the region $0\leq x\leq L$, and the uniform thermal conductivity k as shown. Let uniform heat generation of q is generated. The temperature on the two faces on the slab be T_w. Consider an element of thickness dx and cross sectional area A.

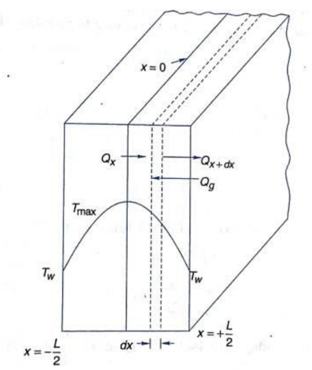


Fig. 1.15 Plane wall with internal heat generation

General heat conduction heat conduction equation for 1D and with internal heat generation is given by

$$\frac{d^2T}{dx^2}+\frac{q}{k}=0 \quad (1.108)$$

Integrating on both sides it gives

$$\frac{dT}{dx}=\frac{-q}{k}x+C_1 \quad (1.109)$$

Again integrating on both sides

$$T=\frac{-q}{k}.\frac{1}{2}.x^2+C_1x+C_2 \quad (1.110)$$

The boundary conditions of the problem is

$T=T_w$ at x=+L/2 and x=-L/2 ------B.C1

& at x=0, $\frac{dT}{dx}$=0. - - - - - - - - - -B.C2 (1.111)

Using the above Boundary conditions, we get C_1=0

$C_2=T_w+\frac{q.L^2}{8.k}$ (1.112)

Substituting the values of C_1&C_2 in eq-1 we get

$T=T_w+\frac{q}{8.k}(L^2 - 4x^2)$ (1.113)

The heat flow rate is given by $Q=-kA\frac{dT}{dx}\Big|_{x=L/2} = -kA(\frac{-q}{8.k}.\frac{8.L}{2}) = \frac{1}{2}q.A.L$ (1.114)

1.14 Hollow cylinder with internal heat generation:

Let us now consider a long hollow cylinder of length L and inside and outside radius r_i& r_o. A constant rate of heat q is generated within the cylinder while the boundary surfaces at $r=r_o$ and $r=r_i$ are kept at uniform temperatures T_i and T_o respectively.

General heat conduction heat conduction equation for 1D and with internal heat generation is given by

$\frac{d}{dr}\left(r.\frac{dT}{dr}\right) + \frac{q.r}{k} = 0$ (1.115)

The boundary conditions are $T=T_i$ at $r=r_i$ and $T=T_o$ at $r=r_o$ (1.116)

Integrating the above equation we get

$\frac{dT}{dr} + \frac{q.r}{2.k} = \frac{C_1}{r}$ (1.117)

$T=\frac{-q.r^2}{4.k} + C_1 lnr + C_2$ (1.118)

Applying the Boundary conditions, we get,

$T_i=\frac{-q.r_i^{\,2}}{4.k} + C_1 lnr_i + C_2$ (1.119)

$T_o=\frac{-q.r_o^{\,2}}{4.k} + C_1 lnr_o + C_2$ (1.120)

Solving the above equations we get,

$C_1=\frac{(T_o-T_i)+\frac{q}{4.k}(r_o^2-r_i^2)}{\ln(\frac{r_o}{r_i})}$ (1.121)

$C_2=T_i+\frac{q.r_i^2}{4.k} - \frac{(T_o-T_i)+\frac{q}{4.k}(r_o^2-r_i^2)}{\ln(\frac{r_o}{r_i})} lnr_i$ (1.122)

Substituting the values of C_1&C_2 in eq-1 we get,

$T=\frac{-q.r^2}{4.k} + \frac{(T_o-T_i)+\frac{q}{4.k}(r_o^2-r_i^2)}{\ln(\frac{r_o}{r_i})} lnr + T_i + \frac{q.r_i^2}{4.k} - \frac{(T_o-T_i)+\frac{q}{4.k}(r_o^2-r_i^2)}{\ln(\frac{r_o}{r_i})} lnr_i$ (1.123)

$$T-T_i=\frac{q}{4.k}(r_i^2-r^2)+\frac{\ln\left(\frac{r}{r_i}\right)}{\ln\left(\frac{r_o}{r_i}\right)}\left[(T_o-T_i)+\frac{q}{4.k}(r_o^2-r_i^2)\right] \quad (1.124)$$

$$\frac{T-Ti}{T_o-T_i}=\frac{q}{4.k}\left[\frac{(r_i^2-r^2)}{T_o-T_i}+\frac{(r_o^2-r_i^2)}{(T_o-T_i)}\frac{\ln\left(\frac{r}{r_i}\right)}{\ln\left(\frac{r_o}{r_i}\right)}\right]+\frac{\ln\left(\frac{r}{r_i}\right)}{\ln\left(\frac{r_o}{r_i}\right)} \quad (1.125)$$

1.15 Sphere with internal heat generation:

Let us now consider a solid sphere with a uniform heat source q. The outside surface at $r=r_o$ is maintained at constant temperature T_o.

General heat conduction heat conduction equation for 1D and with internal heat generation is given by

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{r^2.q}{k}=0 \quad (1.126)$$

Boundary conditions are

$\frac{dT}{dr}=0$ at $r=0$ and $T=T_o$ at $r=r_o$ (1.127)

Integrating the eq-1 we get

$$r^2\frac{dT}{dr}=\frac{-3.q.r^3}{3.k}+C_1 \quad (1.128)$$

$$\frac{dT}{dr}=\frac{-q.r^2}{3.k}+\frac{C_1}{r^2} \quad (1.129)$$

Again integrating we get

$$T=\frac{-q.r^2}{6.k}-\frac{C_1}{r}+C_2 \quad (1.130)$$

Applying B.C-1 in eq-2 we get $C_1=0$

Applying B.C-2 in eq-3 we get $T_o=\frac{-q.{r_o}^2}{6.k}+C_2$

$$C_2=To+\frac{q.{r_o}^2}{6.k} \quad (1.131)$$

Substituting the values of C_1&C_2 in eq-3 we get

$$T=T_o+\frac{q}{6.k}(r_o^2-r^2) \quad (1.132)$$

Assignment-Cum-Tutorial Questions

A. Questions at remembering / understanding level:

I) Objective Questions

1. Which of the following statement(s) is/are correct? [CO1] [BL2]
   a. Conduction heat transfer takes place from one particle of the body to another without the actual motion of the particles.
   b. Conduction heat transfer takes place from one particle of the body to another by actual motion of the heated particles.
   c. Convective heat transfer takes place between a surface and the surrounding fluid.
   d. Convective heat transfer takes place within the fluid medium.

   A) a,c & d B) a & d C) a & c D) b,c & d

2. Which of the following statement(s) is/are correct? [CO1] [BL1]
   a. The rate of heat transfer by conduction is proportional to the temperatue gradient in the direction of heat flow and area normal to the heat flow direction.
   b. The rate of heat transfer by conduction is proportional to the temperatue gradient in the direction of heat flow and inversely proportional to the area normal to the heat flow direction.

c. The thermal conductivtity of matiral can be defined as the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.
d. Conduction heat transfer can takes place in solids, liquids and gases.
A) a & c B) a,c & d C) a & d D) b,c & d

3. Which of the following statement(s) is/are correct? [CO1] [BL2]
a. Thermal conductivity of liquids increases with decrase in temperature (water as exception)
b. Themal conductivity of metallic solids decreases with increase in temperture.
c. Themal conductivity of gases increases with increase in temperature.
d. Themal conductivity non metallic solids increases with increas in temperature.
A) a & c B) a,b & c C) a,c & d D) a,b,c & d

4. Which of the following statement(s) is/are correct? [CO1] [BL2]
a. The heat transfer between a solid surface and an adjustant fluid takes place by convection.
b. The rate of heat transfer by convection is proportional the surface area and temperature difference between the surface and the fluid.
c. Radiation is the energy emitted by matter in the form of eletromagnetic waves (or photons) and dose not required the presence of intervening medium.
d. Stefan-Boltzmann law of radiation states that the the radiation energy emitted by a black surface per unit area is proportional to the fourth power of absolute temperatue of the surface.
A) a & c B) a,b & c C) a,c & d D) a,b,c & d

5. Which of the following statement(s) is/are correct? [CO1] [BL2]
a. Themal diffusivity is a property of a material and its unit is m^2/s.
b. A material that has a high themal conductivity or a low heat capacity will have a large themal diffusivity.
c. The large the thermal diffusivity, the faster the propagation of heat through the medium.
d. A small value of themal diffusivity means that heat is mostly absorbed by the material and a small amount of heat is conducted further.
A) a & c B) a,b & c C) a,c & d D) a,b,c & d

6. Match the following [CO1][BL1]

a. Convection heat transfer	i. Heat conducted to heat stored per unit volume
b. Themal diffusivity	ii. Heat transfer between a solid surface and surrounding fluid
c. Themal conductance	iii. Electromagnetic phenomenon
d. Radiation heat transfer	iv. Reciprocal conduction resistance

A) a-ii, b-i,c-iv,d-iii B) a-iv, b-iii,c-i,d-ii C) a-i, b-iv,c-ii,d-iii D) a-ii, b-i,c-iii,d-iv

7. Match the following [CO1][BL2]

a. Laplas equation	i. $\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_{gen}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$
b. Diffusion equation	ii. $\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_{gen}}{k}=0$
c. Fourier-Biot equation	iii. $\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

d. Poission equation iv. $\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_{gen}}{k}=0$

A) a-ii, b-i,c-iv,d-iii B) a-iv, b-iii,c-i,d-ii C) a-i, b-iv,c-ii,d-iii D) a-ii, b-i,c-iii,d-iv

8. Match the following [CO1][BL2]
   a. Specified temperature boundary conditions i. $-k\frac{\partial T(0,t)}{\partial x}=h_1[T_{\infty 1}-T(0,t)]$
   b. Insulated Boundary conditions ii. $k\frac{\partial T(0,t)}{\partial x}=0,\frac{\partial T(L,t)}{\partial x}=0$
   c. Themal symmetry boundary conditions iii. $-k\frac{dT(0)}{dx}=qo$
   d. Heat flux boundary conditions iv. $\frac{\partial T(L/2,t)}{\partial x}=0$
   e. Convection boundary conditions v. $T(0,t)=T_1$, $T(L,t)=T_2$

   A) a-ii, b-iii,c-v,d-iv,e-i B) a-iv, b-i,c-ii,d-iii,e-ii
   C) a-v, b-ii,c-iv,d-iii,e-i D) a-iii, b-iv,c-ii,d-iv,e-i
9. Which of the following statement(s) is/are correct? [CO1] [BL2]
   a. In the case of steam carrying pipes, the outer radius of pipe must be greater than the critical radius of insulation.
   b. In the case of steam carrying pipes, the outer radius of pipe must be less than the critical radius of insulation.
   c. In the case of electric conductors, the wire radius must be less than the critical radius of insulation.
   d. In the case of electric conductors, the wire radius must be greater than the critical radius of insulation.

   A) a & d B) b & d C) a & c D) b & c
10. As the thickness of insulation on a cylindrical tube increases [CO1] [BL2]
    a. The conduction resistance increases while the convection resistance decreases.
    b. The conduction resistance decreases while the convection resistance increases.
    c. Both conduction and convection resistances decrease.
    d. Both conduction and convection resistances increase.

II) Descriptive Questions

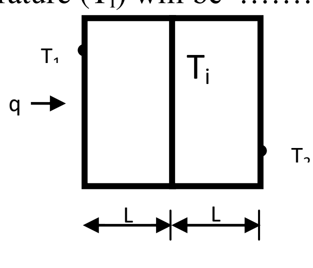
1. Describe the mechanisms of conduction, convection and radiation and state the laws governing them. [CO1] [BL2]
2. Explain the mechanisms of heat conduction in gases, liquids and solids. [CO1][BL2]
3. Explain initial and boundary conditions applied to heat conduction. [CO1] [BL2]
4. Explain the concept of critical radius of insulation applied to steam carrying pipes and electric conductors. [CO1] [BL2]

B. Questions at applying/analyzing level:

I) Multiple choice questions.

1. A composite wall is made up of two different materials arranged in series of the same thickness and having thermal conductivities of k1 & k2 respectively. The equivalent thermal conductivity (k) is [CO1] [BL3]

   a) $k=k_1+k_2$ b) $k=\frac{2k_1k_2}{k_1+k_2}$ c) $k=k_1-k_2$ d) $k=\frac{k_1k_2}{k_1+k_2}$
2. A composite plane wall shown in Figur 1 has the themal conductivities 0.3k and 3k. The interface temperature (T_i) will be ……… [CO1] [BL3]



a) $T_i = \frac{T1+T2}{2}$ b) $T_i > \frac{T1+T2}{2}$ c) $T_i < \frac{T1+T2}{2}$ d) $T_i = \frac{T1+T2}{2} + T_1$

3. Furnace is made of fire brick having thickness x=0.6 m & themal conductivity k=0.8 W/m-K. For the same heat loss (W/m^2) and temparature drop, another meterial having k=0.16 W/m-K will have its thickness .... [CO1] [BL3]
   a) 0.06 m (b) 0.12 m (c) 0.25 m (d) 0.48 m
4. Heat is lost through a brick wall (k =0.72 W/m·K), which is 4 m long, 3 m wide, and 25 m thick at a rate of 500 W. If the inner surface of the wall is at 22°C, the temperature at the mid-plane of the wall is [CO1] [BL3]
   (a) 0°C (b) 7.5°C (c) 11.0°C (d) 14.8°C (e) 22°C
5. Common data questions:
   A 0.8 m high and 1.5 m wide double-pane window consisting of two 4 m thick layers of glass (k = 0.78 W/m-^{0}c) separated by a 10 mm wide stagnant air space (k=0.026 W/m-^{0}c). The room is maintained at 20^{0}C, while the temperature of the outdoors is -10^{0}C. The convective heat transfer coefficients on the inner and outer surfaces of the window to be h_i = 10 W/m^2 – ^{0}c and h_0 = 40 W/m^2 – ^{0}C. [CO1] [BL3]
   i) The steady rate of heat transfer through the double – pare window is
   a) 50.2 w b) 40.2 w c) 89.2 w d) 69.2 w
   ii) The inner surface temperature of double – pane window is
   a) 14.2 ^{0}C b) 16.2 ^{0}C c) 12.2 ^{0}C d) 10.2 ^{0}C
6. A 40-cm-long, 0.4-cm-diameter electric resistance wire submerged in water is used to determine the convection heat transfer coefficient in water during boiling at 1 atm pressure. The surface temperature of the wire is measured to be 114°C when a wattmeter indicates the electric power consumption to be 7.6 kW. The heat transfer Coefficient is [CO1] [BL3]
   (a) 108 kW/m2 (b) 13.3 kW/m2·K (c) 68.1 kW/m2·K
   (d) 0.76 kW/m2·K (e) 256 kW/m2·K.
7. It is intended to reduce the heat loss from a steam pipe of diameter 5cm by applying an insulation(k=0.72 W/m - K) over the outer surface of the pipe. The outer covective heat transfer coefficient is 12 W/m^2 - K. Which of the following statements is corerct? [CO1] [BL4]
   a. The rate of heat transfer increases up to critical radius and then decreases.
   b. The rate of heat transfer increases with increase in insulation thickness.
   c. The rate of heat transfer decreases with increase in insulation thickness.
   d. The rate of heat transfer decreases up to critical radius and then increases.
   A) b B) a C) d D) c
8. A circular plate of 0.2 m diameter has one of its surfaces is insulated; the other is maintained at 550 K. If the hot surface has an emissivity of 0.9 and is exposed to the air at 300 k. Calculate the heat loss by radiation from the Plate to the air. [CO1] [BL3]
   a) 133.76 W b) 163.76 W c) 113.76 W d) 93.76 W
9. Determine the maximum current that a 1mm diameter bare aluminum (k=204W/m-K) wire can carry without exceeding a temperature of 200^{0}C. The wire is suspended in air at temperature 25^{0}C and h = 10 W/m^2k. The electrical resistance of this wire per unit length is 0.037Ω/m. [CO1] [BL3]
   a) 15.2 A b) 12.2 A c) 18.2 A d) 16.2 A

10. Calculate the heat loss per unit length from the surface of a pipe of 75 mm diameter laid in a room at 20^0 C. The emissivity of the pipe material is 0.85 and its surface temperature is 220^0 C. The convective heat transfer coefficient may be assumed as 16 W/m^2-K. [CO1] [BL3]
    a) 750.8 W b) 600.8 W c) 780.9 W d) 500.8 W
11. Consider a steam pipe of length L = 9 m, inner radius r_1 = 5 cm, outer radius r_2 = 6 cm, and thermal conductivity k = 12.5 W/m . ^oC. Steam is flowing through the pipe at an average temperature of 120^oC, and the average convection heat transfer coefficient on the inner surface is given to be h = 70 W/m^2 . ^oC. If the average temperature on the outer surfaces of the pipe is $T_2 = 70^oC$. The rate of heat loss from the steam through the pipe is [CO1] [BL3]
    a) 9415 W b) 941.5 W c) 94.15 W d) 94150 W
12. A steam pipe of outer diameter 5cm is covered with a layer of insulation (k=0.05 w/m-^{0}c) and is exposed to outer ambient at a temperature 20^oc with heat transfer coefficient 5 w/m^2-^{o}c. Which of the following statement is correct? [CO1] [BL4]
    a) Insulation increases the rate of heat transfer
    b) Insulation reduces the rate of heat transfer
    c) Insulation increases the rate of heat transfer up to certain radius and then decreases
    d) Insulation does not alter the rate of heat transfer
13. In a nuclear reactor, heat is generated in 1 cm diameter cylindrical uranium fuel rods at a rate of 4 X 10^7 W/m^3. The temperature difference between the center and the surface of the fuel rod is [CO1] [BL3]
    a) 90^oC b) 45^oC c) 22.5^oC d) 9.0^oC
14. A 2-kW resistance heater wire whose thermal conductivity is k=15 W/m.K has a diameter of D=4 mm and a length of L=0.5 m, and is used to boil water. If the outer surface temperature of the resistance wire is $T_s=105^0C$, determine the temperature at the center of the wire. [CO1] [BL3]
    a) 115^oC b) 126^oC c) 120.5^oC d) 131.5^oC
15. A tube having outside diameter 2cm is maintained at a uniform temperature T and is covered with an insulation (k=0.2 w/m-k). Heat is dissipated from outer surface of insulation by natural convection with h=15 w/m^2-^{o}c in to the ambient air at T_α. What is the percentage change in heat loss when the thickness of insulation is equal to critical thickness? [CO1] [BL4]
    a) The heat loss is decreased by 3.5% b) The heat loss is decreased by 35%
    c) The heat loss is increased by 3.5% d) The heat loss is increased by 35%
16. An electric wire of 1 mm diameter as covered with a 2 mm thick layer of plastic insulation (k=0.5 w/m-k). Air surrounding the wire is at 25^oC with h=10 w/m^2-k. The wire temperature is 100^oC. [CO1] [BL4]
    i) What is the radius of insulation when the rate of heat dissipation is maximum?
    a) 75 mm b) 100 mm c) 50 mm d) 25 mm
    ii) What is the maximum value of this heat dissipation per meter length?
    a) 12 w/m b) 48 w/m c) 24 w/m d) 96 w/m

II) Derivations / Problems.

1. Derive the general heat conduction equation in Cartesian coordinates and deduce it in one dimensional steady state condition with no internal heat generation. [CO1] [BL3]
2. Derive, starting from fundamentals, general heat conduction equation in cylindrical coordinates. [CO1] [BL3]

3. Derive, starting from fundamentals, general heat conduction equation in spherical coordinates. [CO1] [BL3]
4. Consider the case of long hallow cylinder (of constant thermal conductivity) of internal and external radius r_i and r_o respectively. If the temperatures at the inside and outside surfaces are maintained at T_i and T_o, write the mathematical formulation of this problem for steady state heat conduction. [CO1] [BL3]
5. A composite wall is made up three layers of thickness 25 cm, 10 cm and 15 cm of material A, B and C respectively. The thermal conductivities of A and B are 1.7 W/m K and 9.5 W/m K respectively. The outside surface is exposed to air at 20°C with a convection coefficient of 15 W/m^2 K and the inside is exposed to gases at 1200°C with a convection coefficient of 25 W/m^2 K and the inside surface is at 1080°C. Determine the unknown thermal conductivity of layer made up of material C. [CO1] [BL3]
6. A steel pipe is carrying steam at a pressure of 30 bar. Its outside diameter is 90 mm and is lagged with a layer of material 45 mm thick (k=0.05 W/m-^{0}C). The ambient temperature is 20°C and the surface of the lagging has a heat transfer coefficient of 8.4 W/m^2-°C. Neglecting resistance due to pipe material and due to steam film on the inside of steam pipe, find the thickness of the lagging (k=0.07 W/m-^{0}C) which must be added to reduce the steam condensation rate by 50 percent if the surface coefficient remains unchanged. [CO1] [BL4]
7. Consider a 2 m high and 0.7m wide bronze plate whose thickness is 0.12 m one side of the plate is maintained at a constant temperature of 600K while the other side is maintained at 400K. The thermal conductivity of the bronze plate can be assumed to vary linearly in that temperature range as $k(T)=K_0(1+\beta T)$, where k_0=38 W/m-K and $\beta=9.21 \times 10^{-4} K^{-1}$. Disregarding the edge effects and assuming steady one-dimensional heat transfer, determine the rate of heat conduction through the plate. [CO1] [BL3]
8. Hot gas at a constant temperature of 400°C is contained in a spherical shell (2000 mm internal diameter, 50 mm thick) made of steel. Mineral wool insulation (k=0.06 W/m K) of thickness 100 mm is wrapped all around it. Calculate the steady rate at which heat will flow out if the outside air is at a temperature of 30°C. Heat transfer coefficient on the inner surface of the steel shell and on the outer surface of the insulation is 15 W/m^{2}K. [CO1] [BL3]
9. A wall of furnace is made up of inside layer of silica brick 120 mm thick covered with a layer of magnetite brick 240 mm thick. The temperatures at the inside surface and of silica brick wall and outside surface of magnetite brick wall are 725°C and 110°C respectively. The contact thermal resistance between the two walls at the interface is 0.0035°C/W per unit wall area. If the thermal conductivities of silica and magnetite bricks are 1.7 W/m °C and 5.8 W/m °C, calculate the rate of heat loss per unit area of walls and temperature drop at the interface. [CO1] [BL3]
10. A Steel tube (k=43.26 W/m K) of 5.05 cm I.D. and 7.62 cm O.D. is covered with a 2.54 cm layer of asbestos insulation (k=0.208 W/m K). The inside surface of the tube receives heat by convection from a hot gas at a temperature of T_a = 316°C with a heat transfer coefficient h_a = 284 W/m^{2}K, while the outer surface of the insulation is exposed to the ambient air at T_b = 38°C with a heat transfer coefficient of h_b = 17 W/m^{2}K. Estimate i) the loss to ambient air for 32 m length of the tube, and ii) the temperature drops across the tube material and insulation layer. [CO1] [BL3]
11. A steam pipe, 10 cm I.D and 11 cm O.D is covered with an insulating substance (K=1 W/m-K). The steam temperature and the ambient temperatures are 200^{0}C and 20^{0}C,

respectively. If the convective heat transfer coefficient between the insulation surface and air is 8 W/m^2-K, find the critical radius of insulation. For this value of r0, calculate the heat loss per meter of pipe. Neglect resistance of the pipe material. [CO1] [BL4]

12. Consider the one-dimensional steady state heat conduction through a copper slab of length 2L submerged in a constant temperature bath at temperature, T_α. An electric current is passed through this slab causing a uniform heat generation, *q* per unit time and volume. The boundary surfaces dissipate heat by convection resulting in a temperature, T_w on both sides. Formulate the problem mathematically. [CO1] [BL3]
13. Derive an expression for steady state temperature distribution in a plane wall of uniform thickness with internal heat generation when its two surfaces are at temperatures $T_1$and T_2 respectively. [CO1] [BL3]
14. Consider a large 3 cm thick stainless steel plate whose thermal conductivity is 15.1 W/m.^{0}C, in which heat is generated uniformly at a rate of 5 x 10^5W/m^3. Both sides of the plate are exposed to an environment at 30^0C with a heat transfer coefficient of 60 W/m^{20}C. Explain where in the plate the highest and the lowest temperatures will occur and determine their values. [CO1] [BL4]
15. Derive the expression for critical radius of insulation (i) in the case of cylinder (ii) in the case of sphere. [CO1] [BL3]
16. Evaluate the thickness of rubber insulation necessary in the case of a 10 mm diameter copper conductor to ensure maximum heat transfer to the atmosphere, given the thermal conductivity of rubber as 0.155 W/m-K and the surface film coefficient as 8.5 W/m^2K. Estimate this maximum heat transfer rate per meter length of the conductor if the temperature of the rubber is not to exceed 65^{o}C (due to heat generated within) while the atmosphere is at 30^{o}C. Discuss the effect of insulation on the bare conductor. [CO1] [BL4]
17. A tube of 2 cm O.D maintained at uniform temperature of Ti is covered with insulation (k=0.2W/m-K) to reduce heat loss to the ambient air at T_α with ha=15 W/m^2K. find (i)the critical thickness of insulation rc (ii)the ratio of heat loss from the tube with insulation to that without insulation (a) if the thickness of insulation equal to rc (b) if the thickness of insulation is (rc+2) cm. [CO1] [BL4]
18. A 1 mm dia electric wire is covered with 2 mm thick layer of insulation (k=0.5W/m-K). Air surrounding the wire is at 25°c and h=25 W/m^2K. The wire temperature is 100°c. (i) the rate of heat dissipation from the wire per unit length with and without insulation. (ii) The critical radius of insulation. (iii) The maximum value of heat dissipation. [CO1] [BL3]
19. Consider a steam pipe of length *L* = 20 m, inner radius *r*1 = 6 cm, outer radius *r*2 =8 cm, and thermal conductivity *k* = 20 W/m· K. The inner and outer surfaces of the pipe are maintained at average temperatures of *T*1 =150°C and *T*2 = 60°C, respectively. Obtain a general relation for the temperature distribution inside the pipe under steady conditions and determine the rate of heat loss from the steam through the pipe. [CO1] [BL4]
20. Consider a spherical container of inner radius *r*1 = 8 cm, outer radius *r*2 = 10 cm, and thermal conductivity *k* = 45 W/m· K,. The inner and outer surfaces of the container are maintained at constant temperatures of *T*1 = 200°C and *T*2 = 80°C, respectively, as a result of some chemical reactions occurring inside. Obtain a general relation for the temperature distribution inside the shell under steady conditions, and determine the rate of heat loss from the container. [CO1] [BL4]

C. Questions at evaluating/creating level:

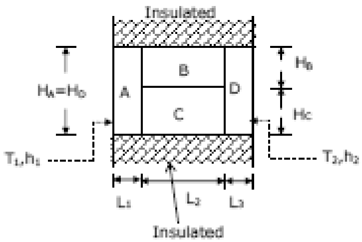
1. A steel pipe line (k= 50 W/m-K) of I.D 100 mm and O.D 110 mm is to be covered with two layers of insulation each having a thickness of 50 mm. The thermal

conductivity of the first insulation material is 0.06 W/m-K and the second is 0.12 W/m-K. Calculate the loss of heat per meter length of pipe and interface temperature between the two layers of insulation when the temperature of the inside tube surface is 250^0C and the outside surface of the insulation is 50^0C. If the order of insulation of materials for steel pipe were reversed, that is, the insulation with a higher value of thermal conductivity was put first , calculate the change in heat loss with all other conditions remaining unchanged. Comment on the result. [CO1] [BL5]

2. Write an interactive computer program to calculate the heat transfer rate and the value of temperature anywhere in the medium for steady one-dimensional heat conduction in a long cylindrical shell for any combination boundary conditions. Run the program for five different sets of specified boundary conditions. [CO1] [BL6]
3. Write an interactive computer program to calculate the heat transfer rate and the value of temperature anywhere in the medium for steady one-dimensional heat conduction in a spherical shell for any combination of specified temperature, specified heat flux, and convection boundary conditions. Run the program for five different sets of specified boundary conditions. [CO1] [BL6]
4. Write an interactive computer program to calculate the heat transfer rate and the value of temperature anywhere in the medium for steady one-dimensional heat conduction in a plane wall whose thermal conductivity varies linearly as $k(T)=k_0(1+\beta T)$ where the constants k_0 and β are specified by the user for specified temperature boundary conditions. [CO1] [BL6]

D. GATE Questions:

1. 1. Air enters a counter-flow heat exchanger at 70^0C and leaves at 40^0C. Water enters at 30^0C and leaves at 50^0C. The LMTD In deg. C is [GATE-2000]
   a) 5.65 b) 4.43 c) 19.52 d) 20.172.
2. A composite wall, having unit length normal to the plane of paper, is insulated at the top and bottom as shown in the figure. It is comprised of four different materials A, B, C and D. [GATE-2001]



The dimensions are: HA = HD = 3m, HB = HC =1.5 m, L1 = L3 = 0.05m, L2= 0.1 m The thermal conductivity of the materials are: KA = KD = 50 W/m-K, KB = 10W/m-k, KC = 1 W/m-k. The fluid temperatures and heat transfer coefficients (see figure) are: T1 = 200°C, h1 = 50 W/m2-K, T2 = 25°C, h2 = 10 W/m2–K.

Assuming one-dimensional conduction,

(a) Sketch the thermal circuit of the system, and

(b) Determine the rate of heat transfer through the wall.

3. A copper tube of 20 mm outer diameter, 1 mm thickness and 20 m long (Thermal conductivity = 400 W/m-K) is carrying saturated steam at 150 C (Convective heat transfer coefficient = 150 W/m2-K). The tube is exposed to an ambient temperature of 27 C. The convective heat transfer coefficient of air is 5 W/m2-K. Glass wool is used for insulation (Thermal conductivity = 0.075 W/m-K). If the thickness of the insulation used is 5 mm higher than the critical thickness of insulation, calculate the

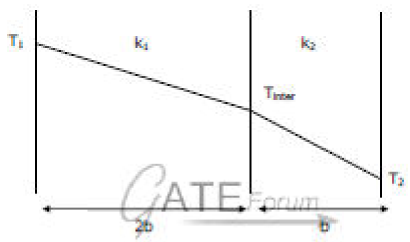
rate of heat lost by the steam and the rate of steam condensation in kg/hr (The enthalpy of condensation of steam = 2230 kJ/kg). [GATE-2002]

4. One dimensional unsteady state heat transfer equation for a sphere with heat generation at the rate of ‘q’ can be written as [GATE-2004]

a) $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{q}{K}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$ b) $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{q}{K}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

c) $\frac{\partial^2 T}{\partial r^2}+\frac{q}{K}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$ b) $\frac{\partial^2}{\partial r^2}(rT)+\frac{q}{K}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

5. In a composite slab, the temperature at the interface ( ) inter T between two materials is equal to the average of the temperatures at the two ends. Assuming steady one-dimensional heat conduction, which of the following statements is true about the respective thermal conductivities? [GATE-2006]



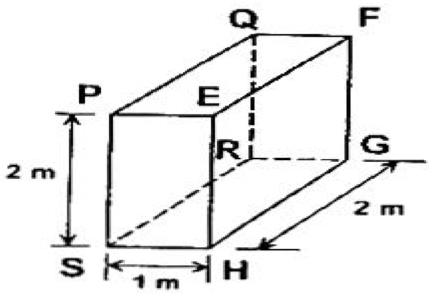
(a) 2k1 = k2 (b) k1 = k2 (c) 2 k1 = 3k2 (d) k1 = 2k2

6. With an increase in thickness of insulation around a circular pipe, heat loss to surroundings due to. [GATE-2006]
   (a) Convection increases, while that due to conduction decreases
   (b) Convection decreases, while that due to conduction increases
   (c) Convection and conduction decreases
   (d) Convection and conduction increases

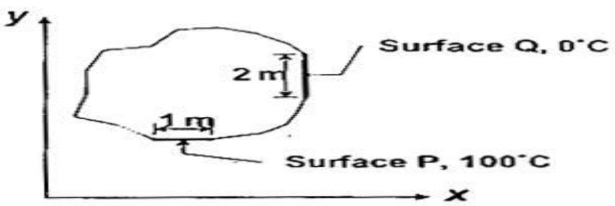
7. Consider steady one-dimensional heat flow in a plate of 20 mm thickness with a uniform heat generation of 80 MW/m^3. The left faces are kept at constant temperature of 160^{0}C and 120^{0}C respectively. The plate has a constant thermal conductivity of 200 W/mK. [GATE-2007]
   1. The location of maximum temperature within the plate from its left face is
      a) 15 mm b) 10 mm c) 5 mm d) 0 mm
   2. The maximum temperature within the plate in ^{0}C is
      a) 160 b) 165 c) 200 d) 250

8. For the three-dimensional object shown in the figure below, five faces are insulted. The sixth face (PQRS), which is not insulated, interacts thermally with the ambient, with a convective heat transfer coefficient of 10 W/m^{2}K. The ambient temperature is 30^{0}C. Heat is uniformly generated inside the object at the rate of 100 W/m^3. Assuming the face PQRS to be at uniform temperature, its steady stat temperature is [GATE-2008]

a) 10^{0}C b) 20^{0}C c) 30^{0}C d) 30^{0}C

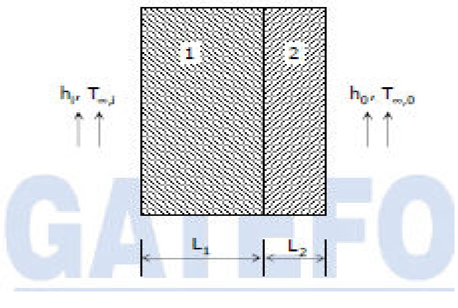


9. Steady two-dimensional heat conduction takes place in the body shown in the figure below. The normal temperature gradients over surfaces P and Q can be considered to be uniform. The temperature gradient $\frac{\partial T}{\partial x}$ at surface Q is equal to 10 K/m. Surface P and Q are maintained at constant temperatures as shown in the figure, while the remaining part of the boundary is insulated. The body has a constant thermal conductivity of 0.1 W/m.K. the values of $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial y}$ at surface P are [GATE-2008]



a) $\frac{\partial T}{\partial x}$ =20 K/m, $\frac{\partial T}{\partial y}$ =0K/m.
b) $\frac{\partial T}{\partial x}$ =0 K/m, $\frac{\partial T}{\partial y}$ =10K/m.
c) $\frac{\partial T}{\partial x}$ =10 K/m, $\frac{\partial T}{\partial y}$ =10K/m.
d) $\frac{\partial T}{\partial x}$ =0 K/m, $\frac{\partial T}{\partial y}$ =20K/m.

10. Consider steady-state heat conduction across the thickness in a plane composite wall (as shown in the figure) exposed to convection conditions on both sides.[GATE-2009]



Given:h_1=20W/m^2K; h_0=50W/m^2K; $T_{\infty,i}$=20^0C; $T_{\infty,0}$=2^0C; k_1=50 w/m K; L_1=0.30m and L_2=0.15m.

Assuming negligible contact resistance between the wall surfaces, the interface temperature, T (in °C), of the two walls will be

(a) -0.50 (b) 2.75 (c) 3.75 (d) 4.50

11. Pipe of 25mm outer diameter carries steam. The heat transfer coefficient between the cylinder and surroundings is 2 5W/m K . It is proposed to reduce the heat loss from

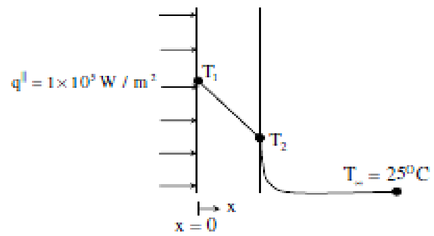
the pipe by adding insulation having a thermal conductivity of 0.05W/mK. Which one of the following statements is TRUE? [GATE-2011]

(a) The outer radius of the pipe is equal to the critical radius
(b) The outer radius of the pipe is less than the critical radius
(c) Adding the insulation will reduce the heat loss
(d) Adding the insulation will increase the heat loss

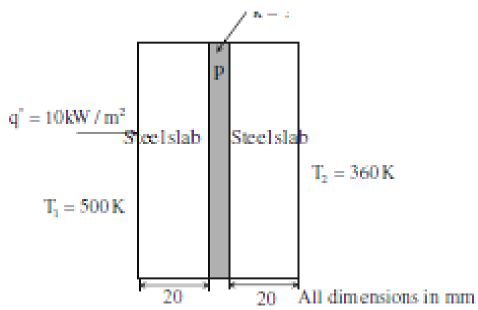
12. Consider one-dimensional steady state heat conduction along x-axis $(0 \leq X \geq L)$, through a plane wall with the boundary surfaces (x = 0 and x = L) maintained at temperatures 0ºC and 100ºC. Heat is generated uniformly throughout the wall. Choose the CORRECT statement. [GATE-2013]

(a) The direction of heat transfer will be from the surface at 100ºC to surface at 0ºC.
(b) The maximum temperature inside the wall must be greater than 100ºC
(c) The temperature distribution is linear within the wall
(d) The temperature distribution is symmetric about the mid-plane of the wall

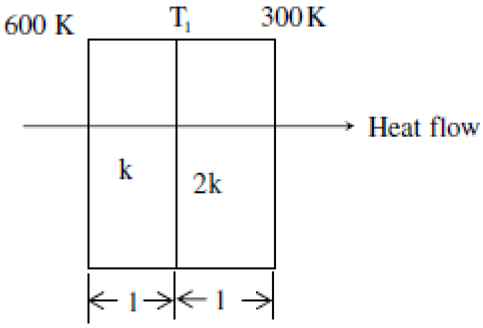
13. Consider one dimensional steady state heat conduction across a wall (as shown in figure below) of thickness 30 mm and thermal conductivity 15 W/m.K. At $x = 0$, a constant heat flux, $q'' = 1 \times 105$ W/m2 is applied. On the other side of the wall, heat is removed from the wall by convection with a fluid at 25OC and heat transfer coefficient of 250 W/m2.K. The temperature (in OC), at $x = 0$ is _____[GATE-2014]



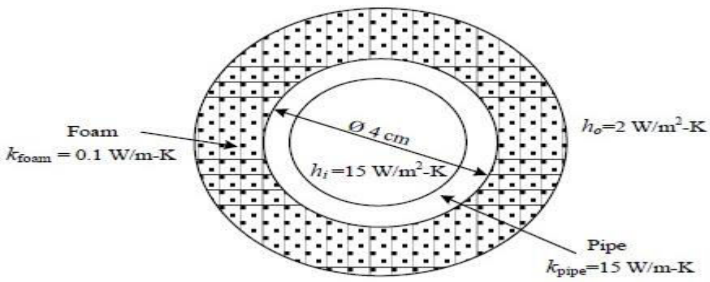
14. A material P of thickness 1 mm is sandwiched between two steel slabs, as shown in the figure below. A heat flux 10 kW/m2 is supplied to one of the steel slabs as shown. The boundary temperatures of the slabs are indicated in the figure. Assume thermal conductivity of this steel is 10 W/m.K. considering one-dimensional steady state heat conduction for the configuration, the thermal conductivity (k, in W/m.K) of material P is _______ [GATE-2014]



15. Heat transfer through a composite wall is shown in figure. Both the sections of the wall have equal thickness (l). The conductivity of one section is k and that of the other is $2k$. The left face of the wall is at 600 K and the right face is at 300 K. The interface temperature Ti (in K) of the composite wall is _______ [GATE-2014]



16. A plane wall has a thermal conductivity of 1.15 W/m.K. If the inner surface is at 1100^0C and the outer surface is at 350^0C, then the design thickness (in meter) of the wall to maintain a steady heat flux of 2500 W/m^2 should be _______ [GATE-2014]
17. A 10 mm diameter electrical conductor is covered by an insulation of 2 mm thickness. The conductivity of the insulation is 0.08 W/m-k and the convection coefficient at the insulation surface is 10 W/m^2-K. Addition of further insulation of the same material will [GATE-2015]
    a) Increase heat loss continuously.
    b) Decrease heat loss continuously.
    c) Increase heat loss to a maximum and then decrease heat loss.
    d) Decrease heat loss to a minimum and then increase heat loss.
18. If a foam insulation is added to a 4 cm outer diameter pipe as shown in the figure, the critical radius of insulation (in cm) is________________ [GATE-2015]



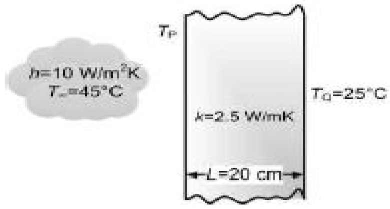
19. A cylindrical uranium fuel rod of radius 2 mm in a nuclear reactor is generating heat at the rate of 4x10^7 W/m^3. The rod is cooled by a liquid (convective heat transfer coefficient 1000 W/m^2-K) of the rod is [GATE-2015]
    a) 308 b) 398 c) 418 d) 448
20. A brick wall $\left(k = 0.9\frac{W}{m.K}\right)$ of thickness 0.18 m separates the warm air in a room from the cold ambient air. On a particular winter day, the outside air temperature is -5^0C and the room needs to be maintained at 27^0C. The heat transfer coefficient associated with outside air is $20\frac{W}{m^2K}$. Neglecting the convective resistance of the air inside the room, the heat loss, in $\left(\frac{W}{m^2}\right)$, is [GATE-2015]
    a) 88 b) 110 c) 128 d) 160

21. A hollow cylinder has length *L*, inner radius *r*1, outer radius *r*2, and thermal conductivity *k*. The thermal resistance of the cylinder for radial conduction is [GATE-2016]

a) $\frac{\ln(r_2/r_1)}{2\pi kL}$ b) $\frac{\ln(r_1/r_2)}{2\pi kL}$ c) $\frac{2\pi kL}{\ln(r_2/r_1)}$ d) $\frac{2\pi kL}{\ln(r_1/r_2)}$

22. Heat is generated uniformly in a long solid cylindrical rod (diameter=10 mm) at the rate of $4x10^7 W/m^3$. The thermal conductivity of the rod material is 25 W/m.K. Under steady state conditions, the temperature difference between the centre and the surface of the rod is ________ ^{0}C. [GATE-2017]

23. A plane slab of thickness *L* and thermal conductivity *k* is heated with a fluid on one side (P), and the other side (Q) is maintained at a constant temperature, *T*Q of 25^0C, as shown in the figure. The fluid is at 45^0C and the surface heat transfer coefficient, *h*, is 10 W/m^2 K . The steady state temperature, *T*P, (in ^{0}C) of the side which is exposed to the fluid is ______(correct to two decimal places). [GATE-2018]



24. A slender rod of length L, diameter d (L>>d) and thermal conductivity k1 is joined with another rod of identical dimensions, but of thermal conductivity K2, to form a composite cylindrical rod of length 2L. The heat transfer in radial direction and contact resistance are negligible. The effective thermal conductivity of the composite rod is [GATE-2019]

a) $k_1 + k_2$ b) $\sqrt{k_1 k_2}$ c) $\frac{k_1 k_2}{k_1 + k_2}$ d) $\frac{2k_1 k_2}{k_1 + k_2}$

25. Three slabs are joined together as shown in the figure. There is no thermal contract resistance at the interfaces. The centre slab experience a non-uniform internal heat generation with an average value equal to 1000 W/m-3, while the left and right slabs have no internal heat generation. All slabs have thickness equal to 1 m and thermal conductivity of each slab is equal to 5 Wm-1K-1. The two extreme faces are exposed to fluid with heat transfer coefficient 100 Wm-1K-1 and bulk temperature 300C as shown. The heat transfer in the slabs is assumed to be one dimensional and steady, and all properties are constant. If the left extreme face temperature T1 is measured to be 1000C, the right extreme face temperature T2 is ________0C [GATE-2019]

26. One-dimensional steady state heat conduction takes place through a solid whose cross-sectional area varies linearly in the direction of heat transfer. Assume there is no heat generation in the solid and the thermal conductivity of the material is constant and independent of temperature. The temperature distribution in the solid is [GATE-2019]

a) Linear b) Quadratic c) Logarithmic d) Exponential

E. IES Questions:

1. A plane wall is 25 cm thick with an area of 1 m2 and has a thermal conductivity of 0.5 W/m-K. If a temperature difference of 60°C is imposed across it, what is the heat flow? [IES-2005]

a) 120W b) 140W c) 160W d) 180W

2. Thermal diffusivity of a substance is: [IES - 2006]

a) Inversely proportional to thermal conductivity
b) Directly proportional to thermal conductivity
c) Directly proportional to the square of thermal conductivity
d) Inversely proportional to the square of thermal conductivity

Which one of the following expresses the thermal diffusivity of a substance in terms of thermal conductivity (k), mass density (p) and specific heat (c)?

a) k2pc b) 1/pkc c) k/pc d) pc/k2

3. Subjected to symmetrical heat transfer from one face of each block. The other face of the block will be reaching to the same temperature at a rate [IES-2006]
   a) Faster in air block
   b) Faster in copper block
   c) Equal in air as well as copper block
   d) Cannot be predicted with the given information.
4. A large concrete slab 1 m thick has one dimensional temperature distribution: T = 4 – 10x + 20x2 + 10x3 Where T is temperature and x is distance from one face towards other face of wall. If the slab material has thermal diffusivity of 2×10-3 m2/hr, what is the rate of change of temperature at the other face of the wall? [IES-2009]

   a) 0.1°C/h b) 0.2°C/h c) 0.3°C/h d) 0.4°C/h

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Academic Year: 2019 – 20 **Semester: II**

Class : III B.Tech **Subject: Heat Transfer**

Learning Material

UNIT - II

Extended Surfaces and Transient Heat Conduction

Syllabus:

Extended surface (fin) Heat Transfer: Long Fins, Fin with insulated tip and Short Fin, Efficiency and effectiveness of fins.

One dimensional Transient Conduction Heat Transfer: Systems with negligible internal resistance, Significance of Biot and Fourier Numbers, Chart solutions of transient conduction systems

2.1 Heat Transfer from Finned Surfaces

2.1.1 Introduction:

Convection heat transfer between a hot solid surface and the surrounding colder fluid is governed by the Newton's cooling law which states that "the rate of convection heat transfer is directly proportional to the temperature difference between the hot surface and the surrounding fluid and is also directly proportional to the area of contact or exposure between them.

Therefore, convection heat transfer can be increased by either of the following ways-

1. Increasing the temperature difference (T_s- T_α) between the surface and the fluid.
2. Increasing the convection heat transfer coefficient by enhancing the fluid flow or flow velocity over the body.
3. Increasing the area of contact or exposure between the surface and the fluid.

Most of the times, to control the temperature difference is not feasible and increase of heat transfer coefficient may require installation of a pump or a fan or replacing the existing one with a new one having higher capacity. The alternative is to increase the surface area by attaching to the surface extended surfaces called fins made of highly conductive materials such as aluminium. Finned surfaces are manufactured by extruding, welding, or wrapping a thin metal sheet on a surface. Fins enhance heat transfer from a surface by exposing a larger surface area to convection and radiation.

- The car radiator is an example of finned surface.

Fins are the extended surface protruding from a surface or body and they are meant for increasing the heat transfer rate between the surface and the surrounding fluid by increasing heat transfer area.

2.1.2 Fin Equation

In the analysis of fins, we consider steady operation with no heat generation in the fin, and we assume the thermal conductivity 'k' of the material to remain constant. We also assume the convection heat transfer coefficient 'h' to be constant and uniform over the entire surface of the fin for convenience in the analysis.

Consider a volume element of a fin location x having a length of Δx, cross-sectional area of A_c, and a perimeter of p, as shown in Fig. 2.1.

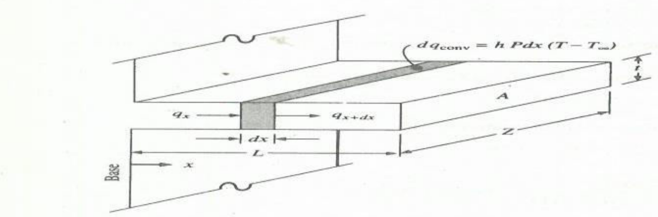


Figure 2.1: Sketch illustrating one-dimensional conduction through a rectangular fin.

Under steady conditions, the energy balance on this volume element can be expressed as

$$\begin{pmatrix} Rate\ of\ heat \\ conduction\ into \\ the\ element\ at\ x \end{pmatrix} - \begin{pmatrix} Rate\ of\ heat \\ conduction\ from \\ the\ element\ at\ x+\Delta x \end{pmatrix} = \begin{pmatrix} Rate\ of\ heat \\ convection\ from \\ the\ element \end{pmatrix}$$

Or

$$Q_{cond,x} - Q_{cond,x+\Delta x} = Q_{conv} \qquad (2.1)$$

$$-kA_c \frac{dT}{dx} + kA_c \left(\frac{dT}{dx} + \frac{d^2T}{dx^2} dx \right) = hPdx(T - T_\infty) \quad (2.2)$$

$$\frac{d}{dx}\left(kA_c \frac{dT}{dx} \right) - hp(T - T_\infty) = 0 \qquad (2.3)$$

Where A_c is the cross-sectional area of the fin at location x.

In general, the cross-sectional area A_c and the perimeter p of a fin vary with x, which makes this differential equation difficult to solve. In the special case of constant cross section and constant thermal conductivity, the differential equation 2.6 reduces to

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad (2.4)$$

Where $m^2 = \frac{hp}{kA_c}$ and $\theta = T - T_\infty$ is the temperature excess.

At the fin base we have $\theta_b = T_b - T_\infty$.

Eq. 2.4 is a linear, homogeneous, second-order differential equation with constant coefficients. The general solution of the differential equation 2.4 is

$$\theta(x) = C_1 e^{-mx} + C_2 e^{mx} \qquad (2.5)$$

Where $C_1$and C_2 are arbitrary constants whose values are to be determined from the boundary conditions at the base and at the tip of the fin.
At the fin base we have a specified temperature boundary condition, expressed as
Boundary condition at fin base: $\theta(0) = \theta_b = T_b - T_\infty$ (2.6)

At the fin tip we have several possibilities, including specified temperature, negligible heat loss (idealized as an adiabatic tip), convection, and combined convection and radiation.

2.1.2.1 Infinitely Long Fin ($T_{fin\ tip}$=T_∞)

For a sufficiently long fin of uniform cross section (A_c=constant), the temperature of the fin at the fin tip approaches the environment temperature T_∞ and thus θ approaches zero. That is

Boundary condition at fin tip: $\theta(L) = T(L) - T_\infty = 0 \quad as \quad L \to \infty$ (2.7)

The variation of temperature along the fin In this case can be expressed as

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-x\sqrt{hp/kA_c}} \qquad (2.8)$$

Note that the temperature along the fin in this case decreases exponentially from T_b to T_∞. The steady rate of heat transfer from the entire fin can be determined from Fourier's law of heat conduction as

$$Q_{long\ fin} = kA_c \frac{dT}{dx}\Big|_{x=0} = \sqrt{hpkA}(T_b - T_\infty) \qquad (2.9)$$

Where p is the perimeter, A_c is the cross-sectional area of the fin, and x is the distance from the fin base.

2.1.2.2 Negligible Heat Loss from the Fin Tip (Adiabatic fin tip, $Q_{fin\ tip}$=0)

Fins are not likely to be so long that their temperature approaches the surrounding temperature at the tip. A more realistic situation is for heat transfer from the fin tip to be negligible since the heat transfer from the fin is proportional to its surface area, and the surface area of the fin tip is usually a negligible fraction of the total fin area. Then the fin tip can be assumed to be adiabatic, and the condition at the fin tip can be expressed as

$$\frac{d\theta}{dx}\Big|_{x=L} = 0 \qquad (2.10)$$

The condition at the fin base remains the same as expressed in Eq. 2.9. The application of these two conditions on the general solution (Eq. 2.8) yields, after some manipulations, the temperature distribution equation:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL} \qquad (2.11)$$

The rate of heat transfer from the fin can be determined again from Fourier's law of heat conduction:

$$Q_{adiabatic\,tip} = kA_c \frac{dT}{dx}\Big|_{x=0}$$
$$= \sqrt{hpkA}(T_b - T_\infty)\tanh mL \qquad (2.12)$$

Note that the heat transfer relations for the very long fin and the fin with negligible heat loss at the tip differ by the factor $\tanh mL$, which approaches 1 as L becomes very large.

2.1.2.3 Convection (Or Combined Convection and Radiation from Fin Tip)

The fin tips, in practice, are exposed to the surroundings, and thus the proper boundary condition for the fin tip is convection that also includes the effects or radiation.

A practical way of accounting for the heat loss from the fin tip is to replace the fin length L in the relation for the insulated tip case (Eq. 2.13 & 2.14) by a corrected length defined as

$$Corrected\ fin\ length: L_c = L + \frac{A_c}{p} \qquad (2.13)$$

Where A_c is the cross-sectional area and p is the perimeter of the fin at the tip.

Therefore, fins subjected to convection at their tips can be treated as fins with insulated tips by replacing the actual fin length by the corrected length.

Using the proper relations for Ac and p, the corrected lengths for rectangular and cylindrical fins are easily determined to be

$$L_{c,rec\tan gular\ fin} = L \ \ and \ \ L_{c,pin\ fin} = L + \frac{D}{4} \qquad (2.14)$$

Where t is the thickness of the rectangular fins and d is the diameter of the cylindrical fins.

2.1.3 Fin Efficiency

We define fin efficiency as

$$\eta_{fin} = \frac{Q_{fin}}{Q_{fin,\max}} = \frac{Actual\,heat\,transfer\,rate\,from\,the\,fin}{\begin{array}{c}Ideal\,heat\,transfer\,rate\,from\,the\,fin\\ if\,the\,entire\,fin\,were\,at\,base\,temperatue\end{array}}$$

For the cases of constant cross section of very long fins and fins with adiabatic tips, the fin efficiency can be expressed as

$$\eta_{long\ fin} = \frac{Q_{fin}}{Q_{fin,\max}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_{fin}(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hp}} = \frac{1}{mL} \qquad (2.15)$$

and

$$\eta_{adiabatic\,tip} = \frac{Q_{fin}}{Q_{fin,\max}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL}{hA_{fin}(T_b - T_\infty)} = \frac{\tanh mL}{mL} \qquad (2.16)$$

Note that fins with triangular and parabolic profiles contain less material and are more efficient than the ones with rectangular profiles, and thus are more suitable for applications requiring minimum weight such as space applications.

- An important consideration in the design of finned surfaces is the selection of the proper fin length L. Normally the longer the fin, the larger the heat transfer area and thus the higher the rate of heat transfer from the fin.
- But also the larger the fin, the bigger the mass, the higher the price, and the larger the fluid friction. Therefore, increasing the length of the fin beyond a certain value cannot be justified unless the added benefits out weight the added cost.
- Also, the fin efficiency decreases with increasing fin length because of the decrease in fin temperature with length.
- Fin lengths that cause the fin efficiency to drop below 60 percent usually cannot be justified economically and should be avoided.
- The efficiency of most fins used in practice is above 90 percent.

2.1.4 Fin Effectiveness

The performance of fins is expressed in terms of the fin effectiveness, ε_{fin} defined as

$$\varepsilon_{fin} = \frac{Q_{fin}}{Q_{no\,fin}} = \frac{Q_{fin}}{hA_b(T_b - T_\infty)} = \frac{\begin{array}{c}Heat\,transfer\,rate\,from\\ the\,fin\,of\,base\,area\,A_b\end{array}}{\begin{array}{c}Heat\,transfer\,rate\,from\\ the\,fin\,of\,area\,A_{fin}\end{array}} \qquad (2.17)$$

The effectiveness of a long fin is determined to be

$$\varepsilon_{long\,fin} = \frac{Q_{fin}}{Q_{no\,fin}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_c(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}} \qquad (2.18)$$

We can draw several important conclusions from the fin effectiveness relation given above for consideration in the design and selection of the fins:

- The thermal conductivity k of the fin material should be as high as possible. So, the fins are made from metals, with copper, aluminium, and iron. Perhaps the most widely used fins are made of aluminium because of its low cost and weight and its resistance to corrosion.
- The ratio of the perimeter to the cross-sectional area of the fin p/A_c should be as high as possible. This criterion is satisfied by thin plate fins and slender pin fins.
- The use of fins is most effective in applications involving a low convection heat transfer coefficient. Thus, the use of fins is more easily justified when the medium is a

gas instead of a liquid and heat transfer is by natural convection instead of by forced convection. Therefore, it is no coincidence that in liquid-to-gas heat exchangers such as the car radiator, fins are placed on the gas side.

2.2 Transient Heat Conduction

2.2.1. Introduction

The temperature of a body, in general, varies with time as well as position. In rectangular coordinates, this variation is expressed as T(x, y, z), where (x, y, z) indicate variation in the x^-,y^-, and z^- directions, and t indicates variation with time.

2.2.2 Lumped Heat Capacity Analysis

In heat transfer analysis, some bodies are observed to behave like a "lump" whose interior temperature remains essentially uniform at all times during a heat transfer process. The temperature of such bodies can be taken to be a function of time only, T(t). Heat transfer analysis that utilizes this idealization is known as lumped system analysis.

Consider a body of arbitrary shape of mass m, volume V, surface area A_s, density ρ, and specific heat c_p initially at a uniform temperature T_i. At time t=0, the body is placed into a medium at temperature T_∞, and heat transfer takes place between the body and its environment, with a heat transfer coefficient h. We assume lumped system analysis to be applicable, so that the temperature remains uniform within the body at all times and changes with time only, T=T(t).

During a differential time interval dt, the temperature of the body decreases by a differential amount dT. An energy balance of the solid for the time interval dt can be expressed as

$$\begin{pmatrix} \text{Heat transfer from} \\ \text{the body during } dt \end{pmatrix} = \begin{pmatrix} \text{The decrease in energy} \\ \text{of the body during } dt \end{pmatrix}$$

Or

$$hA_s\left(T-T_\infty\right)\,dt = mc_p dT \qquad (2.19)$$

Noting that $m = \rho V$ and $dT = d\left(T-T_\infty\right)$, equation 2.19 can be rearranged as

$$\frac{d(T-T_\infty)}{T-T_\infty} = -\frac{hA_s}{\rho V c_p}dt \qquad (2.20)$$

Integrating equation 2.20 and applying the initial condition, i.e. $T=T_i$ at time t=0, we get

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = -\frac{hA_s}{\rho V c_p}t \qquad (2.21)$$

Taking the exponential of both sides and rearranging, we obtain

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \qquad (2.22)$$

Where,

$$b = \frac{hA_s}{pVc_p} = B_i F_0 \qquad (2.23)$$

is a positive quantity whose dimension is $(\text{time})^{-1}$. The reciprocal of b has time unit (usually s), and is called the time constant.

F_0 is the Fourier number and is defined as

$$F_0 = \frac{\begin{array}{c}\textit{The rate at which heat is conducted}\\ \textit{across L of a body of volume } L^3\end{array}}{\begin{array}{c}\textit{The rate at which heat is stored}\\ \textit{in a body of volume } L^3\end{array}} = \frac{\alpha t}{L^2} \qquad (2.24)$$

- Equation 2.23 enables us to determine the temperature T (t) of a body at time t, or alternatively, the time t required for the temperature to reach a specified values T (t).
- The temperature of a body approaches the ambient temperature T_∞ exponentially. The temperature of the body changes rapidly at the beginning, but rather slowly later on.
- A large value of b indicates that the body approaches the environment temperature in a short time. The larger the value of the exponent b, the higher the rate of decay in temperature.
- b is proportional to the surface area, but inversely proportional to the mass and the specific heat of the body. Hence, it takes longer to heat or cool a larger mass, especially when it has a large specific heat.

The total amount of heat transfer between the body and the surrounding medium over the time interval t=0 to t is simply the change in the energy content of the body:

$$Q = mc_p[T_i - T(t)] \qquad (2.25)$$

The amount of heat transfer reaches its upper limit when the body reaches the surrounding temperature T_∞. Therefore, the maximum heat transfer between the body and its surroundings is

$$Q_{\max} = mc_p[T_i - T_\infty] \qquad (2.26)$$

2.2.1.1 Criteria for Lumped System Analysis

The first step in establishing a criterion for the applicability of the lumped system analysis is to define a characteristic length as

$$L_c = \frac{V}{A_s} \qquad (2.27)$$

and a Biot number Bi as

$$Bi = \frac{Conduction\ resistance\ within\ the\ body}{Covection\ resistance\ at\ the\ surface\ of\ the\ body} = \frac{L_c|k}{1|h} = \frac{hL_c}{k} \quad (2.28)$$

- The Biot number is the ratio of the internal resistance of a body to heat conduction to its external resistance to heat convection.

- Therefore, a small Biot number represents small resistance to heat conduction, and thus small temperature gradients within the body.

- Lumped system analysis assumes a uniform temperature distribution throughout the body, which is the case only when the thermal resistance of the body to heat conduction (the conduction resistance) is zero.

- Thus, lumped system analysis is exact when Bi=0 and approximate when Bi>0. Of course, the smaller the Bi number, the more accurate the lumped systems analysis.
- It is generally accepted that lumped system analysis is applicable if $B_i < 0.1$.
- Therefore, small bodies with high thermal conductivity are good candidates for lumped systems analysis, especially when they are in a medium that is a poor conductor of heat (such as air or another gas) and motionless.
- Thus, the hot small copper ball placed in quiescent air is most likely to satisfy the criterion for lumped system analysis.

2.2.2 Transient Heat Conduction in Large Plane Walls, Long Cylinders, and Spheres with Spatial Effects

Consider a plane wall of thickness 2L, a long cylinder of radius r_0, and a sphere of radius r_0 initially at a uniform temperature T_i as shown in Fig. 2.2. At time t=0, each geometry is placed in a large medium that is at a constant temperature T_∞ and kept in that medium for t > 0. Heat transfer takes place between these bodies and their environments by convection with a uniform and constant heat transfer coefficient h. Note that all three cases possess geometric and thermal symmetry: the plane wall is symmetric about its centre plane (x=0), the cylinder is symmetric about its centreline (r=0), and the sphere is symmetric about its centre point (r=0).

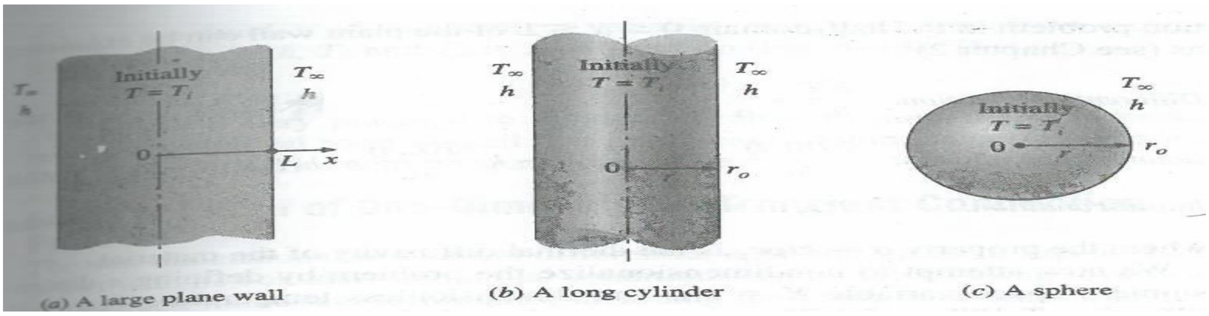


Figure 2.2: Schematic of the simple geometries in which heat transfer is one-dimensional.

The variation of the temperature profile with time in the plane wall is illustrated in Figure 2.3. When the wall is first exposed to the surrounding medium at $T_\infty < T_i \;\; at \; t = 0$, the entire wall is at its initial temperature T_i. But, the wall temperature near the surface starts to drop as a result of heat transfer from the wall to the surrounding medium. This creates a temperature gradient in the wall and initiates heat conduction from the inner parts of the wall toward its outer surface.

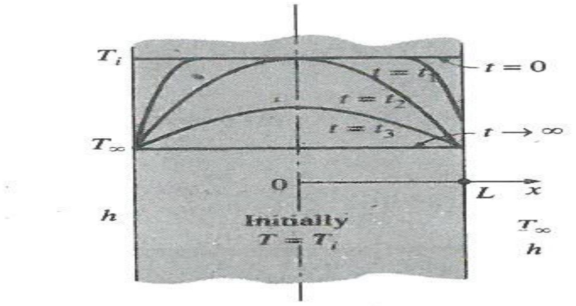


Figure 2.3: Transient temperature profiles in a plane wall exposed to convection from its surfaces for $T_i > T_\infty$

The transient temperature charts in Figures: 2.4, 2.5, and 2.6 for a large plane wall, long cylinder, and sphere were presented by M.P.Heisler in 1947 and are called Heisler charts. There are three charts associated with each geometry: the first chart is to determine the temperature T_0 at the centre of the geometry at a given time t. The second chart is to determine the temperature at other locations at the same time in terms of T_0. The third chart is to determine the total amount of heat transfer up to the time t. These plots are valid for $\tau > 0.2$.

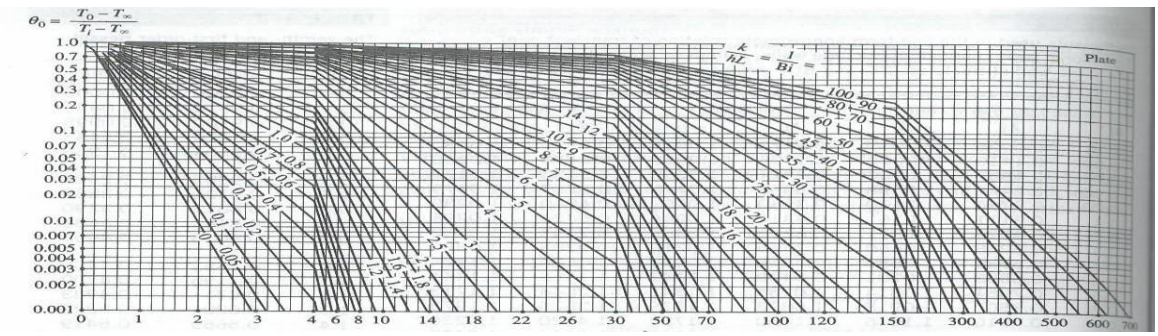


Figure 2.4 (a): Mid-plane temperature.

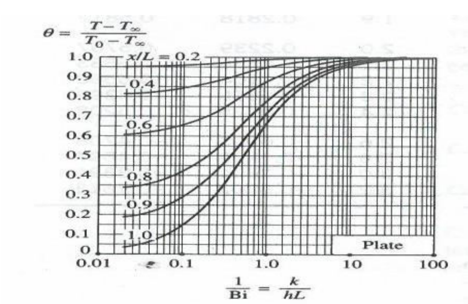


Figure 2.4 (b): Temperature distribution.

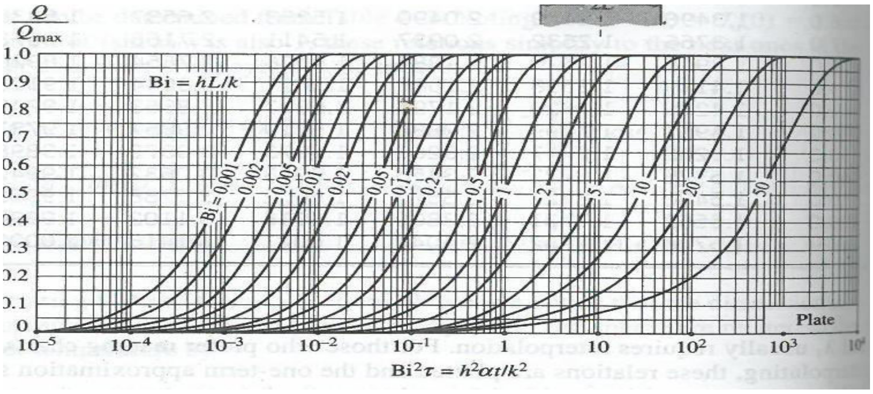


Figure 2.4 (c): Heat Transfer.

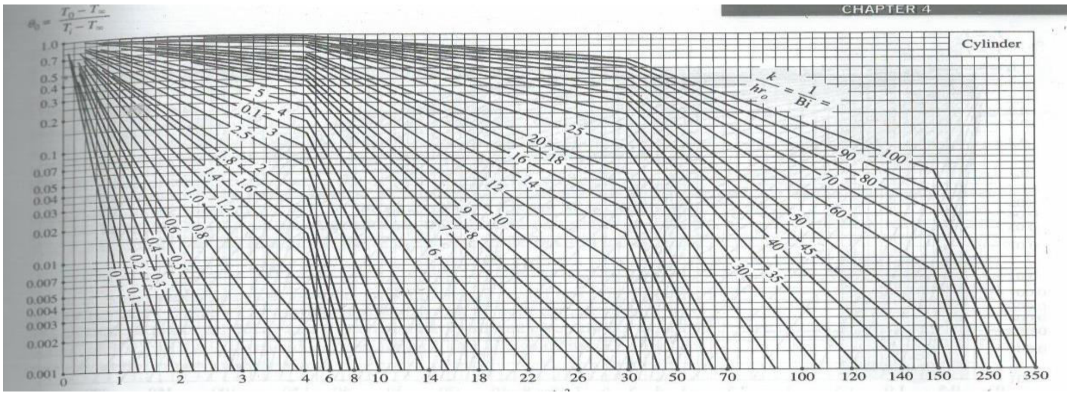


Figure 2.5 (a): Centreline temperature.

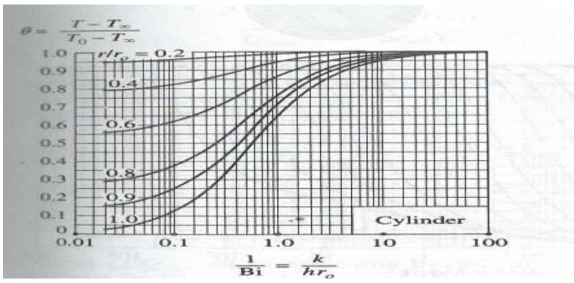


Figure 2.5 (b): Temperature distribution.

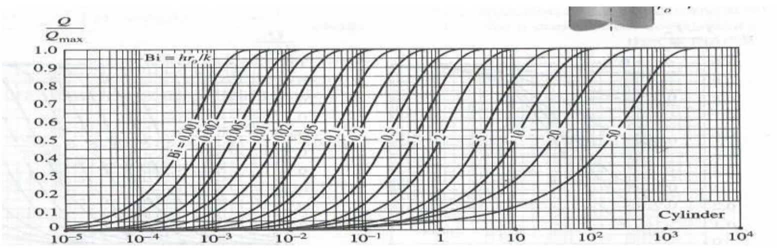


Figure 2.5 (C): Heat transfer.

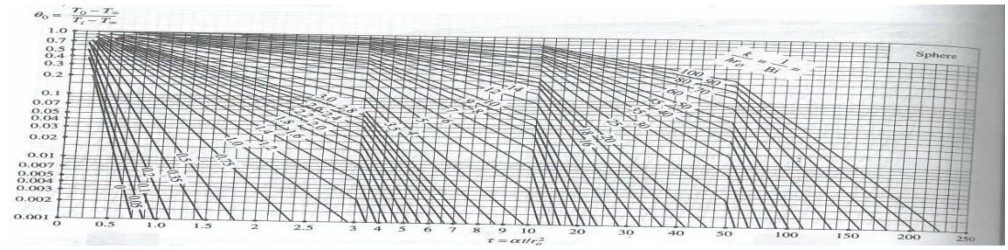


Figure 2.6 (a): Midpoint temperature.

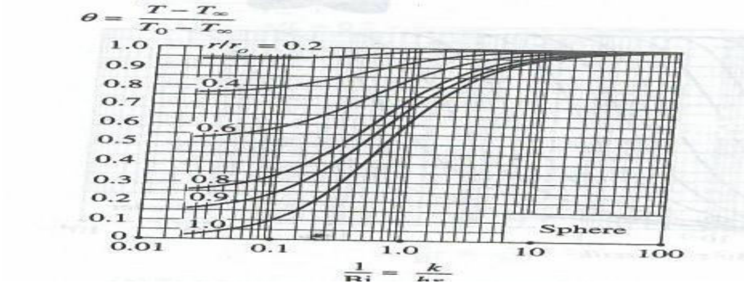


Figure 2.6 (b): Temperature distribution.

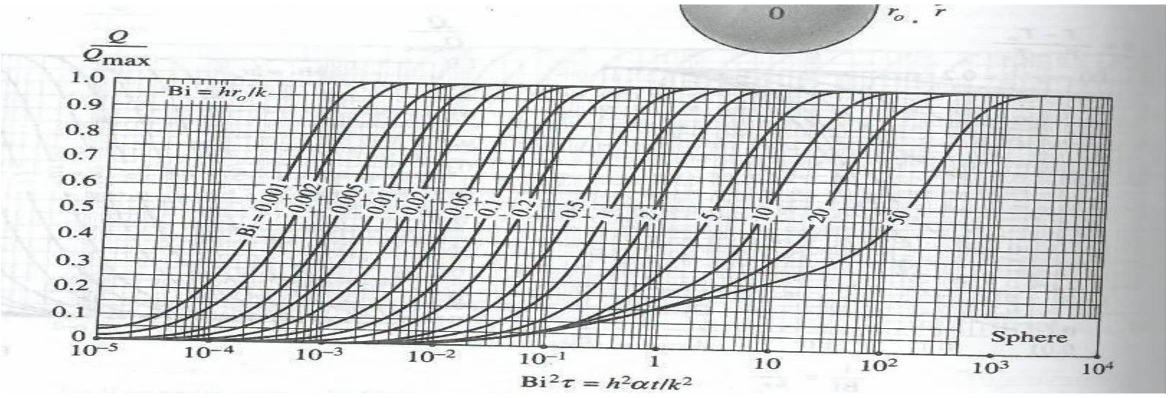


Figure 2.6 (C): Heat transfer.

Assignment-Cum-Tutorial Questions

A. Questions at remembering / understanding level:

I) Objective Questions

1. The fin efficiency is defined as the ratio of the actual heat transfer from the fine to [CO2] [BL2]
   (a) The heat transfer from the same fin with an adiabatic tip.
   (b) The heat transfer from an equivalent fin which is infinitely long.
   (c) The heat transfer from the same fin if the temperature along the entire length of the fin is the same as the base temperature.
   (d) The heat transfer through the base area of the same fin.
   (e) None of the above.
2. Two finned surfaces with long fins are identical, except that the convection heat transfer coefficient for the first finned surface is twice that of the second one. What statement below is accurate for the efficiency and effectiveness of the first finned surface relative to the second one? [CO2] [BL2]
   (a) Higher efficiency and higher effectiveness
   (b) Higher efficiency but lower effectiveness
   (c) Lower efficiency but higher effectiveness
   (d) Lower efficiency and lower effectiveness
   (e) Equal efficiency and equal effectiveness
3. Fins are more effective when [CO2] [BL2]
   (i) the thermal conductivity of fin material is large
   (ii) the ratio of perimeter to cross sectional area of fin is large
   (iii) the convective heat transfer coefficient of the fluid is low
   (a) i only (b) i & iii (c) i, ii & iii (d) ii & iii
4. Which of the following statement(s) is/are true? [CO2] [BL2]
   (i) The shorter the length of fin, the greater the fin efficiency.
   (ii) The longer the length of fin, the greater the fin efficiency.
   (iii) The lower the surface heat transfer coefficient, the higher the fin efficiency.
   (iv) The higher the surface heat transfer coefficient, the lower the fin efficiency.
   (a) i & iii (b) ii & iv (c) i only (d) iv only
5. A rectangular fin of thickness t is axially cut into two halves along the centre plane such that the thickness of each is t/2. If these two halves are separated by a finite distance and are placed parallel to each other, the rate of heat transfer, when compared to the initial one, will [CO2] [BL2]
   (a) be doubled (b) be halved (c) remain the same (d) tripled
6. The Biot number can be thought of as the ratio of [CO2] [BL2]
   (a) internal conduction thermal resistance to the convective thermal resistance.
   (b) convective thermal resistance to the internal conduction thermal resistance.
   (c) the thermal energy storage capacity to the conduction thermal resistance.
   (d) the thermal energy storage capacity to the convection thermal resistance.
7. Which of the following statement(s) is/are true? [CO2] [BL2]
   (i) The Lumped system analysis of transient heat conduction is perfectly valid when the Biot number is zero.
   (ii) The Lumped system analysis of transient heat conduction is approximately valid when the Biot number is greater than 0.1.
   (iii) The Lumped system analysis of transient heat conduction is approximately valid when the Biot number is less than 0.1.

(iv) The Lumped system analysis of transient heat conduction is perfectly valid when the ratio of

(a) i & iii (b) ii & iv (c) i only (d) iv only

8. Solution of transient one dimensional heat conduction problems by Heisler charts is applicable when the Biot number is [CO2] [BL2]

(i) internal conduction resistance to surface convective resistance is small.

(ii) internal conduction resistance to surface convective resistance is large.

(iii) surface convective heat transfer coefficient to thermal conductivity is very small.

(iv) surface convective heat transfer coefficient to thermal conductivity is very large.

(a) i & iii (b) ii & iv (c) i & iv (d) ii & iii

9. Does the (a) efficiency and (b) effectiveness of a fin increase or decrease as the fin length is increased? [CO2] [BL2]

(a) Efficiency increases and effectiveness decreases.
(b) Efficiency decreases and effectiveness increases.
(c) Both efficiency and effectiveness increase.
(d) Both efficiency and effectiveness decrease.

10. Two pin fins are identical, except that the diameter of one of them is twice the diameter of the other. For which fin is the (a) fin effectiveness and (b) fin efficiency higher? [CO2] [BL2]

(a) Effectiveness is higher for first one and efficiency is higher for second one.
(b) Effectiveness is higher for second one and efficiency is higher for first one.
(c) Effectiveness and efficiency of a fin do not depend on the diameter.

II)Descriptive Questions

1. Sketch various types of fin configurations? [CO2] [BL1]
2. Derive an expression for the temperature distribution and the rate of heat transfer for a straight rectangular fin of uniform cross section when the tip of the fin is insulated. [CO2] [BL3]
3. Define fin efficiency and fin effectiveness. Derive the expressions for fin efficiency and fin effectiveness for a very long fin. [CO2] [BL3]
4. Write the general boundary conditions for a longitudinal fin for the cases of (a) Long fin,(b) Fin with insulated tip (c) Fin with convection off the end [CO2] [BL2]
5. Derive an expression for the temperature distribution in one-dimensional transient heat conduction using lumped heat capacity analysis. Also state the criterion for the validity of lumped heat capacity analysis. [CO2] [BL3]
6. What are Biot and Fourier numbers? Explain their physical significance. [CO2] [BL2]

B. Question at applying / analyzing level

I) Multiple choice questions.

Common data for questions 1 and 2.

A 1-cm-diameter, 30-cm-long fin made of aluminum (k = 237 W/m·K) is attached to a surface at 80°C. The surface is exposed to ambient air at 22°C with a heat transfer coefficient of 11 W/m^2·K. If the fin can be assumed to be very long,

1. the rate of heat transfer from the fin is [CO2] [BL3]
(a) 2.2 W (b) 3 W (c) 3.7 W (d) 4 W (e) 4.7W
2. the efficiency of fin is [CO2] [BL3]
(a) 0.60 (b) 0.67 (c) 0.72 (d) 0.77 (e) 0.88
3. A hot surface at 80°C in air at 20°C is to be cooled by attaching 10-cm-long and 1-cm-diameter cylindrical fins. The combined heat transfer coefficient is 30 W/m^2·K, and the heat transfer from the fin tip is negligible. If the fin efficiency is 0.75, the rate of heat loss from 100 fins is [CO2] [BL3]
(a) 325 W (b) 707 W (c) 566 W (d) 424 W (e) 754 W

4. A cylindrical pin fin of diameter 0.6 cm and length of 3 cm with negligible heat loss from the tip has an efficiency of 0.7. The effectiveness of this fin is [CO2] [BL3]
   (a) 0.3 (b) 0.7 (c) 2 (d) 8 (e) 14

Common data for questions 5 and 6

Consider a very long rectangular fin attached to a flat surface such that the temperature at the fin is essentially that of the surrounding air, i.e. 20^0C. Its width is 5.0 cm; thickness is 1.0 mm; thermal conductivity is 200 W/m.K; and base temperature is 40^0C. The heat transfer coefficient is 20 W/m^2.K.

5. What is the temperature at a distance of 5cm from the base?
   (a) 17.8^0C (b) 20.8^0C (c) 23.8^0C (d) 29.8^0C
6. What is the rate of heat loss from the entire fin?
   (a) 2.86 J (b) 2.46 J (c) 2.26 J (d) 2.06 J
7. A triangular shaped fin a motorcycle engine is 0.5-cm thick at its base and 3-cm long (normal distance between the base and the tip of the triangle), and is made of aluminium (k=150 W/m.K). This fin is exposed to air with a convective heat transfer coefficient of 30 W/m^2. K acting on its surfaces. The efficiency of the fin is 50 percent. If the fin base temperature is 130^0C and the air temperature is 25^0C, the beat transfer from this fin per unit width is [CO2] [BL3]
   (a) 32 W/m (b) 47 W/m (c) 68 W/m (d) 82 W/m (e) 95 W/m
8. An 18-cm-long, 16-cm-wide, and 12-cm-high hot iron block (ρ= 7870 kg/m^3, c_p = 447 J/kg-K) initially at 20°C is placed in an oven for heat treatment. The heat transfer coefficient on the surface of the block is 100 W/m^2·K. If it is required that the temperature of the block rises to 750°C in a 25-min period, the oven must be maintained at [CO2] [BL3]
   (a) 750°C (b) 830°C (c) 875°C (d) 910°C (e) 1000°C
9. Copper balls (ρ = 8933 kg/m^3, k = 401 W/m-K, c_p = 385 J/kg-°C, $\alpha = 1.166 x 10^{-4}$ m^2/s) initially at 200°C are allowed to cool in air at 30°C for a period of 2 minutes. If the balls have a diameter of 2 cm and the heat transfer coefficient is 80 W/m^2-K, the centre temperature of the balls at the end of cooling is [CO2] [BL3]
   (a) 104°C (b) 87°C (c) 198°C (d) 126°C (e) 152°C
10. A potato may be approximated as a 5.7-cm-diameter solid sphere with the properties ρ=910 kg/m^3, c_p=4.25 kJ/kg.^{0}C, k=0.68 W/m.^{0}C, and α=1.76 x $10^{-7} m^2/s$. Twelve such potatoes initially at 25^0C are to be cooked by placing them in an oven maintained at 250^0C with a heat transfer coefficient of 95 $W/m^{2.0}$C. The amount of heat transfer in kJ to the potatoes during a 30- minute period is
    [CO2] [BL3]
    (a) 77 (b) 483 (c) 927 (d) 970 (e) 1012

Common data for question 11 and 12

A steel ball [c=0.46 kJ/kg.K, k=35 W/m.K] 5.0 cm in diameter and initially at a uniform temperature of 450^0C is suddenly placed in a controlled environment in which the temperature is maintained at 100^0C. The convection heat-transfer coefficient is 10 W/m^2 -K.

11. The time in hours required for the ball to attain a temperature of 150^0C is [CO2] [BL3]
    (a) 1.42 (b) 1.69 (c) 1.22 (d) 0.82
12. The total heat lost from the steel ball during this time interval is
    (a) 53.8 J (b) 43.8 J (c) 73.8 J (d) 63.8 J

II) Problems

1. Aluminum fins of rectangular profile are attached on a plane wall with 5 mm spacing. The fins have thickness 1 mm, length = 20 mm and the thermal conductivity k = 200 W/m-K. The wall is maintained at a temperature of 250^0C and the fins dissipate heat by convection into ambient air at 30^0C, with heat transfer coefficient = 60 W/m^{2-}-K. Find the heat loss. [CO2] [BL3]
2. A steel rod (k = 32 W/m-K), 12mm in diameter and 60mm long, with an insulated tip is to be used as a spine. It is exposed to surroundings with a temperature of 60^0C and a heat transfer coefficient of 55 W/m^2-K. The temperature at the base of fin is 95^0C. Determine [CO2] [BL3]
   i. Efficiency of fin and ii. Heat dissipation from the fin.
3. A 4-mm-dimater and 10-cm-long aluminium fin (k=237 $W/m.^0C$) is attached to a surface. If the heat transfer coefficient is 12 $W/m^2.^0C$, determine the percent error in the rate of heat transfer from the fin when the infinitely long fin assumption is used instead of the adiabatic fin tip assumption. [CO2] [BL4]
4. A cylinder of 1 m long and 5 cm diameter is placed in an atmosphere at 45^0C. It is provided with 10 longitudinal straight fins of material having k = 10 W/m-K. The height of 0.76 mm thick fins is 1.27cm from the cylinder surface. The heat transfer coefficient between cylinder and atmosphere is 17 W/m^2-K. Calculate the rate of heat transfer, if surface temperature of the cylinder is 150^0C [CO2] [BL3]
5. A stainless – steel rod (18% Cr, 8%Ni) 6.4 mm is diameter is initially at a uniform temperature of 50^0C and is suddenly immersed in a liquid at 200^0C with h=120 W/m^2.K. Using the lumped-capacity method of analysis, calculate the time necessary for the rod temperature to reach 120^0C. Also calculate the amount of heat transferred to the steel rod during the above time interval. [CO2] [BL3]
6. A 40×40 cm copper slab 5mm thick is at a uniform temperature of 250^0C. Suddenly its surface temperature is lowered to 30^0C. Find the time at which the slab temperature becomes 90^0C, ρ = 9000 kg/m^3, Cp = 0.38 kJ/kg.k, K =370 W/mk and h = 90 W/m^2k. [CO2] [BL3]
7. A large slab of aluminium at a uniform temperature of 200^0C is suddenly exposed to a convective surface environment of 70^0C with a heat transfer coefficient of 525 W/m^2k. Estimate the time required at a point 4cm from the surface to come up to a temperature level of 120^0C. (Take k = 215 W/m-K, $\alpha = 8.4 \times 10{-}5\ m^2/s$).[CO2] [BL3]
8. An aluminium alloy plate of 400mm × 400mm × 4mm size at 200^0C is suddenly quenched into liquid oxygen at -183^0C. Determine the time required for the plate to reach a temperature of -70^0C. Assume h = 20000kJ/m^2-hr-K, Cp =0.8 kJ/kg-K and ρ = 3000 kg/m^3. Take K for aluminium as 770.4 kJ/m-hr-K. [CO2] [BL3]
9. A slab of Aluminium 10cm thick is originally at a temperature of 500^0C. It is suddenly immersed in a liquid at 100^0C resulting it a heat transfer coefficient of 1200 W/m^2-K. Determine the temperature at the centreline and the surface1 min after the immersion. Also, the total thermal energy removal per unit area of slab during this period. The properties of aluminum for the given condition are: $\alpha = 8.4\times10^{-5} m^2$ /s, k=215 W/m-K, ρ = 2700 kg/m^3, c_p= 0.9 kJ/kg. [CO2] [BL4]
10. A long 20-cm-diameter cylindrical shaft made of stainless steel 304 comes out of an oven at a uniform temperature of 600°C. The shaft is then allowed to cool slowly in an environment chamber at 200°C with an average heat transfer coefficient of h = 80 W/m2·K. Determine the temperature at the centre of the shaft 45 min after the start of the cooling process. Also, determine the heat transfer per unit length of the shaft during this time period. [CO2] [BL4]

11. A steel ingot (large in size) heated uniformly to 745^{0}C is hardened by quenching it in an oil bath maintained at 20^{0}C. Determine the length of time required for temperature to reach 595^{0}C at a depth of 12mm. The ingot may be approximated as a flat plate. For steel ingot take, = 1.2×10^{-5} m^2/s. [CO2] [BL4]
12. A long steel cylinder 12.0 cm in diameter and initially at 20^{0}C is placed into a furnace at 820^{0}C with the local heat transfer coefficient h = 140 W/m^2 K. Calculate the time required for the axis temperature to reach 800^{0}C. Also calculate the corresponding temperature at a radius of 5.4 cm at that time. The physical properties of steel are k =21 W/m K and $\alpha = 6.11\times 10^{-6}$ m^2/sec. [CO2] [BL4]

C. Questions at evaluating / creating level:

Steam in a heating system flow through tubes whose outer diameter is 5 cm and whose walls are maintained at a temperature of 180^{0}C. Circular aluminium alloy 2024-T6 fins (k=186 W/m.^{0}C) of outer diameter 6 cm and constant thickness 1 mm are attached to the tube as shown in figure 2.7. The space between the fins is 3 mm, and thus there are 250 fins per meter length of the tube. Heat is transferred to the surrounding air at T_∞=25^{0}C, with a heat transfer coefficient of 40 W/m^2.^{0}C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins. ***Answer: 2639W*** [CO2] [BL5]

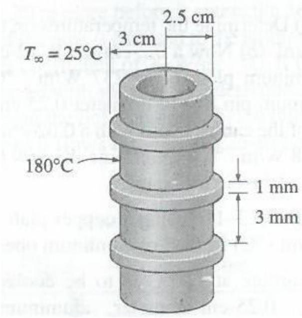


Figure 2.7

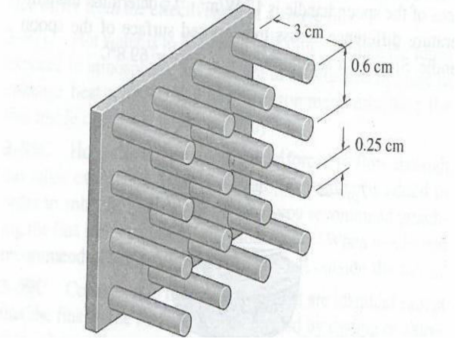


Figure 2.8

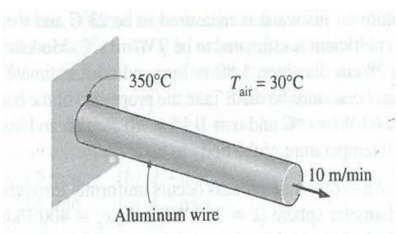


Figure 2.9

1. A hot surface at 100^{0}C is to be cooled by attaching 3-cm-long, 0.25-cm-diamter aluminum pin fins (k=237 W/m.^{0}C) to it, with a centre-to-centre distance of 0.6 cm as shown in figure 2.8. The temperature of the surrounding medium is 30^{0}C, and the heat transfer coefficient on the surface is 35 W/m^2.^{0}C. Determine the rate of heat transfer from the surface for a 1-m x 1-m section of the plate. Also determine the overall effectiveness of the fins. [CO2] [BL5]
2. Consider a sphere and cylinder of equal volume made of copper. Both sphere and cylinder are initially at the same temperature and are exposed to convection in the same environment. Which do you think will cool faster, the cylinder or the sphere? why? [CO2] [BL5]
3. Long aluminum wires of diameter 3 mm (ρ=2702 kg/m^3, c_p=0.896 kJ/kg.^{0}C, k=236 W/m.^{0}C, and α=9.75 x 10^{-5}m^2/s) as shown in figure 2.9 are extruded at a temperature of 350^{0}C and exposed to atmospheric air at 30^{0}C with a heat transfer coefficient of 35 W/m^2.^{0}C (a) Determine how long it will take for the wire temperature to drop to 50^{0}C. (b) If the wire is extruded at a velocity of 10 m/min, determine how far the wire travels after extrusion by the time its temperature drops to 50^{0}C. What change in the cooling process would you propose to shorten this distance? (c) Assuming the aluminum wire leaves the extrusion room at 50^{0}C, determine the rate of heat transfer from the wire to the extrusion room. [CO2] [BL5]

Answers: (a) 144 s, (b) 24m, (c) 856W

D. GATE Questions:

1. Lumped heat transfer analysis of a solid object suddenly exposed to a fluid medium at a different temperature is valid when. [GATE-2001]
   a) Biot number <0.1 b) Biot number > 0.1
   c) Fourier number < 0.1 d) Fourier number > 0.1
2. The value of Biot number is very small (less than 0.01) when [GATE-2002]
   (a) The convective resistance of the fluid is negligible
   (b) The conductive resistance of the fluid is negligible
   (c) The conductive resistance of the solid is negligible
   (d) None of these
3. 3. A fin has 5mm diameter and 100mm length. The thermal conductivity of fin material is 400Wm−1K−1. One end of the fin is maintained at 130ºC and its remaining surface is exposed to ambient air at 30ºC. if the convective heat transfer coefficient is 40Wm-2K-1, the heat loss (in W) from the fin is [GATE-2010]
   (A) 0.08 (B) 5.0 (C) 7.0 (D) 7.8
4. Pipe of 25mm outer diameter carries steam. The heat transfer coefficient between the cylinder and surroundings is 2 5W/m K . It is proposed to reduce the heat loss from the pipe by adding insulation having a thermal conductivity of 0.05W/mK. Which one of the following statements is TRUE? [GATE-2011]
   (A) The outer radius of the pipe is equal to the critical radius
   (B) The outer radius of the pipe is less than the critical radius
   (C) Adding the insulation will reduce the heat loss
   (D) Adding the insulation will increase the heat loss
5. Which one of the following configurations has the highest fin effectiveness? [GATE-2012]
   A) Thin, closely spaced fins B) Thin, widely spaced fins
   C) Thick widely spaced fins D) Thick, closely spaced fins
6. A steel ball of diameter 60 mm is initially in thermal equilibrium at 1030°C in a furnace. It is suddenly removed from the furnace and cooled in ambient air at 30°C , with convective heat transfer coefficient h=20 2 W /m K. The thermo-physical properties of steel are: density 3 r = 7800 kg /m , conductivity k=40 W/m-K and specific heat c=600 J/kgK. The time required in seconds to cool the steel ball in air from 1030°C to 430°C is [GATE-2013]
   (A) 519 (B) 931 (C) 1195 (D) 2144
7. Biot number signifies the ratio of [GATE-2014]
   (A) Convective resistance in the fluid to conductive resistance in the sold
   (B) Conductive resistance in the solid to convective resistance in the fluid
   (C) Inertia force to viscous force in the fluid
   (D) Buoyancy force to viscous force in the fluid
8. A 10 mm diameter electrical conductor is covered by an insulation of 2 mm thickness. The conductivity of the insulation is 0.08 W/m-k and the convection coefficient at the insulation surface is 10 W/m2-K. Addition of further insulation of the same material will [GATE-2015]
   A) Increase heat loss continuously.
   B) Decrease heat loss continuously.
   C) Increase heat loss to a maximum and then decrease heat loss.
   D) Decrease heat loss to a minimum and then increase heat loss.
9. A steel ball of 10 mm diameter at 1000 K is required to be cooled to 350 K by immersing it in a water environment at 300 K. The convective heat transfer

coefficient is 1000 W/m2-K. Thermal conductivity of steel is 40 W/m-K. The time constant for the cooling process is 16 s. The time required (in s) to reach the final temperature is __________ [GATE-2016]

10. The heat loss from a fin 6 W. the effectiveness and efficiency of the fin are 3 and 0.75 respectively. The heat loss (in W) from the fin keeping the entire fin surface at base temperature is __________ [GATE-2017]

11. A metal ball of diameter 0 mm is initially at 220^0C. The ball is suddenly cooled by an air jet of 20^0C. The hat transfer coefficient is 200 W/m^2K. The specific heat, thermal conductivity and density of the metal ball are 400 J/kg.K, 400 W/m.K and 9000 kg/m^3, respectively. The ball temperature (in ^{0}C) after 90 seconds will be approximately. [GATE-2017]

    A) 141 B) 163 C) 189 D) 210

E. IES / IAS Questions

1. On a heat transfer surface, fins are provided [IES-2010]
   (a) to increase temperature gradient so as to enhance heat transfer
   (b) to increase turbulence in flow for enhancing heat transfer.
   (c) to increase surface area to promote the rate of heat transfer.
   (d) to decrease the pressure drop of the fluid
2. The temperature distribution in a stainless fin (thermal conductivity 0.17 W/cm^{0}C) of constant cross-sectional area of 2 cm^2 and length of 1-cm, exposed to ambient of 40^0C (with surface heat transfer coefficient of 0.0025 W/cm^2-^{0}C) is given by $(T-T_\infty)$ =$3x^2$-5x+6, where T is in ^{0}C and x is in cm. if the base temperature is 100^0C, then the heat dissipated by the fin surface will be: [IES-1994]
   (a) 6.8 W (b) 3.4 W (c) 1.7 W (d) 0.17 W
3. The insulated tip temperature of a rectangular longitudinal fin having an excess (over ambient) root temperature of θ_0 is [IES-2004]
   (a) $\theta_0 \tan h(ml)$ (b) $\frac{\theta_0}{\sin h(ml)}$ (c) $\frac{\theta_0 \tan h(ml)}{(ml)}$ (d) $\frac{\theta}{\cos h(ml)}$
4. The efficiency of a pin fin with insulated tip is : [IES-2001]
   (a) $\frac{\tan hmL}{(hA/kP)^{0.5}}$ (b) $\frac{\tan hmL}{mL}$ (c) $\frac{mL}{\tan hmL}$ (d) $\frac{(hA/kP)^{0.5}}{\tan hmL}$
5. A fin of length l protrudes from a surface held at temperature T_0; it being higher than the ambient temperature T_a. The heat dissipation from the free end of the fin is stated to be negligibly small, what is the temperature gradient $\left(\frac{dT}{dx}\right)_{x=1}$ at the tip of the fin? [IES-2008]
   (a) Zero (b) $\frac{T_0-T_1}{l}$ (c) $h(T_0-T_a)$ (d) $\frac{T_l-T_a}{T_0-T_a}$
6. Which one of the following is correct? [IES-2008]
   The effectiveness of a fin will be a maximum in an environment with
   (a) Free convection (b) Forced convection

(c) Radiation (d) Convection and radiation

7. Which one of the following is correct? [IES-2008]

Fins are used to increase the heat transfer from a surface by

(a) Increasing the temperature difference
(b) Increasing the effective surface area
(c) Increasing the convective heat transfer coefficient
(d) None of the above

8. Fins are made as thin as possible to: [IES-2010]

(a) Reduce the total weight
(b) Accommodate more number of fins
(c) Increase the width for the same profile area
(d) Improve flow of coolant around the fin

9. In order to achieve maximum heat dissipation, the fin should be designed in such a way that : [IES-2005]

(a) It should have maximum lateral surface at the root side of the fin
(b) It should have maximum lateral surface towards the tip side of the fin
(c) It should have maximum lateral surface near the centre of the fin
(d) It should have minimum lateral surface near the centre of the fin

10. A finned surface consists of root or base area of 1 m^2 and fin surface area of 2 m^2. The average heat transfer coefficient for finned surface is 20 W/m^2K. Effectiveness of fins provided is 0.75. If finned surface with root or base temperature of 50^0C is transferring heat to a fluid at 30^0C, then rate of heat transfer is: [IES-2003]

(a) 400 W (b) 800 W (c) 1000 W (d) 1200 W

11. Consider the following statement pertaining to large heat transfer rate using fins: [IES-2002]

1. Fins should be used on the side where hat transfer coefficient is small
2. Long and thick fins should be used
3. Short and thin fins should be used
4. Thermal conductivity of fin material should be large

Which of the above statements are correct?

(a) 1, 2 and 3 (b) 1, 2 and 4 (c)2, 3 and 4 (d) 1, 3 and 4

12. Assertion (A): in a liquid-to-gas heat exchanger fins are provided on the gas side.

Reason (R): The gas offers less thermal resistance than liquid [IES-2002]

(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

13. Extended surfaces are used to increase the rate of heat transfer. When the convection heat transfer coefficient h=mk, the addition of extended surface will: [IES-2010]

(a) increase the rate of heat transfer
(b) decrease the rate of heat transfer
(c) not increase the rate of heat transfer
(d) increase the rate of heat transfer when the length of the fin is very large

14. A metallic rod of uniform diameter and length L connects two heat sources each at 500^0C. The atmospheric temperature is 30^0C. The temperature gradient $\frac{dT}{dL}$ at the centre of the bar will be: [IAS-2001]

    (a) $\frac{500}{L/2}$ (b) $-\frac{500}{L/2}$ (c) $-\frac{470}{L/2}$ (d) zero

15. Assertion (A): Lumped capacity analysis of unsteady heat conduction assumes a constant uniform temperature throughout a solid body.

    Reason (R): The surface convection resistance is very large compared with the internal conduction resistance. [IES-2010]

16. Which one of the following statements is correct? [IES-2004]

    The curve for unsteady state cooling or heating of bodies

    (a) Parabolic curve asymptotic to time axis
    (b) Exponential curve asymptotic to time axis
    (c) Exponential curve asymptotic both to time and temperature axis
    (d) Hyperbolic curve asymptotic both to time and temperature axis

17. Assertion (A): In lumped heat capacity systems, the temperature gradient within the system is negligible [IES-2004]

    Reason (R): In analysis of lumped capacity systems, the thermal conductivity of the system material is considered very high irrespective of the size of the system

    (a) Both A and R are individually true and R is the correct explanation of A
    (b) Both A and R are individually true but R is not the correct explanation of A
    (c) A is true but R is false
    (d) A is false but R is true

18. A solid copper ball of mass 500 grams, when quenched in a water bath at 30^0C, cools from 530^0C to 430^0C in 10 seconds. What will be the temperature of the ball after the next 10 seconds? [IES-1997]

    (a) 300^0C (b) 320^0C (c) 350^0C

    (b) (d) Not determinable for want of sufficient date

19. Heisler charts are used to determine transient heat flow rate and temperature distribution when: [IES-2005]

    (a) Solids possess infinitely large thermal conductivity
    (b) Internal conduction resistance is small and convective resistance is large
    (c) Internal conduction resistance is large and the convective resistance is small
    (d) Both conduction and convection resistance are almost of equal significance

Prepared by: Dr.P.Nageswara Reddy

Academic Year: 2019 – 20 **Semester: II**

Class : III B.Tech **Subject: Heat Transfer**

Learning Material

UNIT-III

Convective Heat Transfer and Forced Convection

Syllabus
Convective Heat Transfer: Classification of Convective Heat Transfer, Dimensional analysis as a tool for experimental investigation – Buckingham Pi Theorem for forced and Natural convection, application for developing semi – empirical non dimensional correlation for convection heat transfer, Significance of non-dimensional numbers. Concepts of Continuity, Momentum and Energy Equations.

Forced Convection: Concept of hydrodynamic and thermal boundary layer, use of empirical correlations for forced convection heat transfer Internal flows and External flows.

3.1 PHYSICAL MECHANISM OF CONVECTION

- Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it.
- The fluid motion enhance heat transfer, since it brings warmer and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in a fluid.
- The rate of heat transfer through a fluid is much higher by convection than it is by conduction. If fact, the higher the fluid velocity, the higher the rate of heat transfer.

Experience shown that convection heat transfer strongly depends on the fluid properties: dynamic viscosity μ, thermal conductivity k, density p, and specific heat c_p, as well as the fluid velocity V. It also depends on the geometry and the roughness of the solid surface, in addition to the type of fluid flow, laminar or turbulent.

The rate of convection heat transfer is observed to be proportional to the temperature difference and is conveniently expressed by Newton's law of cooling as

$$Q_{conv} = hA_s(T_s - T_\infty) \qquad (3.1)$$

Where

h=convection heat transfer coefficient, $W/m^{2.0}C$
A_s=heat transfer surface area, m^2
T_s=temperature of the surface, ^0C
T_∞ = temperature of the fluid sufficiently far from the surface, ^0C

The convection heat transfer coefficient h can be defined as the rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference.

Heat transfer from the solid surface to the fluid layer adjacent to the surface is by pure conduction, since the fluid layer is motionless, and can be expressed as

$$q_{conv} = q_{cond} = -k_{fluid} \left.\frac{\partial T}{\partial y}\right|_{y=0} \quad (W/m^2) \qquad (3.2)$$

Where T represents the temperature distribution in the fluid and $(\partial T / \partial y)_{y=0}$ is the temperature gradient at the surface. Heat is then convected away from the surface as a result of fluid motion.

Nusselt Number

It is also common practice to nondimensionalize the heat transfer coefficient h with the Nuselt number, defined as

$$Nu = \frac{hL_c}{k} \tag{3.3}$$

Where k is the thermal conductivity of the fluid L_c is the characteristic length.

- Nusselt number is viewed as the dimensionless convection heat transfer coefficient.
- It is defined as the ratio of heat transfer through the fluid layer by convection to the heat transfer by conduction when the fluid layer is motionless.

Heat flux (the rate of heat transfer per unit surface area) in either case is

$$q_{conv} = h\Delta T \quad \text{and}$$

$$q_{conv} = k\frac{\Delta T}{L}$$

Taking their ratio gives

$$Nu = \frac{q_{conv}}{q_{cond}} = \frac{h\Delta L}{k\Delta T / L} = \frac{hL}{k} \tag{3.4}$$

- Therefore, the Nusselt number represents the enhancement of the transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer.
- The larger the Nusselt number, the more effective the convection.
- A Nusselt number of Nu=1 for a fluid layer represents heat transfer across the layer by pure conduction.

3.2 CLASSIFICATION OF FLUID FLOWS

a) Viscous versus inviscid regions of flow.

Flows in which the frictional effects are significant are called viscous flows. However, in many flows of practical interest, there are negligible small compared to inertial or pressure forces. Neglecting the viscous terms in such in viscid flow regions greatly simplifies the analysis without much loss in accuracy.

b) Internal Versus External Flow

A fluid flow is classified as being internal or external, depending on whether the fluid is forced to flow in a confined channel or over a surface.

- The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is external flow. Air flow over a ball or over an exposed pipe during a windy day is an example of external flow.
- The flow in pipe or duct is internal flow if the fluid is completely bounded by solid surfaces. Water flow in a pipe is an example of internal flow.

c) Laminar Versus Turbulent Flow

The highly ordered fluid motion characterized by smooth layers of fluid is called laminar.

The highly disordered fluid motion that typically occurs at high velocities and is characterized by velocity fluctuations is called turbulent.

d) Natural (or Unformed) versus Forced Flow
A fluid flow is said to be natural or forced, depending on how the fluid motion is initiated.
- In forced flow, a fluid is forced to flow over a surface or in a pipe by external means such as a pump of a fan.
- In natural flows, any fluid motion is due to natural means such as the buoyancy effect due to density difference caused by temperature difference.

e) Steady versus Unsteady Flow
The term steady implies no change at a point with time. The opposite of steady is unsteady. The term uniform implies no change with location over a specified region.

3.3 DIMENSIONAL ANALYSIS
Dimensional analysis is a method by which we deduce information about a phenomenon from the single premise that the phenomenon can be described by a dimensionally correct equation among certain variables.

The method can be applied to all types of fluid resistances, heat flow problems and many other problems in fluid mechanics and thermodynamics.
The equation produced by this method also gives non-dimensional constants which govern the problem under consideration.

Buckingham- π -Theorem
Buckingham- π -theorem states that if a physical phenomenon involves n variables and if these variables can be expressed in terms of m fundamental dimensions then the physical law describing the phenomenon can be expressed as a function of (n-m) independent dimensionless products of variables called π terms.

3.3.1 Dimensional Analysis Applied to Forced Convection
The convective heat transfer coefficient in forced convection can be expressed as

$$h = f(\rho, L, \mu, u_\infty, C_p, k)$$

$$f(h, \rho, L, \mu, u_\infty, C_p, k) = 0 \qquad (3.5)$$

This problem is controlled by 7 physical quantities containing 4 fundamental dimensions, so according to Buckingham theorem, 3 dimensionless variables, π_1, π_2, π_3, can be formed as given below.

$$\therefore \pi_1 = h(\rho)^a (L)^b (\mu)^c (k)^d = M^0 L^0 t^0 T^0 \qquad (3.6)$$

$$\therefore (Mt^3 T^{-1})(ML^{-3})^a (L)^b (ML^{-1}t^{-1})^c (MLt^{-3}T^{-1})^d = M^0 L^0 t^0 T^0 \qquad (3.7)$$

Equating the power of M, L, t and T of both sides, we get the following equations:

$$a + c + d + 1 = 0$$
$$-3a + b - c + d = 0$$
$$-c - 3d - 3 = 0$$
$$-d - 1 = 0$$

Solving the above equations, we get the following values
$a = 0, b = 1, c = 0, d = -1$

$$\pi_1 = \frac{hL}{k} = \text{Nusselt number (Nu)} \qquad (3.8)$$

Similarly $\pi_2 = C_p (\rho)^{a1} (L)^{b1} (\mu)^{c1} (k)^{d1} = M^0 L^0 t^0 T^0$ (3.9)

$$\therefore (L^2 t^{-2} T^{-1})(ML^{-3})^{a1} (L)^{b1} \left(ML^{-1}t^{-1}\right)^{c1} \left(MLt^{-3}T^{-1}\right)^{d1} = M^0 L^0 t^0 T^0 \qquad (3.10)$$

Equating the powers of M, L, t and T of both sides, we get the following equations:

$$a_1 + c_1 + d_1 = 0$$

$$-3a_1 + b_1 - c_1 + d_1 + 2 = 0$$
$$-c_1 - 3d_1 - 2 = 0$$
$$-d_1 - 1 = 0$$

Solving the above equations, we get eh following values:

$$a_1 = 0, \quad b_1 = 0, \quad c_1 = 1, \quad d_1 = -1$$

$$\pi_2 = \frac{\mu C_p}{k} = \text{Prandtl number (Pr)} \tag{3.11}$$

Similarly $\pi_3 = u_\infty (\rho)^{a2} (L)^{b2} (\mu)^{c2} (K)^{d2} = M^0 L^0 t^0 T^0$ (3.12)

$$\therefore (Lt^{-1})(ML^{-3})^{a2} (L)^{b2} \left(ML^{-1}t^{-1}\right)^{c2} \left(MLt^{-3}T^{-1}\right)^{d2} = M^0 L^0 t^0 T^0 \tag{3.13}$$

Equating the powers of M, L, t and T of both sides, we get the following equations:

$$a_2 + c_2 + d_2 = 0$$
$$1 - 3a_2 + b_2 - c_2 + d_2 = 0$$
$$-1 - c_2 - 3d_2 = 0$$
$$-d_2 = 0$$

Solving the above equations, we get the following values

$$a_2 = 1, b_2 = 1, c_2 = -1, d_2 = 0$$

$$\pi_3 = \frac{\rho L u_\infty}{\mu} = \text{Reynolds number (Re)} \tag{3.14}$$

According to Buckingham theorem

$$\therefore f\left(\pi_1 \pi_2 \pi_3\right) = 0 \tag{3.15}$$

$$\pi_3 = f(\pi_2, \pi_3) \tag{3.16}$$

$$Nu = C(\text{Re})^m (\text{Pr})^n \tag{3.17}$$

Where C, n and m are constants and Nu, Re and Pr are known as Nesselt, Reynold and Prandtl numbers respectively.

The constants are calculated with the help of experiments.

The value of C, m and n are co-related for different flow conditions and different geometric configurations. A few of them are listed below.

(i) Flow of fluid over a flat surface at constant temperature.

$$Nu = 0.664\, Re^{0.5} Pr^{1/3}$$

This equation is valid for laminar flow over the flat plate for which, the required condition is $Re < 5x10^5$

If the flow condition on the flat plate is partly laminar and partly turbulent then the heat flow for the turbulent region is given by the following equation:

$$Nu = 0.036(\text{Re})^{0.8} (\text{Pr})^{1/3}$$

This equation is valid for $2X10^5 < \text{Re} < 10^7$.

(ii) Fluid is flowing inside the tube or through the annulus

$$Nu = 0.023(\text{Re})^{0.8} (\text{Pr})^{0.4}$$

This equation is valid for $2300 < \text{Re} < 1X10^4$ and $0.7 < \text{Pr} < 120,\ \ and\ \frac{L}{D} < 60.$

3.3.2 Dimensional Analysis Applied to Natural or Free Convection

Natural convection is caused due to buoyancy force which is the effect of the decrease in density due to heating.

The net force causing the upward flow is given as $\beta g\left(\Delta T\right)$.

The convective heat transfer coefficient in natural convection can be expressed as

$$h = f(\rho, L, \mu, C_p, k, \beta g \Delta T) \tag{3.18}$$

Where, $\beta g\left(\Delta\theta\right)$ will be considered as one physical factor which is responsible for causing flow.

We can write down the above equation as follows.

$$f[\rho, L, \mu, k, h, C_p, (\beta g \Delta T)] = 0 \tag{3.19}$$

This problem is also controlled by 7 physical quantities containing 4 fundamental dimensions, so according to Buckingham theorem, this problem can be controlled by (7-4) =3 non-dimensional groups, π_1, π_2, π_3, as given below.

$$\therefore \pi_1 = h\left(\rho\right)^a\left(L\right)^b\left(\mu\right)^c\left(k\right)^d = M^0L^0t^0T^0 \tag{3.20}$$

$$\therefore (Mt^3T^{-1})(ML^{-3})^a(L)^b(ML^{-1}t^{-1})^c(MLt^{-3}T^{-1})^d = M^0L^0t^0T^0 \tag{3.21}$$

Equating the power of M, L, t and T of both sides, we get the following equations:

$$a+c+d+1=0$$
$$-3a+b-c+d=0$$
$$-c-3d-3=0$$
$$-d-1=0$$

Solving the above equations, we get the following values

$a=0, b=1, c=0, d=-1$

$$\pi_1 = \frac{hL}{k} = \text{Nusselt number (Nu)} \tag{3.22}$$

Similarly $\pi_2 = C_p\left(\rho\right)^{a1}\left(L\right)^{b1}\left(\mu\right)^{c1}\left(k\right)^{d1} = M^0L^0t^0T^0$ (3.23)

$$\therefore (L^2t^{-2}T^{-1})(ML^{-3})^{a1}(L)^{b1}\left(ML^{-1}t^{-1}\right)^{c1}\left(MLt^{-3}T^{-1}\right)^{d1} = M^0L^0t^0T^0 \tag{3.24}$$

Equating the powers of M, L, t and T of both sides, we get the following equations:

$$a_1+c_1+d_1=0$$
$$-3a_1+b_1-c_1+d_1+2=0$$
$$-c_1-3d_1-2=0$$
$$-d_1-1=0$$

Solving the above equations, we get eh following values:

$a_1=0,\quad b_1=0,\quad c_1=1,\quad d_1=-1$

$$\pi_2 = \frac{\mu C_p}{k} = \text{Prandtl number (Pr)} \tag{3.25}$$

Similarly $\pi_3 = (\beta g.\Delta T)\left(\rho\right)^{a2}\left(L\right)^{b2}\left(\mu\right)^{c2}\left(k\right)^{d2} = M^0L^0t^0T^0$ (3.26)

$$\therefore \left(Lt^{-2}\right)(ML^{-3})^{a2}(L)^{b2}(ML^{-1}t^{-1})^{c2}\left(MLt^{-3}T^{-1}\right)^{d2} = M^0L^0t^0T^0 \tag{3.27}$$

Equating the powers of M, L, t and T both sides, we get the following equations

$$a_2+c_2+d_2=0$$
$$-3a_2+b_2-c_2+d_2+1=0$$
$$-c_2-3d_2-2=0$$
$$-d_2=0$$

Solving the above equations, we get the following values.

$$a_2 = 2, \quad b_2 = 3, \quad c_2 = -2 \; and \quad d_2 = 0$$

$$\pi_3 = \frac{(\beta g.\Delta T)\rho^2 L^3}{\mu^2} = \frac{(\beta g.\Delta T)L^3}{v^2}$$ = Grassoff-Number (Gr)

According to Buckingham π theorem

Solving the above equations, we get the following values.

$$\pi_1 = f(\pi_2.\pi_3)$$

$$Nu = C(\mathrm{Pr})^n.(Gr)^m \qquad (3.28)$$

$$\frac{hL}{K} = C\left(\frac{\mu C_p}{K}\right)^n \left(\frac{\beta g.\Delta T}{v^2}\right)^m \qquad (3.29)$$

Where C, n and m are the constant.

The following few correlations for different flow conditions and different geometric configurations are given by the following equations:

(i) The heat transfer coefficient on vertical plate or vertical cylinder is given by the following equation.

$$Nu = 0.56(Gr.\mathrm{Pr})^{0.25}, when\, 10^5 < Gr\,\mathrm{Pr} < 10^8$$

The characteristic length is the height of the plate of cylinder.

(ii) The heat transfer coefficient for horizontal pipe is given by the following equation.

$$Nu = 0.47(Gr.\mathrm{Pr})^{0.25}, when\, 10^5 < Gr\,\mathrm{Pr} < 10^8$$

The characteristic length is a diameter of the pipe.

(iii) Vertical cylinder of plate

$$Nu = 0.13(Gr.\mathrm{Pr})^{0.33}$$

(iv) Horizontal cylinder

$$Nu = 0.10(Gr.\mathrm{Pr})^{0.33}$$

3.3.3 Limitations of Dimensional Analysis

- The most serious lamination of dimensional analysis is that it gives no information about the nature of the phenomenon.
- A complete solution is not obtained nor is the inner mechanism of a phenomenon revealed by dimensional analysis.

3.4 LAMINAR BOUNDARY LAYER ON A FLAT PLATE

3.4.1 Momentum equation

We derive the equation of motion for the boundary layer by making a force-and-momentum balance on an elemental control volume shown in figure 3.1. To simplify the analysis we assume:

1. The fluid is incompressible and the flow is steady.
2. There are no pressure variations in the direction perpendicular to the plate.
3. The viscosity is constant.
4. Viscous-shear forces in the y direction are negligible.

We apply Newton's second law of motion.

$\sum F_x$ =increase in momentum flux in x direction

$$\sum F_x = \frac{d(mV)_x}{d\tau} \qquad (3.30)$$

The above form of Newton’s second law of motion applies to a system of constant mass. For this system the force balance is then written as

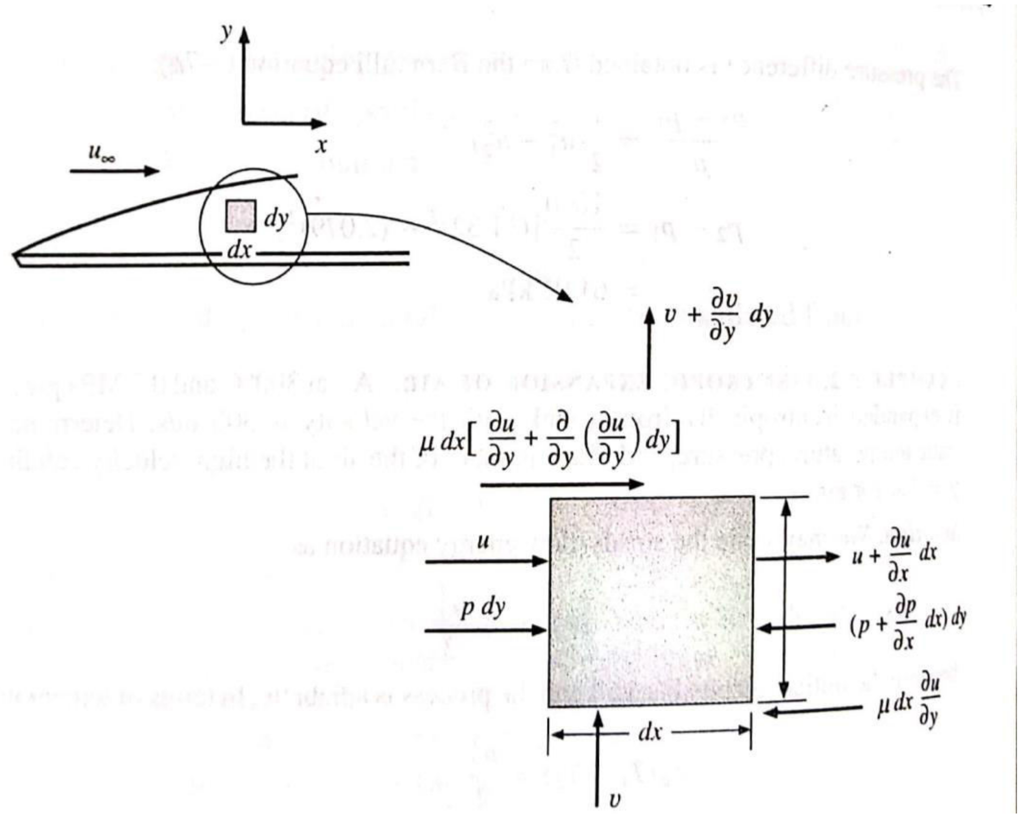


Figure 3.1: Element control volume for force balance on Laminar boundary layer

Figure 3.2 Elemental volume for force and momentum balance of laminar boundary layer. The momentum flux in the x direction is the product of the mass flow through a particular side of the control volume and the x component of velocity at that point. The mass entering the left face of the element per unit time is

$\rho u\, dy$

If we assume unit depth in z direction, thus the momentum entering the left face per unit time is

$$\rho u\, dy u = \rho u^2 dy \tag{3.31}$$

The mass flow leaving the right face is

$$\rho\left(u + \frac{\partial u}{\partial x} dx\right) dy \tag{3.32}$$

And the momentum leaving the right face is

$$\rho\left(u + \frac{\partial u}{\partial x} dx\right)^2 dy \tag{3.33}$$

The mass flow entering the bottom face is

$$\rho u\, dx$$

And the mass flow leaving the top face is

$$\rho\left(\upsilon + \frac{\partial \upsilon}{\partial x} dy\right) dx \tag{3.34}$$

A mass balance on the element yields

$$\rho u\, dy + \rho \upsilon\, dx = \rho\left(u + \frac{\partial u}{\partial x} dx\right) dy + \rho\left(\upsilon + \frac{\partial \upsilon}{\partial y} dy\right) dx \tag{3.35}$$

Or $$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{3.36}$$

This is the mass continuity equation for the boundary layer.

Returning to the momentum-and-force analysis, the momentum in the x direction which enters the bottom face is

$\rho uv\, dx$

And the momentum in the x direction which leaves the top face is

$$\rho\left(u+\frac{\partial u}{\partial y}dy\right)dy+\rho\left(\upsilon+\frac{\partial \upsilon}{\partial y}dy\right)dx \tag{3.37}$$

We are interested only in the momentum in the x direction because the forces considered in the analysis are those in the x direction. These forces are those due to viscous shear and the pressure forces on the element. The pressure force on the left face is ρdy, and that on the right is $-\left[p+(\partial p/\partial x)dx\right]dy$, so that the net pressure force in the direction of motion is

$$-\frac{\partial p}{\partial x}dx\,dy \tag{3.38}$$

The viscous-shear force on the bottom face is

$$-\mu\frac{\partial u}{\partial y}dx \tag{3.39}$$

And the shear force on the top is

$$\mu dx\left[\frac{\partial u}{\partial y}+\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)dy\right] \tag{3.40}$$

The net viscous-shear force in the direction of motion is the sum of the above:

Net viscous-shear force= $\mu\frac{\partial^2 u}{\partial y^2}dx\,dy$ (3.41)

Equating the sum of the viscous-shear and pressure forces to the net momentum transfer in the x direction, we have

$$\mu\frac{\partial^2 u}{\partial y^2}dx\,dy-\frac{\partial p}{\partial x}dx\,dy=\rho\left(u+\frac{\partial u}{\partial x}dx\right)^2dy-\rho u^2dy+\rho\left(v+\frac{\partial v}{\partial y}dy\right)\left(u+\frac{\partial u}{\partial y}dy\right)dx-\rho vu\,dx \tag{3.42}$$

Clearing terms, making use of the continuity relation (5-12) and neglecting second order differentials, gives

$$\rho\left(u+\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial p}{\partial x} \tag{3.43}$$

This is the momentum equation of the laminar boundary layer with constant properties

3.4.2 Energy Equation

Consider the elemental control volume shown in figure: 3.2. To simplify the analysis, we assume.

1. Incompressible steady flow
2. Constant viscosity, thermal conductivity, and specific heat
3. Negligible heat conduction in direction of flow (x direction)

The energy balance may be written as

Energy convected in left face + energy convected in bottom face

+ heat conducted in bottom face + net viscous work done on element.

=energy covected out right face + energy covected out top face + heat conducted out top face

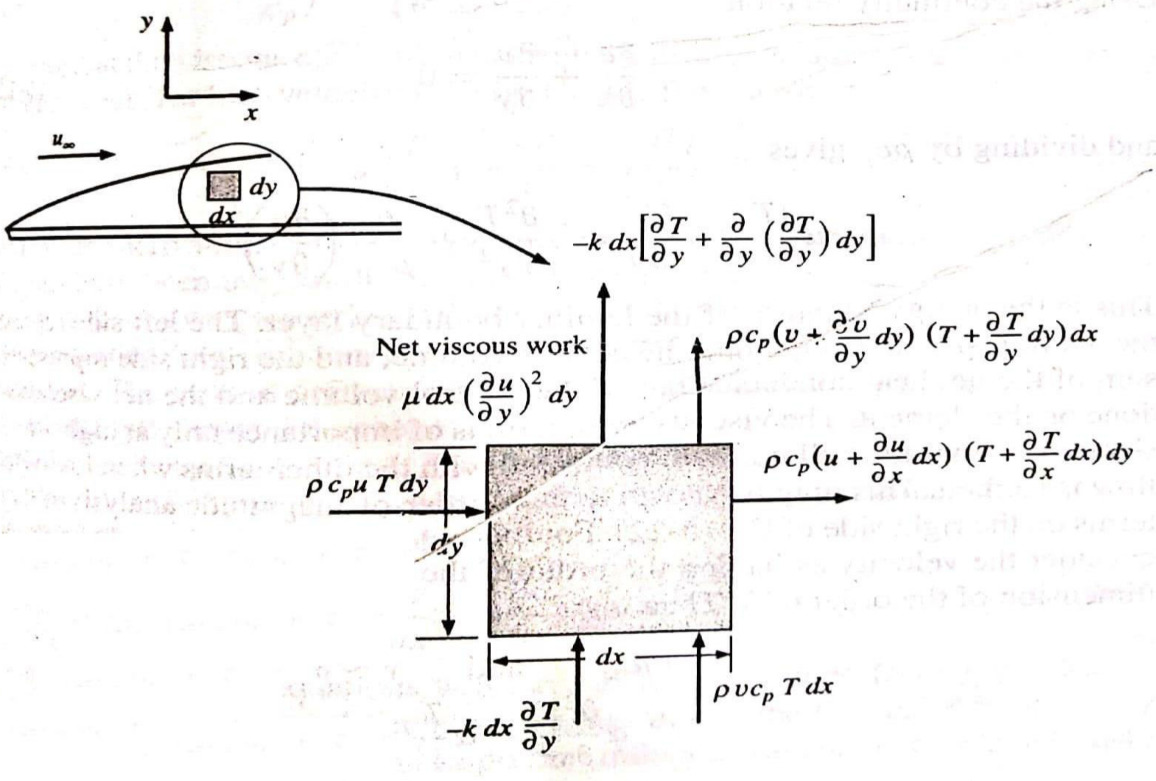


Figure 3.2: Elemental control volume for energy analysis of laminar boundary layer.

The convective and conduction energy quantities are indicated in Figure 3.2.

The viscous work may be computed as a product of the net viscous-shear force and the distance this force moves in unit time. The viscous-shear force is the product of the shear-stress and the area dx.

$$\mu\frac{\partial u}{\partial y}dx \tag{3.44}$$

And the distance through which it moves per unit time in respect to the elemental control volume dx dy is

$$\frac{\partial u}{\partial y}dx \tag{3.45}$$

So that the net viscous energy delivered to the element is

$$\mu\left(\frac{\partial u}{\partial y}\right)^2 dx\ dy \tag{3.46}$$

Writing the energy balance corresponding to the quantities shown in Figure. 3.2 by asumming unit depth in the z direction, and neglecting second-order differentials yields.

$$\rho c_p\left[u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+T\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]dx\ dy = k\frac{\partial^2 T}{\partial y^2}dx\,dy = +\mu\left(\frac{\partial u}{\partial y}\right)^2 dx\,dy \tag{3.47}$$

Using the continuity relation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3.48}$$

And dividing by ρc_p gives

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (3.49)$$

This is the energy equation of the laminar boundary layer.
For low-velocity incompressible flow, we have

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \qquad (3.50)$$

There is a striking similarity between Eq.(5.25) and the momentum equation for constant pressure,

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \qquad (3.51)$$

The solution to the two equations will have exactly the same form when $\alpha = \upsilon$.

3.5 FORCED CONVECTION

3.5.1 Velocity Boundary Layer

Consider the parallel flow of a fluid over a flat plate, as shown in Figure 3.3. The x-coordinate is measured along the plate surface form the leading edge of the plate in the direction of the flow, and y is measured form the surface in the normal direction. The fluid approaches the plate in the x-direction with a uniform velocity, u_∞ which is practically identical to the free-steam velocity over the plate away from the surface.

The velocity of the particles in the first fluid layer adjacent to the plate becomes zero because of the no-slip condition. This motionless layer slows down the particles of these two adjoining fluid layers at different velocities. As a result, the x-component of the fluid velocity, u, varies from 0 at y=0 to nearly u_∞ at y=δ.

The region of the flow above the plate bounded by δ in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the velocity boundary layer. The boundary layer thickness, δ_v, is typically defined as the distance y from the surface at which u=0.99u_∞.

Friction force per unit area is called shear stress, and is denoted by τ_s. Experimental studies indicate that the shear stress for most fluids is proportional to the velocity gradient, and the shear stress at the wall surface is expressed as

$$\tau_s = \mu\left.\frac{\partial u}{\partial y}\right|_{y=0} \quad (\mathrm{N/m^2}) \qquad (3.52)$$

Where the constant of proportionally μ is the dynamic viscosity of the fluid, whose unit is kg/m.s (or equivalently, N.s/m^2, Pa.s, or poise=0.1 Pa.s). The fluids that obey the linear relationship above are called **Newtonian fluids.**

In fluid flow and heat transfer studies, the ratio of dynamic viscosity to density appears frequently. This ratio is give the name kinematic viscosity ν and is expressed as $\nu = \mu / \rho$. Two common units of kinematic viscosity are m^2/s and stoke (1 stoke =1 cm^2/s=0.0001 m^2/s).

The viscosity of a fluid is a measure of its resistance to deformation, and it is a strong function of temperature. The viscosities of liquids decrease with temperature, whereas the viscosities of gases increase with temperature.

3.5.2 Thermal Boundary Layer

A thermal boundary layer develops when a fluid at a specified temperature flows over a surface that is at a different temperature

Consider the flow of a fluid at a uniform temperature of T_∞ over an isothermal flat plate at temperature T_s. the fluid particles in the layer adjacent to the surface reach thermal equilibrium with the plate and assume the surface temperature T_s. These fluid particles then exchange energy with the particles in the adjoining-fluid layer, and so on. As a result, a temperature profile develops in the flow field that ranges from T_s at the surface to T_∞ sufficiently far from the surface.

The flow region over the surface in which the temperature variation in the direction normal to the surface is significant is the thermal boundary layer. The thickness of the thermal boundary layer δ_t at any location along the surface is defined as the distance from the surface at which the temperature difference T-T_s equals 0.99 $\left(T_\infty - T_s\right)$.

Prandtl Number

The relative thickness of the velocity and the thermal boundary layers is best described by the dimensionless parameter Prandtl number, defined as

$$\Pr = \frac{Molecular\ diffusivity\ of\ momentum}{Molecular\ diffusivity\ of\ heat} = \frac{v}{\alpha} = \frac{\mu c_p}{k} \qquad (3.53)$$

- The Prandtl numbers of fluids range from less than 0.01 for liquid metals to more than 100,000 for heavy oils.
- The Prandtl number is in the order of 10 for water.
- The Prandtl numbers of gases are about 1, which indicates that both momentum and heat dissipate through the fluid at about the same rate.
- Heat diffuses very quickly in liquid metals $\left(\Pr << 1\right)$ and very slowly in oils (Pr>>1) relative to momentum. Consequently the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layers.

3.5.3 Laminar and Turbulent Flows

- The flow regime in the first case is said to be laminar, characterized by smooth streamlines and highly-ordered motion, and turbulent in the second case where it is characterized by velocity fluctuations and highly-disordered motion.
- The transition from laminar to turbulent flow does not occur suddenly; rather, it occurs over some region in which the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent.
- Most flow encountered in practice are turbulent. Laminar flow is encounter when highly viscous fluids such as oils flow in small pipes or narrow passages.
- The velocity profile in turbulent flow is much fuller than that in laminar flow, with a sharp drop near the surface.
- The turbulent boundary layer can be considered to consist of four regions, characterized by the distance from the wall. The very thin layer next to the wall where viscous effects are dominant is the viscous sub-layer. The velocity profile in this layer is very nearly linear, and the flow is streamlined. Next to the viscous sub-layers is the buffer layer, in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects. Above the buffer layer is the overlap layer, in which the turbulent effects are much more significant, but still not dominant. Above that is the turbulent layer in which turbulent effects dominate over viscous effects.
- Both the friction and heat transfer coefficients reach maximum values when the flow becomes fully turbulent.

Reynolds Number

The transition from laminar to turbulent flow depends on the surface geometry, surface roughness, flow velocity, surface temperature, and type of fluid, among other things. Reynolds discovered that the flow regime depends mainly on the ratio of the inertia forces to viscous forces in the fluid. This ratio is called the Reynolds number, which is a dimensionless quantity, and is expressed for external flows.

$$\mathrm{Re} = \frac{Inertia\ forces}{Viscous\ forces} = \frac{u_\infty L_c}{\nu} = \frac{\rho u_\infty L_c}{\mu} \qquad (3.54)$$

Where V is the upstream velocity (equivalent to the free-stream velocity for a flat plate), L_c is the characteristic length of the geometry, and $\nu = \mu / \rho$ is the kinematic viscosity of the fluid. For a flat plate, the characteristic length is the distance x from the leading edge.

3.5.4 Parallel Flow over Flat Plates

Consider the parallel flow of a fluid over a flat plate of length L in the flow direction. The x-coordinate is measured along the plate surface from the leading edge in the direction of the flow. The fluid approaches the plate in the x-direction with a uniform velocity V and temperature T_∞. The flow in the velocity boundary layers starts out as laminar, but if the plate is sufficiently long, the flow becomes turbulent at a distance x_{cr} from the leading edge where the Reynolds number reaches its critical value for transition.

The Reynolds number at a distance x from the leading edge of a flat plate

$$\mathrm{Re}_x = \frac{\rho V x}{\mu} = \frac{V x}{\nu} \qquad (3.55)$$

Note that the value of the Reynolds number varies for a flat plate along the flow, reaching $\mathrm{Re}_L = u_\infty L/\nu$ at the end of the plate.

A generally accepted value for the critical Reynolds number is

$$\mathrm{Re}_{cr} = \frac{\rho u_\infty x_{cr}}{\mu} = 5\mathrm{X}10^5 \qquad (3.56)$$

Based on analysis, the boundary layer thickness and the local friction coefficient at location x for laminar flow over a flat plate were determined.

$$\text{Laminar: } \delta_{v,x} = \frac{5x}{\mathrm{Re}_x^{1/2}} \text{ and } C_{f,x} = \frac{0664}{\mathrm{Re}_x^{1/2}} \text{ for } \mathrm{Re}_x < 5\mathrm{X}10^5 \qquad (3.57)$$

The corresponding relations for turbulent flow are

$$\text{Turbulent: } \delta_{v,x} = \frac{0.38x}{\mathrm{Re}_x^{1/5}} \text{ and } C_{f,x} = \frac{0.059}{\mathrm{Re}_x^{1/5}} \text{ for } 5\mathrm{X}10^5 \le \mathrm{Re}_x\, 10^7 \qquad (3.58)$$

Where x is the distance from the leading edge of the plate and $\mathrm{Re}_x = u_\infty x/\nu$ is the Reynolds number at location x.

The relationship between velocity and thermal boundary layers in laminar region can be expressed as

$$\frac{\delta_{t,x}}{\delta_{v,x}} = \frac{1}{\mathrm{Pr}^{1/3}} \qquad (3.58a)$$

Constant Wall Temperature

The average friction coefficient over the entire plate is determined by substituting the relations above into and performing the integrations. We get

$$\text{Laminar: } C_f = \frac{1.33}{\mathrm{Re}_L^{1/2}} \text{ for } \mathrm{Re}_L < 5\mathrm{X}10^5 \qquad (3.59)$$

$$\text{Turbulent: } C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} \text{ for } 5\mathrm{X}10^5 \le \mathrm{Re}_L \le 10^7 \qquad (3.60)$$

The local Nusselt number at a location x for laminar flow over a flat plate was determined by solving the differential energy equation:

$$\text{Laminar: } Nu_x = \frac{h_x x}{k} 0.332\,\text{Re}_x^{0.5}\,\text{Pr}^{1/3} \text{ for Pr} > 0.6 \tag{3.61}$$

The corresponding relation for turbulent flow is

$$\text{Turbulent: } Nu_x = \frac{h_x x}{k} 0.0296\,\text{Re}_x^{0.8} \text{ for Pr}^{1/3} \quad 0.6 \le \text{Pr} \le 60 \; 5\text{X}10^5 \le \text{Re}_x \le 10^7 \tag{3.62}$$

The average Nusselt number over the entire plate is determined by performing the integrations. We get

$$\text{Laminar: } Nu = \frac{hL}{k} 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} \text{ for Re}_L < 5\text{X}10^5 \tag{3.63}$$

$$\text{Turbulent: } Nu = \frac{hL}{k} 0.037\,\text{Re}_L^{0.8} \text{for Pr}^{1/3} \quad 0.6 \le \text{Pr} \le 60 \; 5\text{X}10^5 \le \text{Re}_L \le 10^7 \tag{3.64}$$

In some cases, a flat plate is sufficiently long for the flow to become turbulent, but not long enough to disregard the laminar flow region. In such cases, the average heat transfer coefficient over the entire plate is determined as

$$h = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x,laminar}dx + \int_0^{x_{cr}} h_{x,turbulent}dx\right) \tag{3.65}$$

Again taking the critical Reynolds number to be $\text{Re}_{cr} = 5 \text{ X } 10^5$ and performing the integrations after substituting the indicated expressions, the average Nusselt number over the entire plate is determined to be

$$Nu = \frac{hL}{k}(0.037\,\text{Re}_L^{0.5} - 871)\,\text{Pr}^{1/3} \quad 0.6 \le \text{Pr} \le 60 \quad 5\text{X}10^5 \le \text{Re}_L \le 10^7 \tag{3.66}$$

Liquid metals such as mercury have high thermal conductivities. The local Nusselt number over the plate is given by

$$Nu_x = 0.565(\text{Re}_x\,\text{Pr})^{1/2} \qquad \text{Pr} < 0.05 \tag{3.67}$$

Constant Heat Flux

When a flat plate is subjected to uniform heat flux instead of uniform temperature, the local Nusselt number is given by

$$\text{Laminar:} \qquad Nu_x = 0.453\,\text{Re}_x^{0.5}\,\text{Pr}^{1/3} \tag{3.68}$$

$$\text{Turbulent:} \qquad Nu_x = 0.0308\,\text{Re}_x^{0.8}\,\text{Pr}^{1/3} \tag{3.69}$$

3.5.5 The Relation between Fluid Friction and Heat Transfer

The frictional resistance may be directly related to heat transfer as given below.

The shear stress at the wall may be expressed in terms of a friction coefficient C_f as

$$\tau_w = C_f \frac{\rho u_\infty^2}{2} \tag{3.70}$$

The shear stress may also be calculated from the relation

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg]_w \tag{3.71}$$

The velocity distribution equation is given by

$$\frac{u}{u_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \tag{3.72}$$

Using the velocity distribution given by Eq.(3-72), we have

$$\tau_w = \frac{3}{2}\frac{\mu u_\infty}{\delta} \tag{3.73}$$

And making use of the relation for the boundary-layer thickness gives

$$\tau_w = \frac{3}{2}\frac{\mu u_\infty}{4.64}\left(\frac{u_\infty}{\nu x}\right)^{\frac{1}{2}} \tag{3.74}$$

Combining Eqs. (3.69) and (3.73) leads to

$$\frac{Cfx}{2} = \frac{3}{2}\frac{\mu u_\infty}{4.64}\left(\frac{u_\infty}{\nu x}\right)^{1/2}\frac{1}{\rho u_\infty^2} = 0.323\,\mathrm{Re}_x^{-1/2} \tag{3.75}$$

The exact solution of the boundary-layer equations yields

$$\frac{Cfx}{2} = 0.323\,\mathrm{Re}_x^{-1/2} \tag{3.76}$$

Equation (3.74) may be rewritten in the following form:

$$\frac{Nu_x}{\mathrm{Re}_x \Pr} = \frac{h_x}{\rho c_p u_\infty} = 0.332\Pr^{-2/3}\mathrm{Re}_x^{-1/2} \tag{3.77}$$

The group on the left is called the Stanton number,

$$St_x = \frac{h_x}{\rho c_p u_\infty}$$

So that

$$St_x \Pr^{2/3} = 0.332\,\mathrm{Re}_x^{-1/2} \tag{3.78}$$

Upon comparing Eqs. (3.74) and (3.77), we get

$$St_x \Pr^{2/3} = \frac{Cf_x}{2} \tag{3.79}$$

Equation (3.78), called the Reynolds-Colburn analogy, expresses the relation between fluid friction and heat transfer for laminar flow on a flat plate. The heat-transfer coefficient thus could be determined by making measurements of the frictional drag on a plate under conditions in which no heat transfer is involved.

3.5.6 Flow Across Cylinders and Spheres

Flow across cylinders and sphere is frequently encountered in practice. For example, the tubes in a shell-and-tube heat exchanger involve both internal flow through the tubes and external flow over the tubes, and both flows must be considered in the analysis of the heat exchanger.

The characteristic length for a circular cylinder or sphere is taken to be the external diameter D. Thus, the Reynolds number is defined as $\mathrm{Re} = u_\infty D/\nu$.

The critical Reynolds for flow across a circular cylinder of sphere is about $\mathrm{Re}_{cr} \cong 2\mathrm{X}10^5$.

The average Nusselt number for flow across cylinders can be expressed compactly as

$$Nu_{cyl} = \frac{hD}{k} = C\ \mathrm{Re}^m\ \Pr^n \tag{3.80}$$

3.5.7 Flow across Tube Banks

Cross-flow over tube banks is commonly encountered in practice in heat transfer equipment such as the condensers and evaporators of power plants, refrigerators, and air conditioners. In such equipment, one fluid moves through the tubes while the other moves over the tubes in a perpendicular direction.

In heat exchanger that involves a tube bank, the tubes are usually placed in a shell.

The tubes in a tube bank are usually arranged either in-line or staggered in the direction of flow. The outer tube diameter D is taken as the characteristic length. The arrangement of the tubes in the tube bank is characterized by the transverse pitch S_T.

As the fluid enters the tube bank, the flow area decreases from $A_1=S_TL$ to $A_T=(S_T-D)L$ between the tubes, and thus flow velocity increases.
Therefore, the Reynolds number is defined on the basis of maximum velocity as

$$\mathrm{Re}_D = \frac{\rho u_{\infty,\max} D}{\mu} = \frac{u_{\infty,\max} D}{\nu} \quad (3.81)$$

Then the maximum velocity becomes $$u_{\infty,\max} = \frac{S_T}{S_T - D} u_\infty \quad (3.82)$$

Staggered and $S_D<(S_T+D)/2$: $$u_{\infty,\max} = \frac{S_T}{2(S_T - D)} u_\infty \quad (3.83)$$

3.5.8 Internal Forced Convection

Liquid or gas flow through pipes or ducts is commonly used in heating and cooling applications. The fluid in such application is forced to flow by a fan or pump through a flow section that is sufficiently long to accomplish the desired heat transfer.
Flow sections of circular cross section are referred to as pipes (especially when the fluid is a liquid), and flow sections of noncircular cross section as ducts (especially when the fluid is a gas). Small-diameter pipes are usually referred to as tubes.
The fluid properties in internal flow are usually evaluated at the bulk mean fluid temperature, which is the arithmetic average of the mean temperatures at the inlet and the exit. That is, $T_b=(T_{m,i}+T_{m,e})/2$.

For flow in a circular tube, the Reynolds number is defined as $\mathrm{Re} = \frac{\rho u_\infty D_i}{\mu} = \frac{u_\infty D_i}{\nu}$

Where u_∞ is the average flow velocity, D_i is the internal diameter of the tube, and $\nu = \mu / \rho$ is the kinematic viscosity of the fluid.
For flow through noncircular tubes, the Reynolds number as well as the Nusselt number, and the friction factor are based on the hydraulic diameter D_h defined as $D_h = \frac{4A_c}{p}$

Where A_c, is the cross sectional area of the true and p is its perimeter. The hydraulic diameter is defined such that it reduces to ordinary diameter D for circular tubes since

Circular tubes: $$D_h = \frac{4A_c}{P} = \frac{4\pi D^2/4}{\pi D} = D \quad (3.84)$$

Under most practical conditions, the flow in a tube is laminar for Re < 2300, fully turbulent for Re > 10,000, and transitional in between.
The thickness of this boundary layer increases in the flow direction until the boundary layer reaches the pipe centre and thus fills the entire pipe. The region from the pipe inlet to the point at which the boundary layer merges at the centreline is called the hydrodynamic entrance region, and the length of this region is called the hydrodynamic entry length L_h.
Now consider a fluid at a uniform temperature entering a circular tube whose surface is maintained at different temperature.
This initiates convection heat transfer in the tube and the development of a thermal boundary layer along the tube. The thickness of this reaches the tube centre and thus fills the entire tube.
The region of flow over which the thermal boundary layer develops and reaches the tube centre is called the thermal entrance region, and the length of this region is called the thermal entry length L_t. flow in the thermal entrance region is called thermally developing flow since this is the region where the temperature profile develops.
The region beyond the thermal entrance region in which the dimensionless temperature profile expressed as $(T_S-T)/(T_s-T_\infty)$ remains unchanged is called the thermally fully developed region.

3.5.8.1 Laminar Flow in Tubes

Constant surface heat flux

Circular tube, laminar (q_s=constant): $Nu = \frac{hD}{k} = 4.36$ (3.85)

Therefore, for fully developed laminar flow in a circular tube subjected to constant surface heat flux, the Nusselt number is a constant. There is no dependence on the Reynolds or the Prandltl numbers.

Constant surface temperature

A similar analysis can be performed for fully developed laminar flow in a circular tube for the case of constant surface temperature T_s.

Circular tube, laminar (T_s=constant): $Nu = \frac{hD}{k} = 3.66$ (3.86)

Developing Laminar Flow in the Entrance Region

For a circular tube of length L subjected to constant surface temperature, the average Nusselt number for the thermal entrance region can be determined form

Entry region, laminar: $Nu = 3.66 + \frac{0.065(D/L)\operatorname{Re}\operatorname{Pr}}{1+0.04[(D/L)\operatorname{Re}\operatorname{Pr}]^{2/3}}$

The average Nusselt number for developing laminar flow in a circular tube in that case can be determined form

$$Nu = 1.86\left(\frac{\operatorname{Re}\ \operatorname{Pr} D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} \quad (3.87)$$

All properties are evaluated at the bulk mean fluid temperature, except for μ_s, which is evaluated at the surface temperature.

The average Nusselt number for the thermal entrance region of flow between isothermal parallel plates of length L is expressed as

Entry region, laminar: $Nu = 7.54 + \frac{0.03(D_h/L)\operatorname{Re}\ \operatorname{Pr}}{1+0.016[(Dh/L)\operatorname{Re}\operatorname{Pr}]^{2/3}}$ (3.88)

Where D_h is the hydraulic diameter, which is twice the spacing of the plates. This relation can be used for Re $\leq$ 2800.

3.5.8.2 Turbulent Flow in Tubes

For smooth tubes, the friction factor in turbulent flow can be determined from the explicit first Petukhov equation Petukhov (1970) given as

Smooth tubes: $f = (0.790\ ln\ Re\ -1.64)^{-2}$ for $3000 < Re < 5 \times 10^6$ (3.89)

The Nusselt number in turbulent flow is related to the friction factor through the Chilton-Colburn analogy expressed as $Nu = 0.125 f \operatorname{Re}\operatorname{Pr}^{1/3}$ (3.90)

For fully developed turbulent flow in smooth tubes,

$$Nu = 0.023\operatorname{Re}^{0.8}\operatorname{Pr}^{1/3}\left(\begin{array}{c}0.7 \leq \operatorname{Pr} \leq 160 \\ \operatorname{Re} > 10{,}000\end{array}\right) \quad (3.91)$$

Equation 3.89 is known as the Colburn equation. The accuracy of this equation can be improved by modifying it as $Nu = 0.023\operatorname{Re}^{0.8}\operatorname{Pr}^{n}$ (3.92)

Where n=0.4 for heating and 0.3 for cooling of the fluid flowing through the tube. This equation is known as the Dittus-Boelter equation.

For liquid metals (0.004 < Pr<0.01), the following relations are recommended by Sleichere and Rouse (1975) for 10^4<Re<10^6

Liquid metals, T_s=constant: $Nu = 4.8 + 0.0156\ Re^{0.85} Pr_s^{0.93}$ (3.93)

Liquid metals, q_s=constant: $Nu = 4.8 + 0.0156\ Re^{0.85} Pr_s^{0.93}$ (3.94)

Where the subscripts indicates that the Prandtl number is to be evaluated at the surface temperature.

A. Questions at remembering / understanding level:

i) Objective Questions

1. Match the following. [CO3][BL2]

a. Nusselt number	i. $\frac{Inertia\ force}{Viscous\ force}$
b. Reynolds number	ii $\frac{heat\ transfer\ through\ the\ fluid\ layer\ by\ convection}{heat\ transfer\ through\ the\ fluid\ layer\ by\ conduction}$
c. Prandtl number	iii. $\frac{Molecular\ diffusivity\ of\ momentum}{Molecular\ diffusivity\ of\ heat}$
d. Grashoff number	iv. $\frac{Buoyancy\ force}{Viscous\ force}$

A) a-ii, b-i,c-iv,d-iii B) a-ii, b-i,c-iii,d-iv C) a-i, b-iv,c-ii,d-iii D) a-ii, b-i,c-iv,d-iii

2. The __________number is a significant dimensionless parameter for forced convection and the _________number is a significant dimensionless parameter for natural convection [CO3][BL1]
   (a) Reynolds, Grashof (b) Reynolds, Mach
   (c) Reynolds, Eckert (d) Reynolds, Schmidt (e)Grashof, sherwood
3. In any forced or natural convection situation, the velocity of the flowing fluid is zero where the fluid wets any stationary surface. The magnitude of heat flux where the fluid wets a stationary surface is given by [CO3][BL1]

   a) $k_{fluid}(T_{Fluid} - T_{Wall})$ (b) $k_{fluid}\frac{dT}{dy}\Big|_{wall}$

   (c) $k_{fluid}\frac{d^2T}{dy^2}\Big|_{wall}$ (d) $h\frac{dT}{dy}\Big|_{wall}$
4. Which of the following is/are true? [CO3][BL2]
   a. A physical phenomenon involving a large number of physical variables can be represented by a small number of dimensionless quantities.
   b. Number of dimensionless numbers that can be formed from n physical variables requireing m fundamental dimensions is (n- m).
   c. Number of dimensionless numbers that can be formed from n physical variables requireing m fundamental dimensions is (m- n).
   d. Without experimental results, the relationship obtained between different dimensionless numbers is of no value.

   P) a & b Q) a & c R) a, b & d S) c & d
5. Which of the following is/are true? [CO3][BL2]
   a. Heat transfer in fluid medium by convection is higher than that by pure conduction.
   b. N_u=1 means, heat transfer in fluid medium by convection is equal to that by pure conduction.
   c. Nusselt number will always be greater than 1.0.
   d. Nusselt number is viewed as the dimensionless convection heat transfer coefficient

   P) a, b, c & b Q) a, c & d R) a, b & d S) a & d
6. Which of the following is/are true? [CO3][BL1]

a. Prandtl number for gases is about 1.0.
b. Prandtl number for liquid mettals is lessthan 0.01.
c. Prandtl number for water is about 10.0.
d. Prandtl number for heavy lubricating oils is in the order of 1,00,000

P) a, b, c & b Q) a, c & d R) a, b & d S) a & d

7. **Assertion(A):** The thermal boundary layer is thicker than the hydrodynamic boundary layer for the liquid metals. [CO3][BL2]
Reasoning(R): Heat energy will diffuse in liquid metals very fast compared to the momentum diffusion.
P) Both A and R are correct and R is the correct explanation for A.
Q) Both A and R are correct and R is not the correct explanation for A.
R) A is correct and R is wrong
S) R is correct and A is wrong

8. **Assertion(A):** The thicknesses of both thermal and velocity boundary layers in gaseous fluids are nearly same. [CO3][BL2]
Reasoning(R): Prandtl number for gases is about 10.0.
P) Both A and R are correct and R is the correct explanation for A.
Q) Both A and R are correct and R is not the correct explanation for A.
R) A is correct and R is wrong
S) R is correct and A is wrong

9. **Assertion(A):** The velocity boundary layer is thicker than the thermal boundary layer for the heavy lubricating oils. [CO3][BL2]
Reasoning(R): Heat energy will diffuse in heavy lubricating oils very fast compared to the momentum diffusion.
P) Both A and R are correct and R is the correct explanation for A.
Q) Both A and R are correct and R is not the correct explanation for A.
R) A is correct and R is wrong
S) R is correct and A is wrong

10. The Reynolds-Colburn analogy is expressed as [CO3][BL1]

a) $St_x \Pr^{1/3} = \frac{Cf_x}{2}$ b) $St_x \Pr^{2/3} = \frac{Cf_x}{2}$

c) $St_x^{2/3} \Pr^{1/3} = \frac{Cf_x}{2}$ d) $St_x^{1/3} \Pr^{2/3} = \frac{Cf_x}{2}$

11. Match the following in respect of critical Reynolds number. [CO3][BL1]

a. Flow through a tube	i. 5 X 10^5
b. Flow over a flat plate	ii 2 X 10^5
c. Flow across a cylinder	iii. 2300

A) a-iii, b-i,c-ii B) a-ii, b-iii,c-i C) a-i, b-iii,c-ii D) a-ii, b-i,c-iii

12. Which of the following expressions for velocity and thermal boundary layers for the flow over a flat plate in laminar region are correct? [CO3][BL1]

a) $\frac{\delta_{v,x}}{x^{1/2}} = \frac{5}{\mathrm{Re}_x^{1/2}}, \frac{\delta_{t,x}}{\delta_{v,x}} = \frac{1}{\Pr^{1/3}}$ b) $\frac{\delta_{v,x}}{x} = \frac{5}{\mathrm{Re}_x}, \frac{\delta_{t,x}}{\delta_{v,x}} = \frac{1}{\Pr^{1/3}}$

c) $\frac{\delta_{v,x}}{x} = \frac{5}{\mathrm{Re}_x^{1/2}}, \frac{\delta_{t,x}}{\delta_{v,x}} = \frac{1}{\Pr^{1/3}}$ d) $\frac{\delta_{v,x}}{x} = \frac{5}{\mathrm{Re}_x^{1/2}}, \frac{\delta_{t,x}}{\delta_{v,x}} = \frac{1}{\Pr^{2/3}}$

13. In flow through pipes, for the same Reynolds number [CO3][BL2]

a) The thermal entry length is longer for low Prandtl number fluids
b) The thermal entry length is longer for high Prandtl number fluids
c) Prandtl number does not influence the thermal entry length.
d) The thermal entry length effect is more pronounced only in turbulent flow.

ii) Descriptive Questions

1. State the advantages and limitation of dimensional analysis. [CO3] [BL2]
2. Derive the relationship between N_u, R_e and P_r for forced convection heat transfer using dimensional analysis. [CO3] [BL3]
3. Derive the relationship between N_u, G_r and P_r for natural convection heat transfer using dimensional analysis. [CO3] [BL3]
4. Derive the momentum equation for laminar boundary layer for a flow over a flat plate. [CO3] [BL3]
5. Derive the energy equation for laminar boundary layer for a flow over a flat plate. [CO3] [BL3]
6. Explain the significance of the following non-dimensional numbers [CO3][BL2]
   a. Nusselt number b. Reynolds number c. Prandtl number d. Grashoff number
7. What do you understand by hydro dynamic and thermal boundary layers? Illustrate with reference to flow over a flat plate. [CO3] [BL2]
8. Distinguish between bulk mean temperature and film temperature and laminar flow and turbulent flow. [CO3] [BL2]
9. Explain the development of velocity and thermal boundary layers for the flow over a flat plate with neat sketches. [CO3] [BL2]
10. What is Reynolds analogy? Describe the relation between fluid friction and heat transfer. [CO3] [BL3]

B. Question at applying / analyzing level

i) Multiple choice questions.

Common data for questions 1 and 2

Air at 20^0C flows over a 4-m long and 3-m wide surface of a plate whose temperature is 80^0C with a velocity of 5 m/s. For air, use k = 0.02735 W/m-K, Pr = 0.7228, ν= 1.798×10^{-5} m^2/s.

1. The length of the surface for which the flow remains laminar is [CO3] [BL3]
   a) 1.5 m (b) 1.8 m (c) 2.0 m
2. The rate of heat transfer from the laminar flow region of the surface is [CO3] [BL3]
   (a) 950 W (b) 1037 W (c) 2074 W d) 1804 W
3. Engine oil at 105^0C flows over the surface of a flat plate whose temperature is 15^0C with a velocity of 1.5 m/s. The local drag force per unit surface area 0.8 m from the leading edge of the plate is (For oil, use ν= 8.565 × 10^{-5} m^2/s, ρ= 864 kg/m^3) [CO3] [BL3]
   a) 21.8 N/m^2 (b) 14.3 N/m^2 (c) 10.9 N/m^2 (d) 8.5 N/m^2 (e) 5.5 N/m^2
4. Air (k =0.028 W/m. K, Pr = 0.7) at 50^0C flows along a 1-m-long flat plate whose temperature is maintained at 20^0C with a velocity such that the Reynolds number at the end of the plate is 10,000. The heat transfer per unit width between the plate and air is [CO3] [BL3]
   (a) 20 W/m (b) 30 W/m (c) 40 W/m (d) 50 W/m (e) 60 W/m
5. Air at 20^0C flows over a 4-m-long and 3-m-wide surface of a plate whose temperature is 80^0C with a velocity of 5 m/s. The rate of heat transfer from the surface is (For air, use k = 0.02735 W/m-K, Pr = 0.7228, ν= 1.798 × 10^{-5} m^2/s) [CO3] [BL3]
   a) 383 W b) 8985 W c) 11,231 W d) 14,672 W

6. Water (μ = 9.0×10^{-4} kg/m-s, ρ =1000 kg/m^3) enters a 2-cm-diameter and 3-m-long tube whose walls are maintained at 100^{0}C. The water enters this tube with a bulk temperature of 25^{0}C and a volume flow rate of 3 m^3/hour. The Reynolds number for this internal flow is [CO3] [BL3]
   (a) 59,000 (b) 105,000 (c) 178,000 (d) 236,000 (e) 342,000
7. Air (cp =1007 J/kg-K) enters a 17-cm-diameter and 4-m-long tube at 65^{0}C at a rate of 0.08 kg/s and leaves at 15^{0}C. The tube is observed to be nearly isothermal at 5^{0}C. The average convection heat transfer coefficient in W/m^2-K is [CO3] [BL3]
   a) 24.5 (b) 46.2 (c) 53.9 (d) 67.6 (e) 90.7

   Common data for questions 8 and 9

   Air at 90^{0}C and atmospheric pressure flows over a horizontal flat plate at 60 m/s. The plate is 60 cm square and is maintained at a uniform temperature of 10^{0}C. For air, use k = 0.02735 W/m-K, Pr = 0.7228, $\nu = 1.798\times10^{-5}$ m^2/s.
8. The heat transfer coefficient is [CO3] [BL3]
   a) (b) (c) (d) (e)
9. The total heat transfer from the plate is [CO3] [BL3]
   a) (b) (c) (d) (e)
10. Air at 27^{0}C and 1 atm flows over a flat plate at a speed of 2 m/s. The velocity and thermal boundary-layer thicknesses at a distance of 20 cm from the leading edge of the plate in meter are (The viscosity of air at 27^{0}C is 1.85×10^{-5} kg/m.s) [CO3] [BL3]
    (a) 0.00559 & (b) 0.00359 &
    (c) 0.00259 & (d) 0.00459 &

Common data for questions 11 and 12

Air at 27^{0}C and 1 atm flows over a flat plate at a speed of 2 m/s.

11. The average friction coefficient on the first 40 cm of the plate is [CO3] [BL3]
    a) 2.05×10^{-3} (b) 1.06×10^{-3} (c) 3.06×10^{-3} (d) 4.06×10^{-3}
12. The drag force exerted on the first 40 cm of the plate in mN is [CO3] [BL3]
    a) 5.44 (b) 6.44 (c) 7.44 (d) 4.44

Common data for questions 13 and 14

Air at 1 atm and 35^{0}C flow across a 5.0-cm-diameter cylinder a velocity of 50 m/s. The cylinder surface is maintained at a temperature of 150^{0}C.

13. The heat transfer coefficient (W/m^2.K) for this system is [CO3] [BL3]
    (a) 151.7 (b) 161.7 (c) 181.7 (d) 171.7
14. The heat transfer per unit length of the cylinder is [CO3] [BL3]
    (a) 4100 W (b) 3100 W (c) 2100 W (d) 1100 W

ii) Problems

1. A 1.0-kW heater is constructed of a glass plate with an electrically conducting film which produces a constant heat flux. The plate is 60 by 60 cm and placed in an airstream at 27^{0}C, 1 atm with u_∞=5 m/s. calculate the average temperature difference along the plate and the temperature difference at eh trailing edge. [CO3] [BL3]
2. Air flows over a flat plate at a constant velocity of 20 m/s and ambient conditions of 20 kPa and 20^{0}C. The plate is heated to a constant temperature of 75^{0}C, starting at a distance of 7.5 cm from the leading edge. What is the total heat transfer from the leading edge to appoint 35 cm from the leading edge? [CO3] [BL3]
3. Air flows across a 20-cm-squre plate with a velocity of 5 m/s. free-stream conditions are 10^{0}C and 0.2 atm. A heater in the plate surface furnishes a constant heat-flux condition at the wall so that the average wall temperature is 100^{0}C. Calculate the surface heat flux and the value of *h* at an *x* position of 10 cm. [CO3] [BL3]
4. Air at 27^{0}C and 1 atm flows over a flat plate at a speed of 2 m/s. calculate the boundary-layer thickness at distances of 20 and 40 cm from the leading edge of the plate. The

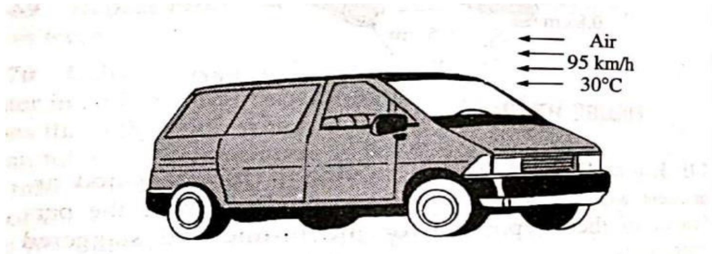
viscosity of air at 27^0C is 1.85×10^{-5} kg/m.s. If the plate is heated over its entire length to a temperature of 60^0C, calculate the heat transferred in (a) the first 20 cm of the plate and (b) the first 40 cm of the plate. Assume unit depth in the z direction. [CO3] [BL4]

5. The bottom of the corn-chip fryer is 3 m long by 0.9 m wide and is maintained at a temperature of 215^0C. Cooking oil flows across this surface at a velocity of 0.3 m/s and has a free-steam temperature of 204^0C. Calculate the heat transfer to the oil and estimate the maximum boundary-layer thickness. Properties of the oil may be taken as $v=2x10^{-6}m^2/s$,k=0.12 W/m.K, and Pr=40. [CO3] [BL4]
6. A fine wire having a diameter of 3.94×10^{-5} m is placed in a 1-atm airstream at 25^0C having a flow velocity of 50 m/s perpendicular to the wire. An electric current is passed through the wire, raising its surface temperature to 50^0C. Calculate the heat loss per unit length. [CO3] [BL3]
7. Air at 1 atm and 10^0C flows across a bank of tubes 15 rows high and 5 rows deep at a velocity of 7 m/s measured at a point in the flow before the air enters the tube bank. The surfaces of the tubes are maintained at 65^0C. The diameter of the tubes is 2.54 cm; they are arranged in an in-line manner so that the spacing in both the normal and parallel directions to the flow is 3.81 cm. Calculate the total heat transfer per unit length for the tube bank and the exit air temperature. [CO3] [BL4]
8. Water at 60^0C enters a tube of 2.54-cm diameter at a mean flow velocity of 2 cm/s. Calculate the exit water temperature if the tube is 3.0 m long and the wall temperature is constant at 80^0C. [CO3] [BL3]
9. Water at the rate of 1 kg/s is forced through a tube with a 2.5-cm ID. The inlet water temperature is 15^0C, and the outlet water temperature is 50^0C. The tube wall temperature is 14^0C higher than the water temperature all along the length of the tube. What is the length of the tube? [CO3] [BL3]
10. Air at 2 atm and 20^0C is heated as it flows through a tube with a diameter of 2.54 cm at a velocity of 10 m/s. Calculate the heat transfer per unit length of tube if a constant-heat-flux condition is maintained at the wall and the wall temperature is 20^0C above the air temperature, all along the length of the tube. How much would bulk temperature over a 3-m length of the tube? [CO3] [BL3]
11. Air at 1 atm and 27^0C enters a 5.0-mm-diameter smooth tube with a velocity of 3.0 m/s. the length of the tube is 10 cm. a constant heat flux is imposed on the tube wall. Calculate the heat transfer if the exit bulk temperature is 77^0C. Also calculate the exit wall temperature and the value of h at exit. [CO3] [BL3]
12. Liquid bismuth flow at a rate of 4.5 kg/s through a 5.0-cm-diameter stainless-steel tube. The bismuth enters at 415^0C and is heated to 440^0C as it passes through the tube. If a constant heat flux is maintained along the tube and the tube wall is at a temperature 20^0C higher than the bismuth bulk temperature, calculate the length of tube required to effect the heat transfer. [CO3] [BL4]
13. An annulus consists of the region between two concentric tubes having diameters of 4 cm and 5 cm. Ethylene glycol flows in this space at a velocity of 6.9 m/s. The entrance temperature is 20^0C, and the exit temperature is 40^0C. Only the inner tube is a heating surface, and it is maintained constant at 80^0C. Calculate the length of annulus necessary to effect the heat transfer. [CO3] [BL4]
14. Air at 1400 kPa enters a duct 7.5 cm in diameter and 6 m long at a rate of 0.5 kg/s. The duct wall is maintained at an average temperature of 500 K. The average air temperature in the duct is 550 K. Estimate the decrease in temperature of the air as it pass through the duct. [CO3] [BL4]

C. Questions at evaluating / creating level:

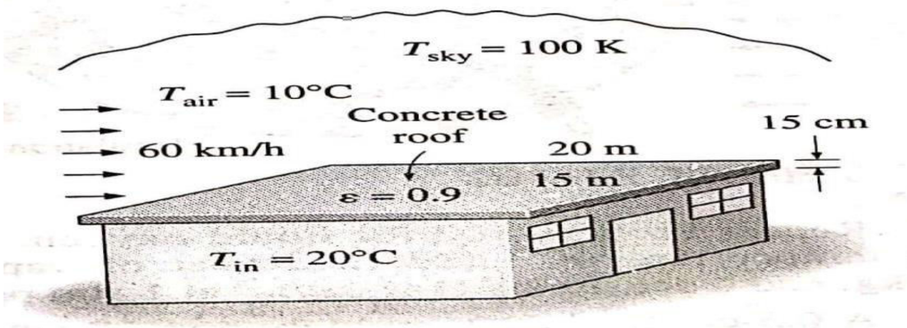
1. Consider the flow of air over a flat plate under laminar flow conditions at 1 atm. Investigate the influence of temperature on the heat transfer coefficient by examining five cases with constant free-steam temperature of 20^0C, constant free-steam velocity and surface temperatures of 50,100,150,250, and 350^0C. What do you conclude from this analysis? From the results, determine an approximate variation of the heat-transfer coefficient with absolute temperature for air at 1 atm. [CO3] [BL3]
2. The passenger compartment of a minivan travelling at 95 km/h can be modelled as a 1.0 m-high, 1.8-m-wide, and 3.4-m-long rectangular box whose walls have an insulating value of R-0.5 (i.e., a wall thickness-to-thermal conductivity ratio of 0.5 $m^2.^0C/W$). The interior of a minivan is maintained at an average temperature of 20^0C during a trip at night while the outside air temperature is 30^0C.

   The average heat transfer coefficient on the interior surfaces of the van is 6.8 $W/m^2.^0C$. The air flow over the exterior surfaces can be assumed to be turbulent because of the intense vibrations involved, and the heat transfer coefficient on the front and back surfaces can be taken to be equal to the on the top surface. Disregarding any heat gain or loss by radiation, determine the rate of heat transfer from the ambient air to the van. [CO3] [BL3]



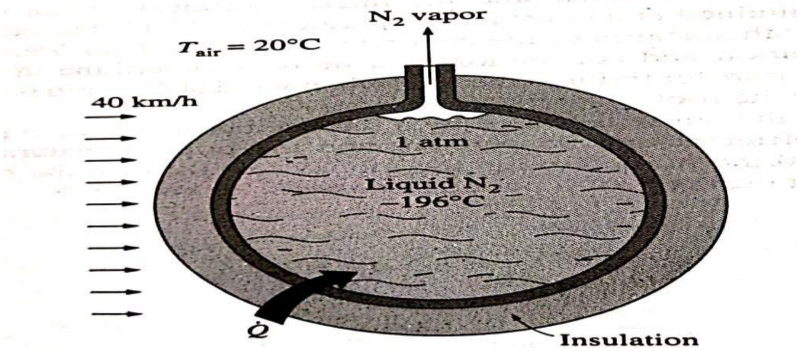
3. The roof of a house consists of a 15-cm-thick concrete slab (k=2 $W/m^2.^0C$) that is 15 m wide and 20 m long. The convection heat transfer coefficient on the inner surface of the roof is 5 $W/m^2.^0C$, while the night sky temperature is 100 K. The house and the interior surfaces of the wall are maintained at a constant temperature of 20^0C. The emissivity of both surfaces of the concrete roof is 0.9. Considering both radiation and convection heat transfer, determine the rate of heat transfer through the roof when wind at 60 km/h is blowing over the roof.

   If the house is heated by a furnace burning natural gas with an efficiency of 85 percent, and the price of natural gas is $1.20/therm, determine the money lost through the roof that night during a 14-h period. [CO3] [BL3]

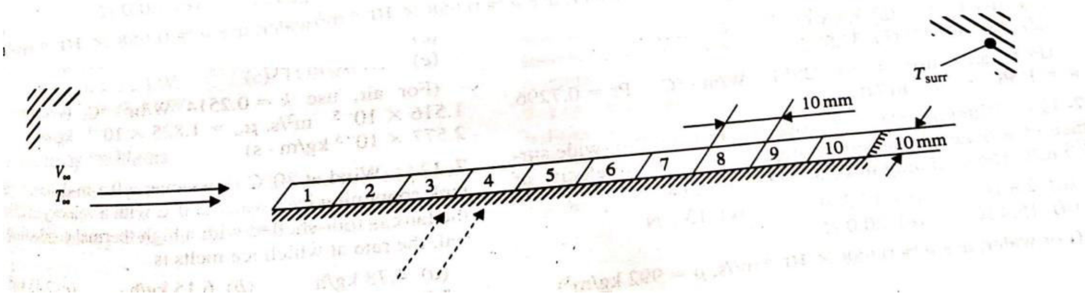


4. The boiling temperature of nitrogen at atmospheric pressure at sea level (1 atm pressure) is -196^0C. Therefore, nitrogen is commonly used in low-temperature scientific studies, since the temperature of liquid nitrogen in a tank open to the atmosphere will remain constant at -196^0C until it is depleted. Any heat transfer to the tank will result in the evaporation of some liquid nitrogen, which has a heat of vaporization of 198 kJ/kg and a density of 810 kg/m^3 at 1 atm.

Consider a 4-m diameter spherical tank that is initially filled with liquid nitrogen at 1 atm and is -196^{0}C. The tank is exposed to 20^{0}C ambient air and 40 km/h winds. The temperature of the thin-shelled spherical tank is observed to be almost the same as the temperature of the nitrogen inside. Disregarding any radiation heat exchange, determine the rate of evaporation of the liquid nitrogen in the tank as a result of heat transfer from the ambient air if the tank is (a) not insulated, (b) insulated with 5-cm-thick fibreglass insulation (k=0.035 W/m^2.^{0}C), and (c) insulted with 2-cm-thick super insulation that has an effective thermal conductivity of 0.00005 W/m^2.^{0}C. [CO3] [BL3]



5. Ten square silicon chips of 10 mm on a side are mounted in a single row on an electronic board that is insulated at the bottom side. The top surface is cooled by air flowing parallel to the row of chips with T_∞=24^{0}C and V=30 m/s. The chips exchange heat by radiation with the surroundings at T_{surr}=-10^{0}C. The emissivity of the chips is 0.85. when in use, the same electrical power is dissipated in each chip. The maximum allowable temperature of the chips is 100^{0}C. Assume that the temperature is uniform within each chip, no heat transfer occurs between adjacent chips, and T_∞ is the same throughout the array.
   (a) Which chip reaches the highest steady operating temperature? Why?
   (b) Determine the maximum electric power that can be dissipated per chip
   (c) Determine the temperature of the 5th chip in the direction of the air flow.
   (d) Consider tow cooling schemes: one used in parts (a)-(c) with the airflow parallel to the array (solid-line arrows), the other with the flow normal to it (dashed-line arrows). Which scheme is more efficient from a cooling point of view? Why? What other difference (s) between the two schemes would you consider when choosing one for a practical application? [CO3] [BL3]



D. GATE Questions:

1. Water (Prandtl number ˇ 6) flows over a flat plate which is heated over the entire length. Which one of the following relationship between the hydrodynamic boundary layer thickness (d) and the thermal boundary layer thickness (dt) is true?

[GATE-2001]

(a) dt > d (b) dt < d (c) dt = d (d) Cannot be predicted

2. The properties of mercury at 300 K are: density = 13529 kg/m^3, specific heat at constant pressure = 0.1393 kJ/kg-K, dynamic viscosity =0.1523×10^{-2} N.s/m^2 and thermal conductivity =8.540 W/m-K. The Prandtl number of the mercury at 300 K is [GATE-2002]

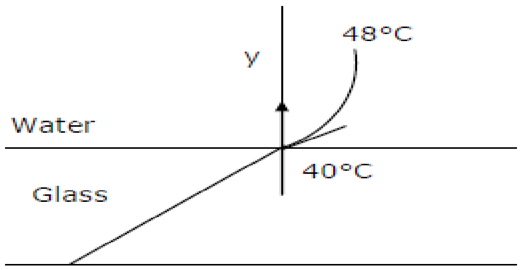
   (a) 0.0248 (b) 2.48 (c) 24.8 (d) 248

3. Consider a laminar boundary layer over a heated flat plate. The free stream velocity is $U\infty$. At some distance x from the leading edge the velocity boundary layer thickness is δ_u and the thermal boundary layer thickness is δ_T. If the Prandtl number is greater than 1, then [GATE-2003]

   (a) $\delta_v > \delta_T$ (b) $\delta_T > \delta_v$ (c) $\delta_v \approx \delta_T (U_\infty X)^{-1/2}$ (d) $\delta_v \approx \delta_T X^{-1/2}$

Common data for Questions 04 and 05

Heat is being transferred by convection from water at 48°C to a glass plate whose surface that is exposed to the water is at 40°C. The thermal conductivity of water is 0.6 W/mK and the thermal conductivity of glass is 1.2 W/mK. The spatial gradient of temperature in the water at the water-glass interface is dT/dy=1×104 K/m.



4. The value of the temperature gradient in the glass at the water-glass interface is K/m is [GATE-2003]

   (a) -2 × 104 (b) 0.0 (c) 0.5 × 104 (d) 2 × 104

5. The heat transfer coefficient h in W/m2 K is [GATE-2003]

   (a) 0.0 (b) 4.8 (c) 6 (d) 750

6. For air flow over a flat plate, velocity (U) and boundary layer thickness (d) can be expressed respectively, as [GATE-2004]

$$\frac{U}{U_\alpha} = \frac{3}{2}\frac{Y}{\delta} - \frac{1}{2}\left(\frac{Y}{\delta}\right)^3; \delta = \frac{4.64x}{\sqrt{\mathrm{Re}_x}}$$

   If the free stream velocity is 2m/s, and air has kinematic viscosity of 1.5×10^{-5} m^2/s and density of 1.23 kg/m^3, the wall shear stress at x = 1m, is

   (a) 2.36 × 10^2 N/m^2 (b) 43.6 × 10^{-3} N/m^2
   (c) 4.36 × 10-3N/m^2 (d) 2.18 × 10^{-3} N/m^2

7. Statement for Linked Answer Questions 85a & 85b: An un-insulated air conditioning duct of rectangular cross section 1 m × 0.5 m, carrying air at 20°C with a velocity of 10 m/s, is exposed to an ambient of 30°C. Neglect the effect of duct construction material. For air in the range of 20-30°C, data are as follows: thermal conductivity =0.025 W/m.K; viscosity = 18 μ Pa.s; Prandtl number =0.73; density = 1.2 kg/m3. The laminar flow Nusselt number is 3.4 for constant wall temperature conditions and, for turbulent flow, Nu=0.023 Re$^{0.8}$ Pr$^{0.33}$. [GATE-2005]

   A) (A) The Reynolds number for the flow is

   (a) 444 (b) 890 (c) 4.44×105 (d) 5.33×105

   (B) The heat transfer per metre length of the duct, in watts, is:

   (a) 3.8 (b) 5.3 (c) 89 (d) 769

8. With an increase in thickness of insulation around a circular pipe, heat loss to surroundings due to. [GATE-2006]

(A) Convection increases, while that due to conduction decreases
(B) Convection decreases, while that due to conduction increases
(C) Convection and conduction decreases
(D) Convection and conduction increases

9. The temperature distribution within the thermal boundary layer over a heated isothermal flat plate is given by $\frac{T-T_W}{T_\infty - T_W} = \frac{3}{2}\left(\frac{y}{\delta_1}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$, where T_w and T_∞ are the temperatures of plate and free stream respectively, and y is the normal distance measured from the plate. The local Nusselt number based on the thermal boundary layer thickness δ_t, is given by [GATE-2007]

a) 1.33 b) 1.50 c) 2.0 d) 4.64

10. Consider steady one-dimensional heat flow in a plate of 20 mm thickness with a uniform heat generation of 80 MW/m^3. The left faces are kept at constant temperature of 160^0C and 120^0C respectively. The plate has a constant thermal conductivity of 200 W/mK. [GATE-2007]

1. The location of maximum temperature within the plate from its left face is

a) 15 mm b) 10 mm c) 5 mm d) 0 mm

2. The maximum temperature within the plate in ^0C is

a) 160 b) 165 c) 200 d) 250

11. 29. A coolant fluid at 30°C flows over a heated flat plate maintained at a constant temperature of 100°C. The boundary layer temperature distribution at a given location on the plate may be approximated as T = 30 + 70exp (−y) where y (in m) is the distance normal to the plate and T is in °C. If thermal conductivity of the fluid is 1.0W/mK, the local convective heat transfer coefficient (in W/m2K) at that location will be [GATE-2009]

(A) 0.2 (B) 1 (C) 5 (D) 10

12. Water flows through a pipe having an inner radius of 10 mm at the rate of 36 kg/hr at 25^0C. The viscosity of water at 25^0C is 0.001 kg/m.s. The Reynolds number of the flow is _______ [GATE-2014]

13. 47. The non-dimensional fluid temperature profile near the surface of a convectively cooled flat plate is given by $\frac{T_w - T}{T_w - T_\infty} = a + b\frac{Y}{L} + c\left(\frac{Y}{L}\right)^2$, where y is measured perpendicular to the plate, L is the plate length, and a, b and c are arbitrary constants. T_W and T_∞ are wall and ambient temperatures, respectively. If the thermal conductivity of the fluid is k and the wall heat flux is q¢¢ , the Nusselt number $Nu = \frac{q}{T_w - T_\infty}\frac{L}{k}$ is equal to [GATE-2014]

(A) a (B) b (C) 2c (D) (b + 2c)

14. For laminar forced convection over a flat plate, if the free stream velocity increases by a factor of 2, the average heat transfer coefficient [GATE-2014]

(A) Remains same (B) Decreases by a factor of $\sqrt{2}$
(C) Rises by a factor of $\sqrt{2}$ (D) Rises by a factor of 4

15. Consider laminar flow of water over a flat plate of length 1 m. If the boundary layer thickness at a distance of 0.25 m from the leading edge of the plate is 8 mm, the boundary layer thickness (in mm), at a distance of 0.75 m, is _______ [GATE-2014]

16. 55. Consider the turbulent flow of a fluid through a circular pipe of diameter, *D*. Identify the correct pair of statements. [GATE-2014]

I. The fluid is well-mixed II. The fluid is unmixed

III. $Re_p < 2300$ IV. $Re_p > 2300$

(A) I, III (B) II, IV (C) II, III (D) I, IV

17. Match Group A with Group B: [GATE-2014]

Group A	Group B
P: Biot number	1: Ratio of buoyancy to viscous force
Q: Grashof number	2: Ratio of inertia force to viscous force
R: Prandtl number	3: Ratio of momentum to thermal diffusivities
S: Reynolds number	4: Ratio of internal thermal resistance to boundary layer

(A) P-4, Q-1, R-3, S-2 (B) P-4, Q-3, R-1, S-2

(C) P-3, Q-2, R-1, S-4 (D) P-2, Q-1, R-3, S-4

18. Water flows through a tube of diameter 25 mm at an average velocity of 1.0m/s. The properties of water p=100kg.m^3, μ=7.25 x10^{-4}N.s/m^2,k=0.625 W/mk, Pr=4.85. Using $Nu = 0.023Re^{0.8} Pr^{0.4}$, the convective heat transfer coefficient (in W/m^2.K) is ______ [GATE-2014]

19. For flow of viscous fluid over a flat plate, if the fluid temperature is the same as the plate temperature, the thermal boundary layer is [GATE-2015]

A) Thinner than the velocity boundary layer.

B) Thicker than the velocity boundary layer.

C) of the same thickness as the velocity boundary layer

D) not formed at all

20. Air (p=1.2 kg/m^3 and kinematic viscosity, v=2x10^{-5}m^2/s) with a velocity of 2 m/s flows over the top surface of a flat plate of length 2.5 m. If the average value of friction coefficient is $C_f = \frac{1.328}{\sqrt{\mathrm{Re}_x}}$ the total drag force (in N) per unit width of the plate is ____ [GATE-2015]

21. In the laminar flow of air (Pr=0.7) over a heated plate, if δ and δ_T denote, respectively, the hydrodynamic and thermal boundary layer thickness, then [GATE-2015]

A) $\delta = \delta_T$ B) $\delta > \delta_T$ C) $\delta < \delta_T$ D) δ =0 but δ_T -≠ 0

22. The ratio momentum diffusivity (v) to thermal diffusivity (α), is called. [GATE-2015]

A) Prandtl number B) Nusselt number

C) Biot number D) Lewis number

23. A fluid (Prandtl number, Pr = 1) at 500 K flows over a flat plate of 1.5 m length, maintained at 300 K. The velocity of the fluid is 10 m/s. Assuming kinematic viscosity, m2/s, the thermal boundary layer thickness (in mm) at 0.5 m from the leading edge is __________ [GATE-2016]

24. For a hydro-dynamically and thermally fully developed laminar flow through a circular pipe of constant cross-section, the Nusselt number at constant wall heat flux (Nuq) and that at constant wall temperature (NuT) are related as [GATE-2019]

A) $Nu_q > Nu_T$ B) $Nu_q < Nu_T$ C) $Nu_q = Nu_T$ D) $Nu_q = (Nu_T)^2$

E. IES / IAS Questions

1. The properties of mercury at 300 K are: density = 13529 kg/m^3, specific heat at constant pressure = 0.1393 kJ/kg-K, dynamic viscosity = 0.1523 x 10-2 N.s/m2 and thermal conductivity = 8.540 W/m-K. The Prandtl number of the mercury at 300 K is (a) 0.0248 (b) 2.48 (c) 24.8 (d) 248 [IES-2002]
2. For the fully developed laminar now and heat transfer in a uniformly heated long circular tube, if the now velocity is doubled and the tube diameter is halved, the heat transfer coefficient will be [IES-2000]
   (a) double of the original value (b) half of the original value
   (c) same as before (d) four times of the original value
3. Assertion (A): According to Reynolds analogy for Prandtl number equal to unity. Stanton number is equal to one half of the friction factor. [IES-2001]
   Reason (R): If thermal diffusivity is equal to kinematic viscosity, the velocity and the temperature distribution in the flow will be the same.
4. The Nusselt number is related to Reynolds number in laminar and turbulent flows respectively as [IES-2000]
   (a) Re-1/2 and Re0.8 (b) Re1/2 and Re0.8 (c) Re-1/2 and Re-0.8 (d) Re1/2 and Re-0.8
5. Match List I with II and select the correct answer using the code given below the Lists:

List I (Non-dimensional Number)	List II (Application)
A. Grashof number	1. Mass transfer
B. Stanton number	2. Unsteady state heat conduction
C. Sherwood number	3. Free convection
D. Fourier number	4. Forced convection

Code: [IES 2007]

	A	B	C	D		A	B	C	D
(a)	4	3	1	2	(b)	3	4	1	2
(c)	4	3	2	1	(d)	3	4	2	1

6. Match List I (Type of heat transfer) with List II (Governing dimensionless parameter) and select the correct answer: [IES-2002]

List I	List II
A. Forced convection	1. Reynolds, Grashof and Prandtl number
B. Natural convection	2. Reynolds and Prandtl number
C. Combined free and forced convection	3. Fourier modulus and Biot number
D. Unsteady conduction with convection at surface	4. Prandtl number and Grashof number

	A	B	C	D		A	B	C	D
(a)	2	1	4	3	(b)	3	4	1	2
(c)	2	4	1	3	(d)	3	1	4	2

7. For steady, uniform flow through pipes with constant heat flux supplied to the wall, what is the value of Nusselt number? [IES 2007]
   (a) 48/11 (b) 11/48 (c) 24/11 (d) 11/24
8. Nusselt number for fully developed turbulent flow in a pipe is given by Nu = $Nu = CR_e^a P_r^b$ The values of a and b are [IES-2001]
   (a) a = 0.5 and b = 0.33 for heating and cooling both
   (b) a = 0.5 and b = 0.4 for heating and b = 0.3 for cooling
   (c) a = 0.8 and b = 0.4 for heating and b = 0.3 for cooling
   (d) a = 0.8 and b = 0.3 for heating and b = 0.4 for cooling
9. Match List - I with List - II and select the correct answer using the code given below the Lists: [IES-2006]

List - I (Phenomenon)	List – II (Associated Dimensionless Parameter)
A. Transient conduction	1. Reynolds number
B. Forced convection	2. Grashoff number
C. Mass transfer	3. Biot number
D. Natural convection	4. Mach number
	5. Sherwood number

	A	B	C	D		A	B	C	D
(a)	3	2	5	1	(b)	5	1	4	2
(c)	3	1	5	2	(d)	5	2	4	1

10. Nusselt number for a pipe flow heat transfer coefficient is given by the equation Nu_D=4.36. Which one of the following combinations of conditions do exactly apply for use of this equation? [IES-2006]
    (a) Laminar flow and constant will temperature
    (b) Turbulent flow and constant wall heat flux
    (c) Turbulent flow and constant wall temperature
    (d) Laminar flow and constant wall heat flux
11. A fluid of thermal conductivity 1.0 W/m-K flows in fully developed flow with Reynolds number of 1500 through a pipe of diameter 10 cm. The heat transfer coefficient for uniform heat flux and uniform wall temperature boundary conditions are, respectively. [IES-2002]
    (a) $36.57\, and\, 43.64\frac{W}{m^2K}$ (b) $43.64\, and\, 36.57\frac{W}{m^2K}$
    (c) $43.64\frac{W}{m^2K}\, for\, both\, the\, cases$ (d) $36.57\frac{W}{m^2K}\, for\, both\, the\, cases$
12. Which one of the following stems is correct? [IES-2004]
    The non-dimensional parameter known as Stanton number (St) is used in
    (a) Forced convection heat transfer in flow over flat plate
    (b) Condensation heat transfer with laminar film layer
    (c) Natural convection heat transfer over flat plate
    (d) Unsteady heat transfer from bodies in which internal temperature gradients cannot be neglected
13. For fully-developed turbulent flow in a pipe with heating, the Nusselt number Nu, varies with Reynolds number R_e and Prandtl number Pr as [IES-2003]
    (a) $R_e^{0.5}P_r^{\frac{1}{3}}$ (b) $R_e^{0.8}P_r^{0.2}$ (c) $R_e^{0.8}P_r^{0.4}$ (d) $R_e^{0.8}P_r^{0.3}$
14. For laminar flow over a flat plate, the local heat transfer coefficient 'h_x' varies as $x^{-1/2}$, where x is the distance from the leading edge (x=0) of the plate. The ratio of the average coefficient 'h_a' between the leading edge and some location 'A' at x=x on the plate to the local heat transfer coefficient 'h_x' at A is [IES-1999]
    (a) 1 (b) 2 (c) 4 (d) 8
15. The ratio of energy transferred by convection to that by conduction is called
    (a) Stanton number (b) Nusselt number (c) Biot number (d) Preclet number

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Academic Year: 2019 – 20 **Semester: II**

Class : III B.Tech **Subject: Heat Transfer**

Learning Material

UNIT-IV

Syllabus

Free Convection: Development of Hydrodynamic and thermal boundary layer along a vertical plate, Use of empirical relations for Vertical plates and cylinders.

Heat Transfer with Phase Change:

Boiling: Pool boiling, Regimes-Nucleate boiling and Film boiling, Critical Heat flux, Flow boiling.

Condensation: Film wise and drop wise condensation, Film condensation on vertical and horizontal cylinders, using empirical correlations.

4.1 PHYSICAL MECHANISM OF NATURAL CONVECTION

- In natural (or free) convection, the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid.
- Many familiar heat transfer applications involve natural convection as the primary mechanism of heat transfer. Some examples are cooling of electronics equipment such as power transistors, TVs, and DVDs; heat transfer from electric baseboard heaters or steam radiators: heat transfer from the refrigeration coils and power transmission lines; and heat transfer from the bodies of animals and human beings.

4.1.1 Natural Convection from a Vertical Plate

Consider a vertical hot flat plate immersed in a quiescent fluid body. We assume the natural convection flow to be steady, laminar, and two-dimensional, and the fluid to be Newtonian with constant properties, including density, with one exception: the density difference $\rho-\rho_\infty$ is to be considered since it is this density difference between the inside and the outside of the boundary layer that gives rise to buoyancy force and sustains flow. We take the upward direction along the plate to be x, and the direction normal to surface to be y, as shown in Fig. 4.1. Therefore, gravity acts in the –x direction.

The velocity and temperature profiles for natural convection over a vertical hot plates are also shown in Fig.4.1. Note that as in forced convection, the thickness of the boundary layer increases in the flow direction. Unlike forced convection, however, the fluid velocity is *zero* at the outer edge of the velocity boundary layer as well as at the surface of the plate. This is expected since the fluid beyond the boundary layer is motionless. Thus, the fluid velocity increases with distance from the surface, reaches a maximum, and gradually decreases to zero at a distance sufficiently far from the surface. At the surface, the fluid temperature is equal to the plate temperature, and gradually decreases to the temperature of the surrounding fluid at a distance sufficiently far from the surface, as show in the figure. In the case of cold surfaces, the shape of the velocity and temperature profiles remains the same but their direction is reversed.

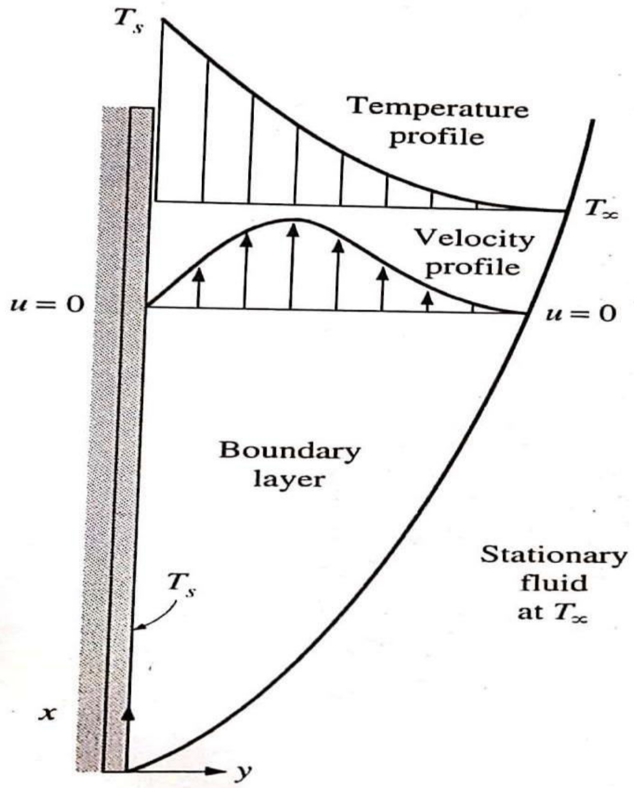


Figure 4.1: Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at temperature T_s inserted in a fluid at temperature T_∞.

4.1.2 Natural Convection from a Horizontal Plate

The rate of heat transfer to or from a horizontal surface depends on whether the surface is facing upward or downward. For a hot surface in a cooler environment, the net force acts upward, forcing rises freely, inducing strong natural convection currents and thus effective heat transfer, as shown in Fig. 4.2. But if the hot surface is facing downward, the plate blocks the heated fluid that tends to rise (except near edges), impending heat transfer. The opposite is true for a cold plate in a warmer environment since the net force (weight minus buoyancy force) in the case acts downward, and the cooled fluid near the plate tends to descend.

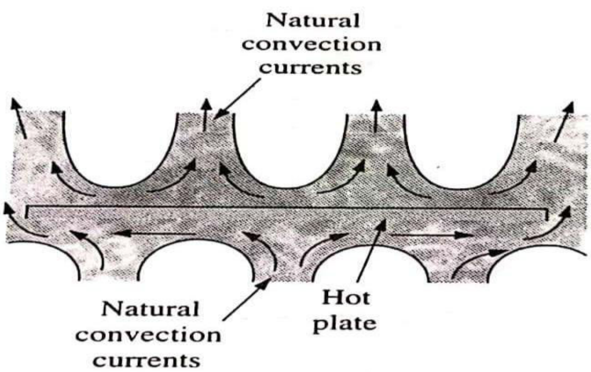


Figure 4.2: Natural convection flows on the upper and lower surfaces of a horizontal hot plate.

4.1.3 Grashof Number

Grashof number Gr_L,

$$Gr_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \quad (4.1)$$

where

g = gravitational acceleration, m/s^2

β = coefficient of volume expansion, 1/K (β =1/T for ideal gases)

T_s = Temperature of the surface, ^0C

T_∞ = temperature of the fluid sufficiently far from the surface, ^0C

L_c = characteristic length of the geometry, m

ν = kinematic viscosity of the fluid, m^2/s

- The flow regime in forced convection is governed by the dimensionless Reynolds number, which represents the ratio of inertial forces to viscous forces acting on the fluid.
- The flow regime in natural convection is governed by the dimensionless Grashof number, which represent the ratio of the buoyancy force to the viscous force acting on the fluid.
- For vertical plates, for example, the critical Grashof number is observed to be about 10^9.
- Therefore, the flow regime on a vertical plate becomes turbulent at Granshof numbers greater than 10^9.

4.1.4 Natural Convection over Surfaces

The simple empirical correlations for the average Nusselt number Nu in natural convection are of the form

$$Nu = \frac{hL_c}{k} = C(Gr_L \Pr)^n = C\ Ra_L^n \quad (4.2)$$

Reyleigh number, which is the product of the Grashof and Prandlt number

$$Ra_L = Gr_L \Pr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\Pr \quad (4.3)$$

The values of the constant C and n depend on the geometry of the surface and the flow regime, which is characterized by the range of the Rayleigh number. The value of n is usually $\frac{1}{4}$ for laminar flow and $\frac{1}{3}$ for turbulent flow. The value of the constant C is normally less than 1.

- All fluid properties are to be evaluated at the film temperature $T_f = \frac{1}{2}(T_S + T_\infty)$.

Vertical plate:

$$Nu=0.59Ra_L^{1/4} \text{ for } 10^4 < Ra < 10^9 \quad (4.4)$$

$$Nu=0.1Ra_L^{1/3} \text{ for } 10^{10} < Ra < 10^{13} \quad (4.5)$$

Horizontal plate:

a) Upper surface of a hot plate or Lower surface of a cold plate:

$$Nu=0.54Ra_L \text{for } ^{1/4}\ 10^4 < Ra < 10^7 \quad (4.6)$$

$$Nu=0.15Ra_L^{1/3} \text{ for } 10^7 < Ra < 10^{11} \quad (4.7)$$

b) Lower surface of a hot plate or upper surface of a cold plate

$$Nu=0.27Ra_L^{1/4} \text{ for } 10^5 < Ra < 10^{11} \quad (4.8)$$

4.2 BOILING HEAT TRANSFER

Many familiar engineering applications involve condensation and boiling heat transfer. In a household refrigerator, for example, the refrigerant absorbs heat from the refrigerated space by boiling in the evaporator section and rejects heat to the kitchen air by condensing in the condenser section (the long coils behind or under the refrigerator). Also, in steam power plants, heat is transferred to the steam in the boiler where water is vaporized, and the waste heat is rejected from the steam in the condenser where the steam is condensed. Some electronic components are cooled by boiling by immersing them in a fluid with an appropriate boiling temperature.

Boiling is a liquid-to-vapour phase changes process just like evaporation, but there are significant different between the two.

- Evaporation occurs at the liquid-vapour interface when the vapour pressure is less than the saturation pressure of the liquid at a given temperature. Water in a lake at 20^0C, for example, evaporates to air at 20^0C and 60 percent relative humidity since the saturation pressure of water at 20^0C is 2.3 kPa and the vapour pressure of air at 20^0C and 60 percent relative humidity is 1.4 kPa. Other examples of evaporation are the drying of clothes, fruits, and vegetables.
- Boiling, on the other hand, occurs at the solid-liquid interface when a liquid is brought into contact with a surface maintained at a temperature T_s sufficiently above the saturation temperature T_{sat} of the liquid (Fig. 10-2). At 1 atm, for example, liquid interface that detach from the surface when they reach a certain size and attempt to rise to the free surface of the liquid.

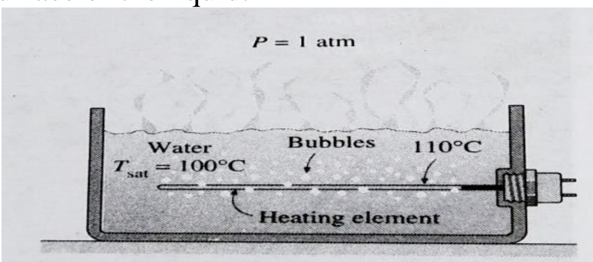


Figure 4.3: Boiling occurs when a liquid is brought into contact with a surface at a temperature above the saturation temperature of the liquid.

As a form of convection heat transfer, the boiling heat flux from a solid surface to the fluid is expressed from Newton's law of cooling as

$$q_{boiling} = h(T_s - T_{sat}) = h\Delta T_{excess} \qquad (W/m^2) \tag{4.9}$$

Where $\Delta T_{excess} = T_s - T_{sat}$ is called the excess temperature, which represents the temperature excess of the surface above the saturation temperature of the fluid.

Boiling is classified as pool boiling or flow boiling, depending on the presence of bulk fluid motion (Fig. 4.4).

- Boiling is called pool boiling in the absence of bulk fluid flow and flow boiling (or forced convection boiling) in the presence of it. In pool boiling, the fluid body is stationary, and any motion of the fluid is due to natural convection currents and the motion of the bubbles under the influence of buoyancy. The boiling of water in a pan on top of a stove is an example of pool boiling. Pool boiling of a fluid can also be achieved by placing a heating coil in the fluid.
- In flow boiling, the fluid is forced to move in a heated pipe or over a surface by external means such as a pump. Therefore, flow boiling is always accompanied by other convection effects.

Pool and flow boiling are further classified as sub-cooled boiling or saturated boiling, depending on the bulk liquid temperature (Fig. 4.5).

- Boiling is said to be sub-cooled (or local) when the temperature of the main body of the liquid is below the saturation temperature T_{sat} (i.e., the bulk of the liquid is sub-cooled) and saturated (or bulk) when the temperature of the liquid is equal to T_{sat} (i.e., the bulk of the liquid is saturated).

4.2.1 Boiling Regimes and the Boiling Curve

The pioneering work on boiling was done in 1934 by S.Nukiyama, who used electrically heated nichrome and platinum wires immersed in liquids in his experiments. Nukiyama notice that boiling takes different forms, depending on the value of the excess temperature ΔT_{excess}. Four different boiling regimes are observed: natural convection boiling, nucleate boiling, transition boiling, and film boiling. These regimes are illustrated on the boiling curve (Fig. 4.6), which is a plot of boiling heat flux versus the excess temperature. Although the boiling curve given in this figure is for water, the general shape of the boiling curve remains the same for different fluids.

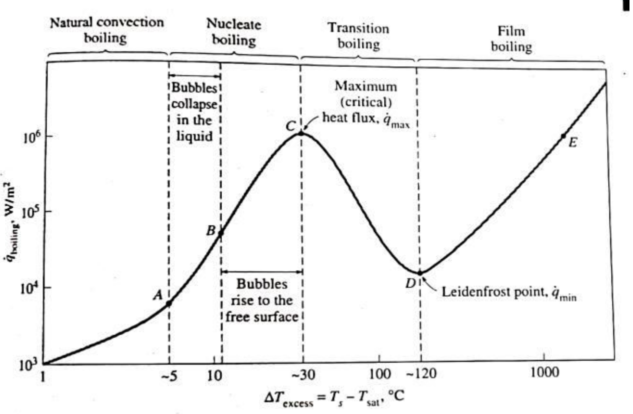


Figure 4.6: Typical boiling curve for water at a 1 atm pressure

Natural convection boiling (to Point A on the Boiling Curve): We known form thermodynamics that a pure substance at a specified pressure starts boiling when it reaches that saturation temperature at that pressure. But in practice we do not see any bubbles forming on the heating surface until the liquid is heated a few degrees above that saturation temperature (about 2 to 6^0C for water). Therefore, the liquid is slightly superheated in this case (a metastable condition) and evaporates when it rises to the free surface. The fluid motion in this mode of boiling is governed by natural convection currents, and heat transfer from the heating surface to the fluid is by natural convection.

Nucleate boiling (between Points A and C): The first bubbles start forming at a point A of the boiling curve at various preferential sites on the heating surface. The bubbles form at an increasing rate at an increasing number of nucleation sites as we move along the boiling curve toward point C.

The nuclear boiling regime can be separated into two distinct regions. In region A-B, isolated bubbles are formed at various preferential nucleation sites on the heated surface. But these bubbles are dissipated in the liquid shortly after they separate from the surface. The space vacated by the rising bubbles is filled by the liquid in the vicinity of the heater surface, and the process is repeated. The stirring and agitation caused by the entrainment of the liquid to the heater surface is primarily responsible for the increased heat transfer coefficient and heat flux in this region of nuclear boiling.

In region B-C, the heater temperature is further increased, and bubbles form at such greater rates at such a large number of nucleation sites that they form numerous continuous columns of vapour in the liquid. These bubbles move all the way up to the free surface, where they break up and release their vapour content. The large heat fluxes obtainable in this region are caused by the combined effect of liquid entertainment and evaporation.

At large values of ΔT_{excess}, the rate of evaporation at the heater surface reaches such high values that a large fraction of the heater surface is covered by bubbles, making it difficult for the liquid to reach the heater surface and wet it. Consequently, the heat flux increases at a lower rate with increasing ΔT_{excess}, and reaches a maximum at point C. the heat flux at his point is called the critical (or maximum) heat flux ΔT_{excess}.

- For water, the critical heat flux exceeds 1 MW/m^2.

Transition boiling (between Points C and D): As the heater temperature and thus the ΔT_{excess} is increased past point C, the heat flux decreases, as shown in Fig. 4.6. This is because a large fraction of the heater surface is covered by a vapour film, which acts as an insulation due to the low thermal conductivity of the vapour relative to that of the liquid. In the transition boiling regime, both nucleate and film boiling partially occur. Nucleate boiling at point C is completely replaced by film boiling regime, which is also called the unstable film boiling regime, is avoided in practice.

- For water, transition boiling occurs over the excess temperature range from about 30^{0}C to about 120^{0}C.

Film boiling (beyond Point D): In this region the heater surface is completely covered by a continuous stable vapour film. Point D, where the heat flux reaches a minimum, is called the Leidenfrost point, in honour of J.C.Leidenfrost, who observed in 1756 that liquid droplets on a very hot surface jump around and slowly boil away. The presence of a vapour film between the heater surface and the liquid is responsible for the low heat transfer rates in the film boiling region. The heat transfer rate increases with increasing excess temperature as a result of heat transfer from the heated surface to the liquid through the vapour film by radiation, which becomes significant at high temperatures.

- Nukiyama noticed, with surprise, that when the power applied to the nichrome wire immersed in water exceeded q_{max} even slightly, the wire temperature increased suddenly to the melting point of the wire and burnout occurred beyond his control.
- The burnout phenomenon in boiling can be explained as follows: In order to move beyond point C where q_{max} occurs, we must increase the heater surface temperature T_s, to increase T_s, however, we most increase the heat flux. But the fluid cannot receive this increased energy at an excess temperature just beyond point C. Therefore, the heater surface ends up absorbing the increased energy, causing the heater surface temperature T_s to rise. But the fluid can receive even less energy at this increased excess temperature, causing the heater surface temperature T_s to rise even further. This is point E on the boiling curve, which corresponds to very high surface temperatures. Therefore, any attempt to increase the heat flux beyond q_{max} will cause the operation point on the boiling curve to jump suddenly from point C to point E. However, surface temperature that corresponds to point E is beyond the melting point of most heater materials, and burnout occurs. Therefore, point C on the boiling curve is also called the burnout point, and the heat flux at this point the burnout heat flux.

4.2.2 Heat Transfer Correlations in Pool Boiling

4.2.2.1 Nuclear boiling

The most widely used correlation for the rate of heat transfer in the nucleate boiling regime was proposed in 1952 by Rohsenow, and expressed as

$$q_{nucleat} = \mu_1 h_{fg}\left[\frac{g\left(\rho_1-\rho_v\right)}{\sigma}\right]^{1/2}\left[\frac{c_{pl}\left(T_s-T_{sat}\right)}{C_{sf}h_{fg}\,\mathrm{Pr}_l^n}\right] \tag{4.10}$$

where

$q_{nucleate}$ = nucleate boiling heat flux, W/m^2

μ_l = viscosity of the liquid, kg/m.s

h_{fg} = enthalpy of vaporization, J/kg

g = gravitational acceleration, m/s^2

ρ_l = density of the liquid, kg/m^3

ρ_v = density of the vapour, kg/ m^3

σ = surface tension of liquid-vapour interface, N/m

c_{pl} = specific heat of the liquid, J/kg.^{o}C

T_s = surface temperature of the heater, ^{o}C

T_{sat} = surface temperature of the fluid, ^{o}C

C_{sf}= experimental constant that depends on surface-fluid combination

Pr_l = Prendtl number of the liquid.

n = experimental constant that depends on the fluid.

Values of the coefficient C_{sf} and n for various fluid-surface combinations are given in Table 4.1.

Table 4.1: Values of the coefficient C_{sf} and n for various fluid-surface combinations

Fluid-Heating surface Combination	$\mathbf{C_{sf}}$	**n**
Water – copper (polished)	0.0130	1.0
Water – copper (scored)	0.0068	1.0
Water – stainless steel (mechanically polished)	0.0130	1.0
Water – stainless steel (ground and polished)	0.0060	1.0
Water – stainless steel (Teflon pitted)	0.0058	1.0
Water – stainless steel (chemically etched)	0.0130	1.0
Water – brass	0.0060	1.0
Water – nickel	0.0060	1.0
Water – platinum	0.0130	1.0
n-Pentane-copper (polished)	0.0154	1.7
n-Pentane-chromium	0.0150	1.7
Benzene –chromium	0.1010	1.7
Ethyl alcohol-chromium	0.0027	1.7
Carbon tetrachloride-copper	0.0130	1.7
Isopropanol-copper	0.0025	1.7

The maximum (or critical) heat flux in nucleate pool boiling was determined theoretically by S.S.Kutateladze in Russia in 1948 and N.Zuber in the United States in 1958 using quite different approaches, and is expressed as

$$q_{mass} = C_{cr}h_{fg}\left[\sigma g \rho_v^2(\rho_l-\rho_v)\right]^{1/4} \tag{4.11}$$

Where C_{cr} is a constant whose value depends on the heater geometry

Table 4.2: Value of the coefficient C_{cr} for use in Eq. 4.1.1 for maximum heat flux (dimensionless parameter $L^* = L[g(\rho_l-\rho_v)/\sigma]^{1/2}$)

Heater Geometry	C_{cr}	Charac. Dimension of Heater, L	Range of L*

Large horizontal flat heater	0.149	Width or diameter	L* > 27
Small horizontal flat hater[1]	18.9 K_1	Width or diameter	9< L* > 20
Large horizontal cylinder	0.12	Radius	L* > 1.2
Small horizontal cylinder	0.12 $L^{*-0.25}$	Radius	0.15 < L* > 27
Large sphere	0.11	Radius	L* > 4.26
Small sphere	0.227 $L^{*-0.5}$	Radius	0.15 < L* > 4.26

4.2.2.2 Film boiling:

The heat flux for film boiling on a horizontal cylinder or sphere of diameter d is given by

$$q_{film} = C_{film}\left[\frac{g k_v^3 \rho_v \left(\rho_l - \rho_v\right)\left[h_{fg} + 0.4 c_{pv}\left(T_s - T_{sat}\right)\right]}{\mu_v D\left(T_s - T_{sat}\right)}\right]^{1/4}\left(T_s - T_{sat}\right) \qquad (4.12)$$

Where k_v is the thermal conductivity of the vapour in W/m.^{0}C and

$$C_{film}\begin{Bmatrix} 0.62 \; for\; horizontal\; cylinder \\ 0.67 \; for\; sphere \end{Bmatrix} \qquad (4.13)$$

4.2.3 Flow Boiling:

In flow boiling, the fluid is forced to move by an external source such as a pump as it undergoes a phase-change process. The boiling in this case exhibits the combined effects of convection and pool boiling. The flow boiling is also classified as either external or internal flow boiling depending on whether the fluid is forced to flow over a heated surface or inside a heated tube.

The different stages encountered in flow boiling in a heated tube are illustrated in Fig. 4.7 together with the variation of the heat transfer coefficient along the tube. Initially, the liquid is sub-cooled and heat transfer to the liquid is by forced convection. Then bubbles start forming on the inner surfaces of the tube, and the detached bubbles are drafter into the mainstream. This gives the fluid flow a bubbly appearance, and thus the name bubbly flow regime. As the fluid is heated further, the bubbles grow in size and eventually coalesce into slugs of vapour. Up to half of the volume in the tube in this slug flow regime is occupied by vapour. After a while the core of the flow consists of vapour only, and the liquid is confined only in the annular space between the vapour core and the tube walls. This is the annular-flow regime, and very high heat transfer coefficients are realized in this regime. As the heating continues, the annular liquid layer gets thinner and thinner, and eventually dry spots start to appear on the inner surfaces of the tube. The appearance of dry spots is accompanied by a sharp decrease in the heat transfer coefficient. This transition regime continues until the inner surface of the tube is completely dry. Any liquid at his moment is in the form of droplets suspended in the vapour core, which resemble a mist, and we have a mist-flow regime until all the liquid droplets are vaporized. At the end of the mist-flow regime we have saturated vapour, which becomes superheated with any further heat transfer.

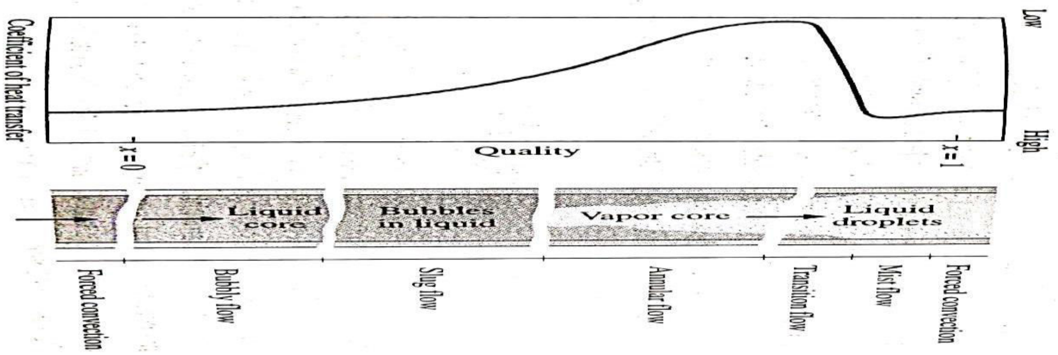


Figure 4.7: Different flow regimes encountered in flow boiling in a tube under forced convection

4.3 CONDENSATION HEAT TRASNFER

Condensation occurs when the temperature of a vapour is reduced below its saturation temperature T_{sat}. This is usually done by brining the vapour into contact with a solid surface whose temperature T_s is below the saturation temperature T_{sat} of the vapour.

Two distinct forms of condensation are observed: drop-wise condensation and film-wise condensation.

- In drop-wise condensation, the condensed vapour forms droplets on the surface instead of a continuous film, and the surface is covered by countless droplets of varying diameters.
- In film condensation, the condensate wets the surface and forms a liquid film on the surface that slides down under the influence of gravity. The thickness of the liquid film increases in the flow direction as more vapour condenses on the film. This is how condensation normally occurs in practice.
- In film condensation, the surface is blanketed by a liquid film of increasing thickness, and this "liquid wall" between solid surface and the vapour serves as a resistance to heat transfer.

4.3.1 Condensation on a vertical plate

Consider a vertical plate of height L and width b maintained at a constant temperature T_s that is exposed to vapour at the saturation temperature T_{sat}. The downward direction is taken as the position x-direction with the origin placed at the top of the plate where condensation initiates, as shown in Fig. 4.8. The surface temperature is below the saturation temperature ($T_s<T_{sat}$) and thus the vapour condenses on the surface. The liquid film flows downward under the influence of gravity. The film thickness δ and thus the mass flow rate of the condensate increases with x as a result of continued condensation on the existing film. Then heat transfer from the vapour to the plate must occur through the film, which offers resistance to heat transfer. Obviously the thicker the film, the larger its thermal resistance and thus the lower the rate of heat transfer.

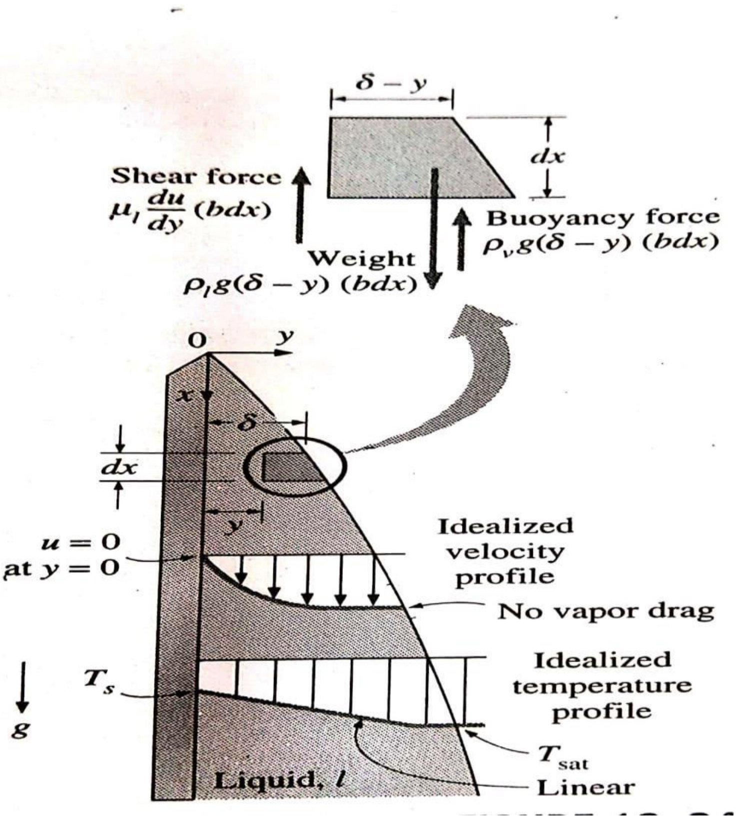


Figure 4.8: Condensation on vertical plate

4.3.2 Film Condensation - Heat Transfer Correlations for Film Condensation

Vertical plates:

The thickness of film at any position x along the plate is given as

$$\delta(x) = \left[\frac{4\mu_1 k_1 (T_{sat} - T_s)x}{g\rho_1(\rho_1 - \rho_v)h_{fg}}\right]^{1/4} \qquad (4.14)$$

The heat transfer rate from the vapour to the plate at a location x in laminar region can be expressed as

$$h_x = \left[\frac{g\rho_l(\rho_l - \rho_v)h_{fg}k_1^3}{4\mu_l(T_{sat} - T_s)x}\right]^{1/4} \qquad (4.15)$$

The average heat transfer coefficient over the entire plate in laminar region is determined from its definition by substituting the h_x relation and performing the integration. It gives

$$h = h_{avg} = \frac{1}{L}\int_0^L h_x dx = \frac{4}{3}h_{x=L} = 0.943\left[\frac{g\rho_l(\rho_l - \rho_v)h_{fg}k_1^3}{\mu_l(T_{sat} - T_s)L}\right]^{1/4} \quad (W/m^2.^0C),\ 0 < \text{Re} < 30 \qquad (4.16)$$

where

- g = gravitation acceleration, m/s^2
- p_l, p_v = densities of the liquid and vapour, respectively, kg/m^3
- μ_l = viscosity of the liquid, kg/m.s
- h^*_{fg} = latent heat of vaporization, J/kg
- k_1 = thermal conductivity of the liquid, W/m.^{0}C
- L = height of the vertical plate, m
- T_s = surface te4mperatur of the plate, ^{0}C
- T_{sat} = saturation temperature of the condensing fluid, ^{0}C

Horizontal Tubes and Spheres

Nusselt's analysis of film condensation on vertical plates can also be extended to horizontal tubes and spheres. The average heat transfer coefficient for film condensation on the outer surfaces of a horizontal tube is determined to be

$$h_{vert} = 0.729\left[\frac{g\rho_l(\rho_l - \rho_v)h_{fg}^{*}k_1^3}{\mu_l(T_{sat} - T_s)D}\right]^{1/4} \qquad (W/m^2.^0C) \qquad (4.17)$$

Where D is the diameter of the horizontal tube, Equation 10-31 can easily be modified for as sphere by replacing he constant 0.729 by 0.815.

A comparison of the heat transfer coefficient relations for a vertical tube of height L and a horizontal tube of diameter D yields.

$$\frac{h_{vert}}{h_{horiz}} = 1.29\left(\frac{D}{L}\right)^{1/4} \qquad (4.18)$$

Horizontal Tube Banks

Horizontal tubes stacked on top of each other as shown in Fig. 10-28 are commonly used in condenser design. The average thickness of the liquid film at the lower tubes is much larger as a result of condensate falling on top of them from the tubes directly above. Therefore, the average heat transfer coefficient at the lower tubes in such arrangements is smaller.

$$h_{horiz,N\,tubes} = 0.729\left[\frac{g\rho_l(\rho_l - \rho_v)h_{fg}^{*}k_1^3}{\mu_l(T_{sat} - T_s)ND}\right]^{1/4} = \frac{1}{N^{1/4}}h_{horiz,1tube} \qquad (4.19)$$

A. Questions at remembering / understanding level:

I) Objective Questions

1. For the natural convection heat transfer from a vertical plate, which of the following is/are true? [CO4][BL2]
   i. The thickness of the boundary layers increases along the plate from the bottom to the top when the surface temperature is less than the fluid temperature.
   ii. The thickness of the boundary layers increases along the plate from the top to the bottom when the surface temperature is less than the fluid temperature.
   iii. The thickness of the boundary layers increases along the plate from the bottom to the top when the surface temperature is more than the fluid temperature.
   iv. The thickness of the boundary layers increases along the plate from the top to the bottom when the surface temperature is more than the fluid temperature.

   a) i & iii b) ii & iv c) ii & iii d) ii only
2. Which of the following statement(s) is/are true for the natural convection heat transfer from a horizontal plate? [CO4][BL2]
   i. The rate of heat transfer is more when the hot surface faces up.
   ii. The rate of heat transfer is less when the hot surface faces up.
   iii. The rate of heat transfer is more when the cold surface faces down
   iv. The rate of heat transfer is less when the cold surface faces down.

   a) ii & iv b) i & iii c) ii & iii d) ii only
3. Consider laminar natural convection from a vertical hot-plate. Which of the following statement is correct? [CO4][BL2]
   a. The heat flux is higher at the top.
   b. The heat flux is higher at the bottom.
   c. The heat flux remains the same along the entire length of the plate.
   d. Depends on fluid temperature.

4. Will a hot horizontal plate whose back side is insulated cool faster or slower or remains same when it's hot surface is facing down instead of up? [CO4][BL2]
    a. Faster b. slower c. remains same d. can't say
5. What is the reason for heater coil burnout in pool boiling? [CO5][BL2]
    a. Rate of heat transfer from the surface is reduced due to formation of liquid film.
    b. Rate of heat transfer from the surface is reduced due to formation of vapour film.
    c. Excessive excess temperature.
    d. None of the above.
6. Assertion (A): In film boiling, the rate of heat transfer decreases with increase in excess temperature when radiation effects are minimal.
   Reason (R): The rate of heat transfer is reduced as the heating surface is blanketed with vapour bubbles [CO5][BL2]
    a. Both A and R are correct. R is the correct explanation of A.
    b. Both A and R are correct. R is not the correct explanation of A.
    c. A is correct and R is wrong.
    d. R is correct and A is wrong.
7. Assertion (A): The heat transfer rates experienced in drop wise condensation are higher than those experienced in film wise condensation.
   Reason (R): In drop wise condensation, the surface of the tube is more exposed to hot steam without any film barrier between the surface and steam. [CO5][BL2]
    a. Both A and R are correct. R is the correct explanation of A.
    b. Both A and R are correct. R is not the correct explanation of A.
    c. A is correct and R is wrong.
    d. R is correct and A is wrong.
8. Assertion (A): The heat transfer rates experienced in film wise condensation are lower than those experienced in drop wise condensation.
   Reason (R): In film wise condensation, a blanket of vapour film is formed over the surface of the tube and hence heat transfer rate is reduced. [CO5][BL2]
    a. Both A and R are correct. R is the correct explanation of A.
    b. Both A and R are correct. R is not the correct explanation of A.
    c. A is correct and R is wrong.
    d. R is correct and A is wrong.

II) Descriptive Questions

1. What is natural convection? How does it differ from forced convection? What force causes natural convection currents? [CO4][BL2]
2. Define the Grashof number. What is its physical significance? How does the Rayleigh number differ from the Grashof number? [CO4][BL2]
3. Explain the development of boundary layers in natural convection from a vertical plate with neat sketches. [CO4][BL2]
4. Draw the boiling curve and identify the different boiling regimes. Also, explain the characteristics of each regime. [CO5][BL2]
5. Distinguish between nucleate and film boiling. [CO5][BL2]
6. Draw the boiling curve and identify the burnout point on the curve. Explain how burnout is caused. Why is the burnout point avoided in the design of boilers? [CO5][BL2]
7. What is the difference between film and drop-wise condensation? Which is a more effective mechanism of heat transfer? [CO5][BL2]

8. Consider film condensation on the outer surfaces of tube whose length is 10 times it diameter. For which orientation of the tube, will the heat transfer rate be the highest: horizontal or vertical? Explain. Disregard the base and top surfaces of the tube. [CO5][BL2]

B. Question at applying / analyzing level

I) Multiple choice questions.

Common data for questions 1, 2 and 3.

A large vertical plate 4.0 m high is maintained at 60^0C and exposed to atmospheric air at 10^0C. [CO4][BL3]

1. The Rayleigh number is ----

   a. 4.62×10^{10} b. 2.62×10^{11} c. 2.62×10^{11} d. 4.62×10^{11}

2. The average heat transfer coefficient (W/m^2.K) is -----

   a. 4.80 b. 3.80 c. 2.80 d. 5.80

3. The rate of heat transfer (W) if the plate is 10 m wide is -----

   a. 8600 b. 6600 c. 7600 d. 9600

Common data for questions 4, 5, and 6.

A vertical square plate, 30 by 30 cm, is exposed to steam at atmospheric pressure. The plate temperature is 98^0C. [CO5][BL3]

4. The condensing heat transfer coefficient (W/m^2.K) is ----

   a. 13150 b. 11150 c. 14150 d. 16150

5. The rate of heat transfer (W) is ---------

   a. 1367 b. 1867 c. 2367 d. 2867

6. The mass of steam condensed in kg per hour is --------

   a. 1.78 b. 2.78 c. 4.78 d. 3.78

Common data for questions 7, 8, and 9.

Water is to be boiled at atmospheric pressure in a mechanically polished stainless steel pan placed on top of a heating unit. The inner surface of the bottom of the pan is maintained at 108^0C. The diameter of the bottom of the pan is 30 cm. [CO5][BL3]

7. The heat flux (W/m^2) in nucleate boiling is ------

   a. 7.21×10^4 b. 5.21×7^4 c. 2.21×10^4 d. 1.21×10^4

8. The rate of heat transfer (W) during nucleate boiling is -----

   a. 2097 b. 3097 c. 5097 d. 7097

9. The rate of evaporation of water (kg/s) is -----------

   a. 1.26×10^{-3} b. 2.26×10^{-3} c. 3.26×10^{-3} d. 4.26×10^{-3}

II) Problems

1. A 40 cm square vertical plate is heated electrically such that a constant-heat-flux condition is maintained with a total heat dissipation of 40 W. The ambient air is at 1 atm and 20^0C. Calculate the value of the heat-transfer coefficient at a height of 15 cm. Also calculate the average heat-transfer coefficient for the plate. [CO4][BL3]
2. A vertical flat plate is maintained at a constant temperature of 50.0^0C and exposed to atmospheric air at 25.0^0C. At a distance of 40.0 cm. from the leading edge of the plate the boundary layer thickness is 2.54 cm. Estimate the thickness of the boundary layer at a distance of 61 cm from the leading edge. [CO4][BL3]
3. A 20 by 20 cm vertical plate is fitted with an electric heater which produces a constant heat flux of 900 W/m^2. The plate is submerged in water at 20^0C. Calculate the heat transfer coefficient and the average temperature of the plate. How much heat would be lost by an isothermal surface at this average temperature? [CO4][BL4]
4. A horizontal pipe 5.0 cm in diameter is located in a room where atmospheric air is at 25^0C. The surface temperature of the pipe is 150^0C. Calculate the free-convection heat loss per meter length of pipe. [CO4][BL3]

5. Consider a 0.5 m × 0.5 m thin square plate in a room at 25^0C. One side of the plate is maintained at a temperature of 100^0C, while the other side is insulated. Determine the rate of heat transfer from the plate by natural convection if the plate is (a) vertical, (b) horizontal with hot surface facing up, and (c) horizontal with hot surface facing down. [CO4][BL4]
6. A large vertical plate 6.0 m high and 1.20 m wide is maintained at a constant temperature of 60^0C and exposed to atmospheric air 5^0C. Calculate the heat lost by the plate. [CO4][BL3]
7. A 30 cm square vertical plat is maintained at 90^0C and exposed to saturated water vapor at 1 atm pressure. Calculate the condensation rate and film thickness at the bottom of the plate. [CO5][BL3]
8. A 5 mm diameter copper heater rod is submerged in water at 1 atm. The temperature excess is 11^0C. Estimate the hat loss per unit length of the rod. [CO5][BL3]
9. Water is boiled at 90^0C by a horizontal brass heating element of diameter 5 mm. determine the maximum heat flux that can be attained in the nucleate boiling regime. [CO5][BL3]
10. Water is boiled at 100^0C by a 0.5 m long and 1.2 cm diameter nickel-plated electric heating element maintained at 125^0C. Determine (a) the boiling heat transfer coefficient, (b) the electric power consumed by the heating element, and (c) the rate of evaporation of water. [CO5][BL4]
11. A 60 cm long 2.5 cm diameter brass heating element is to be used to boil water at 120^0C. If the surface temperature of the heating element is not to exceed 130^0C, determine the highest rate of steam production in the boiler, in kg/h. [CO5][BL3]
12. Saturated steam at 50^0C is to be condensed at a rate of 8 kg/h on the outside of a 2.5 cm outer diameter vertical tube whose surface is maintained at 40^0C by the cooling water. Determine the required tube length. [CO5][BL3]
13. The condenser of a steam power plant operates at a pressure of 4.50 kPa. The condenser consists of 100 horizontal tubes arranged in a 10 x 10 square array. The tubes are 6 m long and have an outer diameter of 2.5 cm. If the tube surfaces are at 22^0C, determine (a) the rate of heat transfer from the steam to the cooling water and (b) the rate of condensation of steam in the condenser. [CO5][BL4]
14. Saturated steam at 45^0C is condensed on a 1.5 m high vertical plate that is maintained at 35^0C. Determine the rate of heat transfer from the steam to the plate and the rate of condensation per meter width of the plate. [CO5][BL3]
15. Consider a 15 cm × 20 cm printed circuit board (PCB) that has electronic component on one side. The board is placed in a room at 20^0C. The heat loss from the back surface of the board is negligible. If the circuit board is dissipating 8 W of power in steady operation, determine the average temperature of the hot surface of the board, assuming the board (a) vertical; (b) horizontal with hot surface facing up; and (c) horizontal with hot surface facing down. Assume the surrounding surfaces to be at the same temperature as the air in the room. [CO4][BL4]

C. Questions at evaluating / creating level:

1. Consider a 1.2 m high and 2 m wide glass window with a thickness of 6 mm, thermal conductivity k = 0.78 W/m.^{0}C, and emissivity ε=0.9. The room and the walls that face the window are maintained at 25^0C, and the average temperature of the inner surface of the outdoors is -5^0C, determine (a) the convection heat transfer coefficient on the inner surface of the window, (b) the rate of total heat transfer through the window, and (c) the rate of the total heat transfer through the window, and (c) the combined

natural convection and radiation heat transfer coefficient on the outer surface of the window. Is it reasonable to neglect the thermal resistance of the glass in this case?

[CO4][BL5]

2. Design the condenser of a steam power plant that has a thermal efficiency of 40 percentage and generates 10 MW of net electric power. Steam enters the condenser as saturated vapor at 10 kPa, and it is to be condensed outside horizontal tubes through which cooling water from a nearby river flows. The temperature rise of the cooling water is limited to 8^0C, and the velocity of the cooling water in the pipes is limited to 6 m/s to keep the pressure drop at an acceptable level. Specify the pipe diameter, total pipe length, and the arrangement of the pipes to minimize the condenser volume.

[CO5][BL6]

3. The technology for power generation using geothermal energy is well established, and numerous geothermal power plants throughout the world are currently generating electricity economically. Binary geothermal plants utilize a volatile secondary fluid such as isobutene, n-pentane, and R-114 in a closed loop. Consider a binary geothermal plant with R-114 as the working fluid that is flowing at a rate of 600 kg/s. The R-114 is vaporized in a boiler at 115^0C by the geothermal fluid that enters at 165^0C and is condensed at 30^0C outside the tubes by cooling water that enters the tubes at 18^0C. Design the condenser of the binary plant.

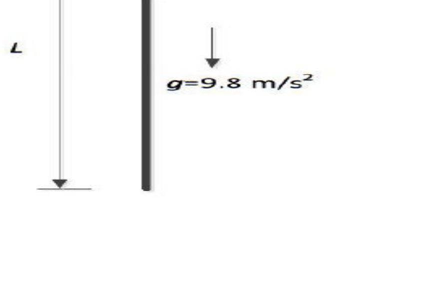
Specify (a) the length, diameter, and number of tubes and their arrangement in the condenser, (b) the mass flow rate of the cooling water, and (c) the flow rate of make-up water needed if a cooling tower is used to reject the waste heat from the cooling water. The liquid velocity is to remain under 6 m/s and the length of the tubes is limited to 8 m. [CO5][BL6]

D. GATE Questions:

1. The average hat transfer coefficient on a thin hot vertical plate suspended in still air can be determined from observations of the change in plate temperature with time as it cools. Assume the plate temperature to be uniform at any instant of time and radiation heat exchange with the surroundings negligible. The ambient temperature is 25^0C, the plate has a total surface area of 0.1 m^2 and a mass of 4 kg. The specific heat of the plate material is 2.5 kJ/kg.K. The convective heat transfer coefficient in W/m^2K, at the instant when the plate temperature is 225^0C and the change in plate temperature with time dT/dt=-K/s, is [GATE-2007]

   a) 200 b) 20 c) 15 d) 10

2. A thin vertical flat plate of height L, and infinite width perpendicular to the plane of the figure, is losing heat to the surroundings by natural convection. The temperatures of the plate and the surroundings, and the properties of the surrounding fluid, are constant. The relationship between the average Nusselt and Rayleigh numbers are the height of the plate. The height of the plate is increased to 16L keeping all other factors constant. [GATE-2019]



If the average heat transfer coefficient for the first plate is h1 and that for the second plate is h2 the value of the ratio h1/h2 is----------

E. IES Question:

1. Assertion (A): A slab of finite thickness heated on one side and held horizontal will lose more heat per unit time to the cooler air it the hot surface faces upwards when compared with the case where the hot surface faces downwards. [IES-1996]
   Reason (R): When the hot surface faces upwards, convection takes place easily whereas when the hot surface faces downwards, heat transfer is mainly by conduction through air.
   (a) Both A and R are true, and R is correct explanation for A
   (b)
2. In respect of free convection over a vertical flat plate the Nusselt number varies with Grashof number 'Gr' as [IES-2000]
   (a) Gr and $Gr^{1/4}$for laminar ad turbulent flows respectively.
   (b) $Gr^{1/2}$ and $Gr^{1/3}$ for laminar and turbulent flows respectively.
   (c) $Gr^{1/4}$ and $Gr^{1/3}$ for laminar and turbulent flows respectively.
   (d) $Gr^{1/3}$ and $Gr^{1/4}$ for laminar and turbulent flows respectively.
3. Heat is lost from a 100 mm diameter steam pipe placed horizontally in ambient at 30^0C. If the Nusselt number is 25 and thermal conductivity of air is 0.03 W/mK, then the heat transfer co-efficient will be [IES-1999]
   (a) 7.5 W/m^2K (b) 16.2 W/m^2K (c) 25.2 W/m^2K (d) $30W/m^2K$
4. Which one of the following non-dimensional numbers is used for transition from laminar to turbulent flow in free convection? [IES-2007]
   (a) Reynolds number (b) Grashof number
   (c) Peclet number (d) Rayleigh number
5. For natural convective now over a vertical flat plate as shown in the given figure, the governing differential equation for momentum is [IES-2001]
   $$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=g\beta(T-T_\infty)+\gamma\frac{\partial^2 u}{\partial y^2}$$
   If equation is non-dimensionalized by $U=\frac{u}{U_\infty}, v\frac{u}{U_\infty} X=\frac{x}{L}, y=\frac{y}{L} and\ \theta=\frac{T-T_\infty}{T_s-T_\infty}$ then the term $g\beta(T-T_\infty)$, is equal to
   (a) Grahof number (b) prandtl number
   (c) Rayleigh number (d) $\frac{Grashof\ number}{(\text{Re}\ ynolds\ number)}$
6. A 320 cm high vertical pipe at 150^0C wall temperature is in a room with still air at 10^0C. This pipe supplies heat at the rate of 8 kW into the room air by natural convection. Assuming laminar flow, the height of the pipe needed to supply 1kW only is [IES-2002]
   (a) 10 cm (b) 20 cm (c) 40 cm (d) 80 cm
7. The average Nusselt number is laminar natural convection from a vertical wall at 180^0C with still air at 20^0C is found to be 48. If the wall temperature becomes 30^0C, all other parameters remaining same, the average Nusselt number will be [IES-2002]
   (a) 8 (b) 16 (c) 24 (d) 32
8. In the case of turbulent flow through a horizontal isothermal cylinder of diameter 'D' free convection heat transfer coefficient from the cylinder will [IES-1997]
   (a) Be independent of diameter (b) vary as $D^{3/4}$(c) vary as $D^{1/4}$ (d) vary as $D^{1/2}$

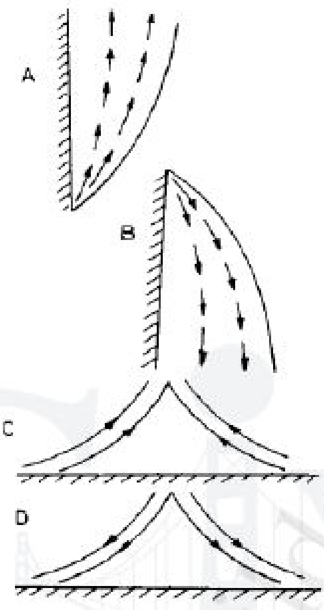
9. Given that N_u=Nusselt number, R_e=Reynolds number, P_r=Prandlt number, S_h=Sherwood number, S_c=Schmidt number and G_r=Grashoff number the functional relationship for free convective mass transfer is given as:
   (a) $N_h = f(G_r, P_r)$ (b) $S_h = f(S_c, G_r)$ (c) $N_u = f(R_r, P_r)$ (d) $S_h = f(R_e, S_c)$

10. A cube at high temperature is immersed in a constant temperature bath. It loses heat from its top, bottom and side surface with heat transfer coefficient of h_1, h_3 and h_3 respectively. The average heat transfer coefficient for the cube is [IES-1996]
   (a) $h_1+h_2+h_3$ (b) $(h_1+h_2+h_3)^{1/3}$ (c) $\frac{1}{h_1}+\frac{1}{h_2}+\frac{1}{h_3}$ (d) none of the above

11. In free convection heat transfer transition from laminar to turbulent flow is governed by the critical value of the [IES-1992]
   (a) Reynolds number (b) Grashoff's number
   (c) Reynolds number, Grashoff number (d) Prandtl number, Granshoff number

12. Match list I with list II and select the correct answer using the codes given below the lists: [IES-1995]

List I (Flow Pattern) List II (Situation)



1. Heated horizontal plate
2. Cooled horizontal plate
3. Heated vertical plate
4. Cooled vertical plate

Codes:	A	B	C	D		A	B	C	D
(a)	4	3	2	1	(b)	3	4	1	2
(c)	3	4	2	1	(d)	4	3	1	2

13. Free convection flow depends on all of the following EXCEPT [IES-1992]
   (a) density (b) coefficient of viscosity (c) gravitational force (d) velocity

14. consider the following statements: [IES-1997]
   The effect of fouling in a water-cooled steam condenser is that is
   1. reduces the overall heat transfer coefficient of water.
   2. Reduces the overall heat transfer coefficient.
   3. Reduces the area available for heat transfer.
   4. Increases the pressure drop of water.
   Of these statements
   (a) 1, 2 and 4 are correct (d) 2, 3 and 4 are correct
   (c) 1 and 4 are correct (d) 1 and 3 are correct

15. Consider the following phenomena: [IES-1997]

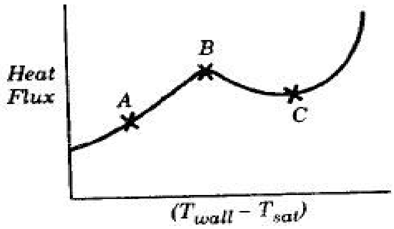
1. Boiling 2. Free convection in air 3. Forced convection 4. Conduction in air.
Their correct sequence in increasing order of heat transfer coefficient is:
(a) 4, 2, 3, 1 (b) 4, 1, 3, 2 (c) 4, 3, 2, 1 (d) 3, 4, 1, 2

16. Consider the following statements regarding condensation heat transfer: [IES-1996]
1. For a single tube, horizontal position is preferred over vertical position for better heat transfer.
2. Heat transfer coefficient decreases if the vapour steam moves at high velocity.
3. Condensation of steam on an oily surface is drop wise.
4. Condensation of pure benzene vapour is always drop wise.

Of these statements
(a) 1 and 2 are correct(b) 2 and 4 are correct
(c) 1 and 3 are correct (d) 3 and 4 are correct

17. Drop wise condensation usually occurs on [IES-1992]
(a) glazed surface (b) smooth surface (c) oily surface (d) coated surface

18. consider the following statements regarding nucleate boiling: [IES-1995]
1. The temperature of the surface is greater than the saturation temperature of the liquid.
2. Bubbles are created by the expansion of entrapped gas or vapour at small cavities in the surface.
3. The temperature is greater than that of film boiling.
4. The heat transfer from the surface to the liquid is greater than that in film boiling.



Of these correct statements are:
(a) 1, 2 and 4 (b) 1 and 3 (c) 1, 2, and 3 (d) 2, 3, and 4

19. The burnout heat flux in the nucleate boiling regime is a function of which of the following properties?
1. Heat of evaporation 2. Temperature difference
3. Density of vapour 4. Density of liquid 5.vapour-liquid surface tension.
Select the correct answer using the codes given below:
Codes: (a) 1, 2, 4 and 5 (b) 1, 2, 3 (c) 1, 3, 4 and 5 (d) 2, 3 and 4

20. The given figure shows a pool-boiling curve. Consider the following statements in this regard:
1. Onset of nucleation causes a marked change in slope.
2. At the point B, hat transfer coefficient is the maximum.
3. In an electrically heated wire submerged in liquid, film heating is difficult to achieve.
4. Beyond the point C, radiation becomes significant.

Of these statements
(a) 1, 2, and 4 are correct (b) 1, 3 and 4 are correct
(c) 2, 3 and 4 are correct (d) 1, 2 and 3 are correct

21. Assertion (A): IF the heat fluxes in pool boiling over a horizontal surface is increased above the critical heat flux, the temperature difference between the surface and liquid decreases sharply [IES-2003]
Reason (R): With increasing heat flux beyond the value corresponding to the critical heat flux, a stage is reached when the rate of formation of bubbles is so high that they start to coalesce and blanket the surface with a vapour film.
(a) Critical heat flux for nucleate pool boiling
(b) Film boiling
(c) Condensation Heat Transfer

22. Saturated steam is condense over a vertical flat surface and the condensate film flows down the surface. The local heat transfer coefficient for condensation [IES-1999]
    (a) remains constant at all locations of the surface.
    (b) decreases with increasing distance from the top of the surface
    (c) increases with increasing thickness of condensate film
    (d) increases with decreasing temperature differential between the surface and vapour.
23. Consider the following statements: [IES-1998]
    1. If a condensing liquid does not wet a surface drop wise, then condensation will take place on it.
    2. Drop wise condensation gives a higher heat transfer rate than film-wise condensation.
    3. Reynolds number of condensing liquid is based on its mass flow rate.
    4. Suitable coating or vapour additive is used to promote film-wise condensation.

    Of these statements:
    (a) 1 and 2 are correct (b) 2, 3 and 4 are correct
    (c) 4 alone is correct (d) 1, 2 and 3 are correct
24. Assertion (A): Even though drop wise condensation is more efficient, surface condensers are designed on the assumption of film wise condensation as a matter of practice [IES-1995]
    Reason (R): Drop wise condensation can be maintained with the use of promoters like oleic acid.
    (a) Both A and R are true and R provides satisfactory explanation for A.
    (b) A and R are true and A provides satisfactory explanation for R.
    (c) Both are true and R provides satisfactory explanation for A.
    (d) A and R are true and R provides satisfactory explanation for A.
25. Assertion (A): The rate of condensation over a rusty surface is less than that over a polished surface [IES-1995]
    Reason (R): The polished surface promotes drop wise condensation which does not wet the surface.
    (a) Both A and R are true and R provides satisfactory explanation for A.
    (b) A and R are true and A provides satisfactory explanation for R.
    (c) Both are true and R provides satisfactory explanation for A.
    (d) A and R are true and R provides satisfactory explanation for A.

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Academic Year: 2019 – 20 **Semester: II**

Class : III B.Tech **Subject: Heat Transfer**

Learning Material

UNIT-V

Heat Exchangers: Classification, overall heat transfer Coefficient, fouling factor, Design of Heat Exchangers – LMTD and NTU methods.

HEAT EXCHANGERS

INTRODUCTION

A heat exchanger is a device that enables heat transfer between two or more fluids that are at different temperatures.

The applications of heat exchangers are multi-fold, which include chemical processing plants, automobile radiators, steam power plants, gas turbine power plants, domestic and commercial refrigerators, freezers, water coolers, air-conditioners, food processing systems, nuclear power plants, space radiators, aeronautical system, and so on.

The heat exchanger design is quite a complex proposition, as it should address rate of heat transfer, pressure drop, sizing and economic aspects mainly, in addition to several other secondary prerequisites. As an example, applications like power plants and chemical processing plants call for cost minimization, while space and aeronautical applications pose restrictions on size and weight factors.

Heat transfer in a heat exchanger usually involves convection in each fluid and conduction through the wall separating the two fluids. In the analysis of heat exchangers, it is convenient to work with an overall heat transfer coefficient U that accounts for the contribution of all these effects on heat transfer. The rate of heat transfer between the two fluids at a location in a heat exchanger depends on the magnitude of the temperature difference at that location, which varies along the heat exchanger.

TYPES OF HEAT EXCHANGERS

The simplest type of heat exchanger consists of two concentric pipes of different diameters, as shown in Fig. 5.1, called the double-pipe heat exchanger. One fluid in a double-pipe heat exchanger flows through the smaller pipe while the other fluid flows through the annular space between the two pipes.

Two types of flow arrangement are possible in a double-pipe heat exchanger: in parallel flow, both the hot and cold fluids enter the heat exchanger at the same end and move in the same direction. In counter flow, on the other hand, the hot and cold fluids enter the heat exchanger at opposite ends and flow in opposite directions.

Another type of heat exchanger, which is specifically designed to realize a large heat transfer surface area per unit volume, is the compact heat exchanger. The ratio of the heat transfer surface area of a heat exchanger to its volume is called the area density β. A heat exchanger with β >700 m^2/m^3 (or 200 ft^2/ft^3) is classified as being compact. Examples of compact heat exchangers are car radiators ($\beta \approx 1000 m^2/m^3$), glass-ceramic gas turbine heat exchangers ($\beta \approx 6000 m^2/m^3$), the regenerator of s Stirling engine ($\beta \approx 15{,}000 m^2/m^3$), and the human lung ($\beta \approx 20{,}000 m^2/m^3$). Compact heat exchangers enable us to achieve high heat transfer between two fluids in a small volume, and they are commonly used in applications with strict limitation s on the weight and volume of heat exchangers.

The large surface area in compact heat exchangers is obtained by attaching closely spaced thin plate or corrugated fins to the walls separating the two fluids. Compact heat exchangers are commonly used in gas-to-gas and gas-to-liquid (or liquid-to-gas) heat exchangers to counteract the low heat transfer coefficient associated with gas flow with increased surface area. In a car radiator, which is a water-to-air compact heat exchanger, for example, it is no surprise that fins are attached to the air side of the tube surface.

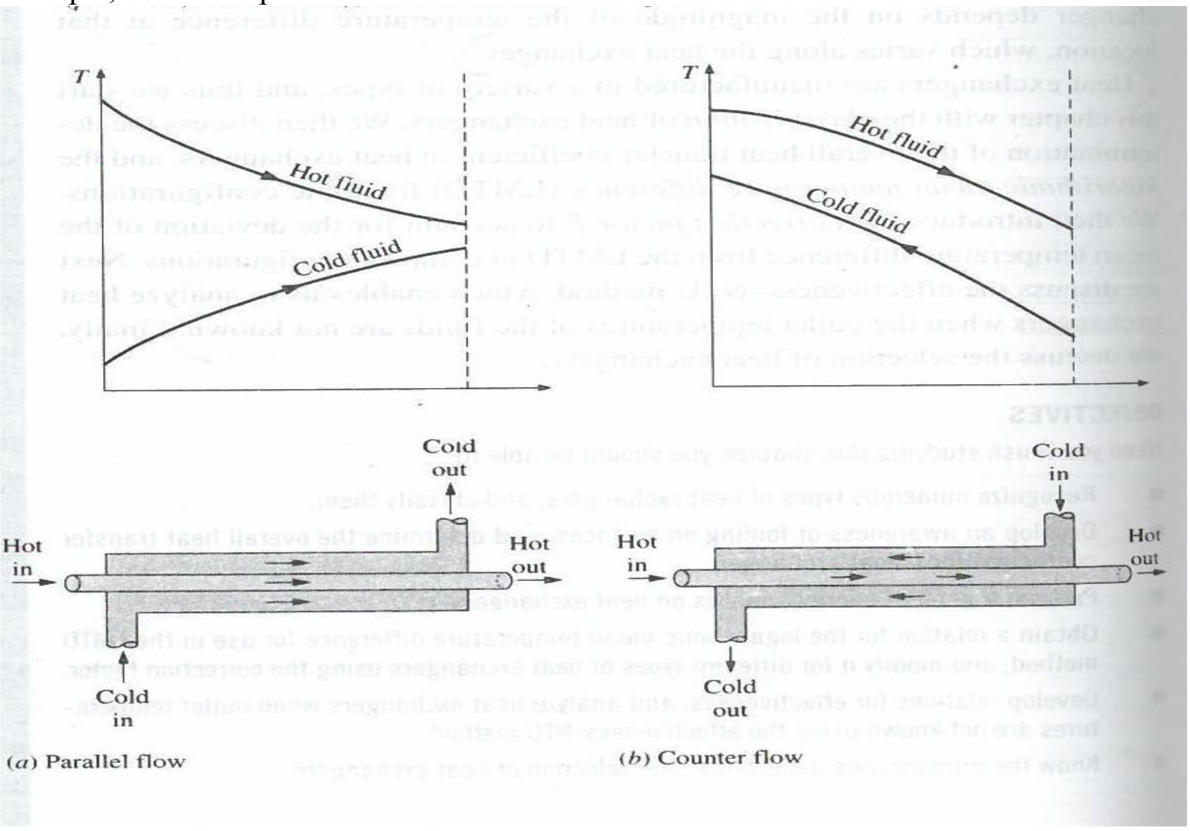


Figure 5.1: Tube-in-Tube Heat Exchanger

In compact heat exchangers, the two fluids usually move perpendicular to each other, and such flow configuration is called cross-flow. The cross-flow is further classified as unmixed and mixed flow, depending on the flow configuration, as shown in Fig. 5.2. In (a) the cross-flow is said to be unmixed since the plate fins force the fluid to flow through a particular inter fin spacing and prevent it from moving in the transverse direction (i.e., parallel to the tubes). The cross-flow in (b) is said to be mixed since the fluid now is free to move in the transverse direction. Both fluids are unmixed in a car radiator.

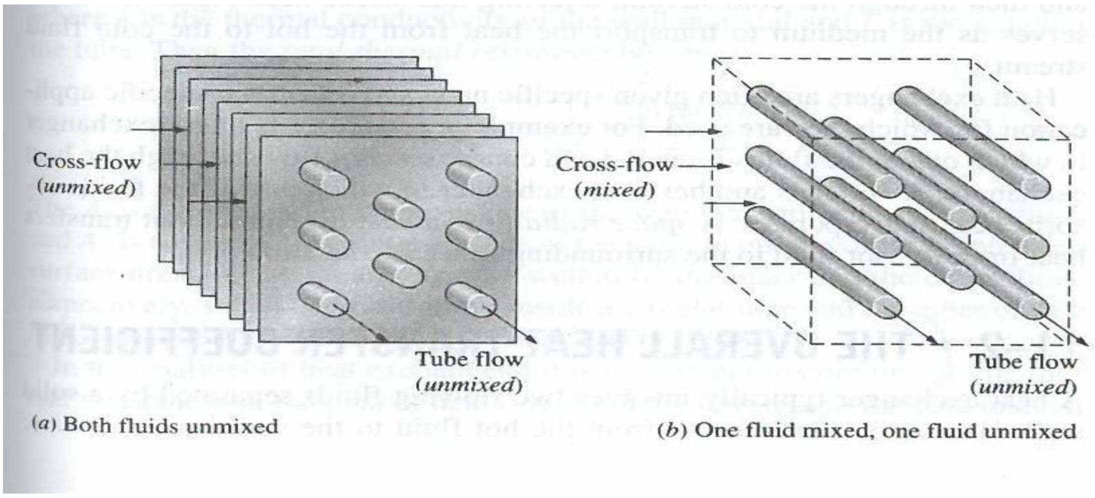


Figure 5.2: Cross flow Heat Exchanger

Perhaps the most common type of heat exchanger in industrial applications is the shell-and-tube heat exchanger, shown in Fig.5.3. Shell-and-tube heat exchangers contain a large number of tubes (sometimes several hundred) packed in a shell with their axes parallel to that of the shell. Heat transfer takes place as one fluid flows inside the tubes while the other fluid flows outside the tubes while the other fluid flows outside the tubes through the shell. Baffles are commonly placed in the shell to force the shell-side fluid to flow across the shell to enhance heat transfer and to maintain uniform spacing between the tubes.

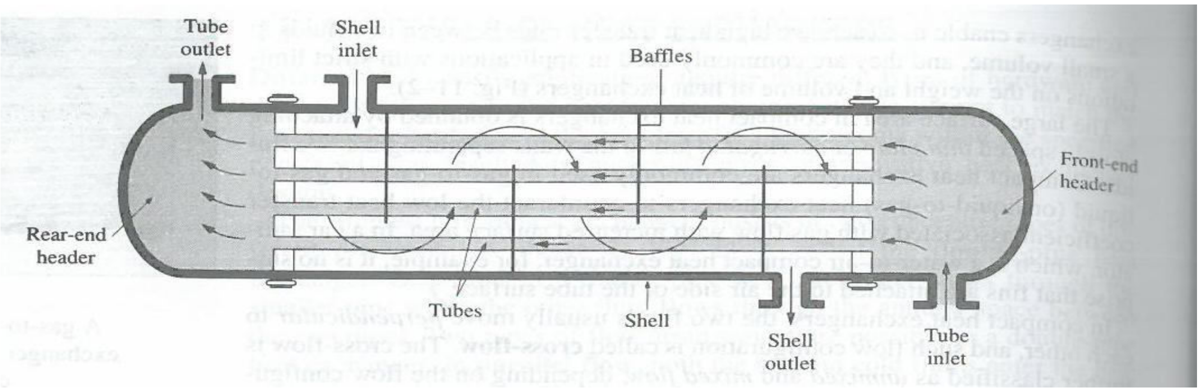
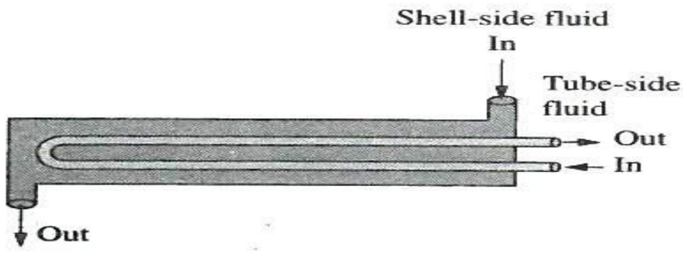
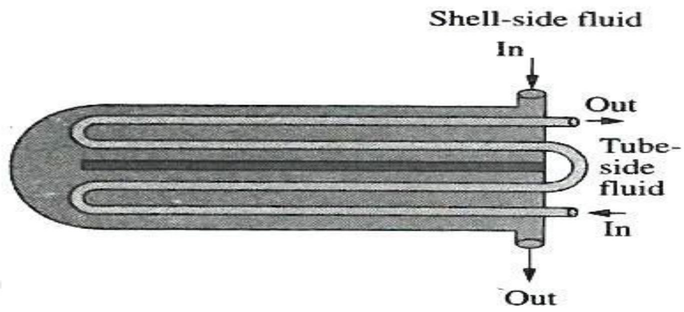


Figure 5.3: Shell and Tube Heat Exchanger

Shell-and-tube heat exchangers are further classified according to the number of shell and tube passes involved. Heat exchangers in which all the tubes make one U turn in the shell, for example, are called one-shell-pass and two-tube-passes heat exchangers (Figure 5.4). Likewise, a heat exchanger that involves two passes in the shell and four passes in the tubes is called a two-shell-passes and four-tube-passes heat exchanger.



(*a*) One-shell pass and two-tube passes



(*b*) Two-shell passes and four-tube passes

Figure 5.4: One-shell pass and Two-tube passes and Two-shell pass and Four-tube Passes Heat Exchangers.

An innovative type foe heat exchanger that has found widespread use is the plate and frame (or just plate) heat exchanger, which consists of a series of plates with corrugated float flow passages (Fig.5.5). The hot and cold fluids flow in alternate passages, and thus each cold fluid stream is surrounded by two hot fluid streams, resulting in very effective heat transfer. Also, plate heat exchangers can grow with increasing demand for heat transfer by simply mounting more plates. They are well suited for liquid-to-liquid heat exchange applications.

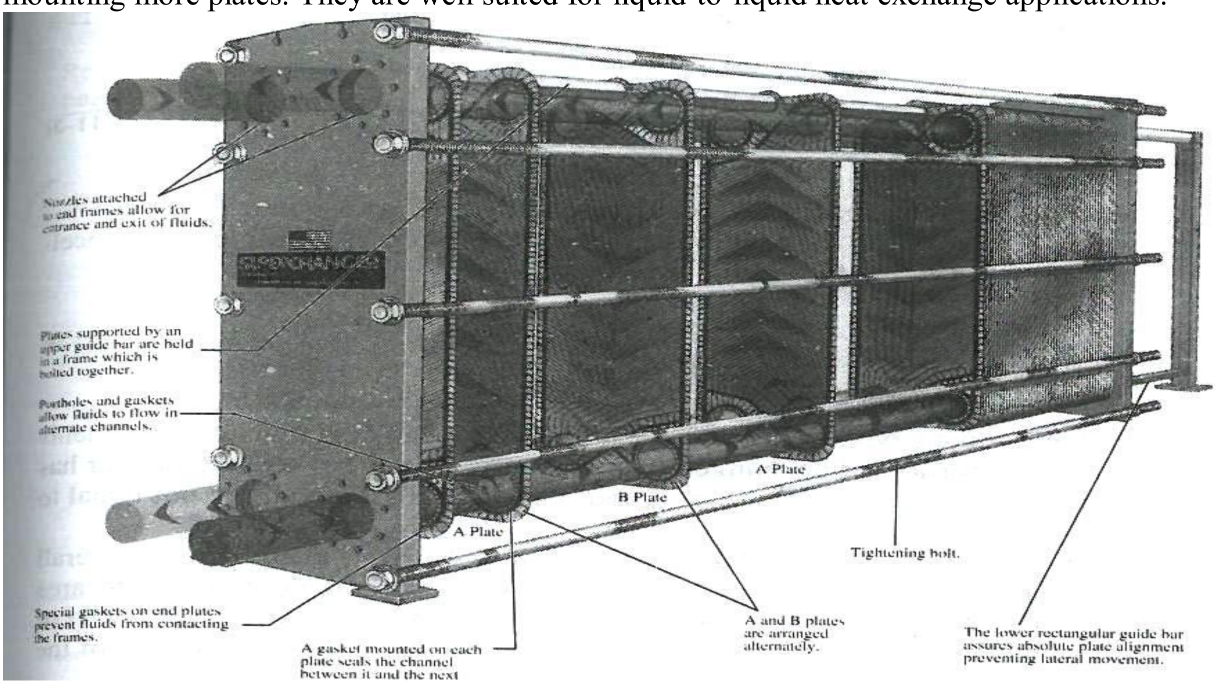


Figure 5.5: A plate-and-frame liquid-to-liquid Heat Exchanger.

Another type of heat exchanger that involves the alternate passage of the hot and cold fluid steams through the same flow area is the regenerative heat exchanger. The static-type regenerative heat exchanger is basically a porous mass that has a large heat storage capacity, such as a ceramic wire mesh. Hot and cold fluids flow through this porous mass alternatively. Heat is transferred from the hot fluid to the matrix to the cold fluid during the flow of the cold fluid. Thus, the matrix serves as a temporary heat storage medium.

The dynamic-type regenerator involves a rotating drum and continuous flow of the hot and cold fluid through different portions of the drum so that any portion of the drum passes periodically through the hot stream, storing heat, and then through the cold stream, rejecting this stored heat. Again the drum serves as the medium to transport the heat from the hot to the cold fluid stream.

A condenser is a heat exchanger in which one of the fluid is cooled and condenses as it flows through the heat exchanger. A boiler is another heat exchanger in which one of the fluids absorbs heat and vaporizes. A space radiator is a heat exchanger that transfers heat from the hot fluid to the surrounding space by radiation.

THE OVERALL HEAT TRANSFER COEFFICIENT

A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by convection, through the wall by conduction, and from the wall to the cold fluid again by convection.

The rate of heat transfer between the two fluids is expressed as

$$Q = UA\Delta T = U_i A_i \Delta T = U_0 A_0 \Delta T \tag{5.1}$$

Where U is the overall heat transfer coefficient, whose unit is $W/m^2.^0C$, which is identical to the unit of the ordinary convection coefficient h.

Typical values of the overall heat transfer are given in Table 5.1

Table 5.1: Representative values of the overall heat transfer coefficients in heat exchangers.

Types of heat exchanger	**U, $W/m^2.^0C$**
Water-to-water	850-1700
Water-to-oil	100-350
Water-to-gasoline or kerosene	300-1000
Feed water heaters	1000-8500
Steam-to-light fuel oil	200-400
Steam-to-heavy fuel oil	50-200
Steam condenser	1000-6000
Freon condenser (water cooled)	300-1000
Ammonia condenser (water cooled)	800-1400
Alcohol condensers (water cooled)	250-700
Gas-to-gas	10-40
Water-to-air in finned tubes (water in tubes)	30-60
	400-850
Steam-to-air in finned tubes (steam in tubes)	30-300
	400-4000

U_i is the overall heat transfer coefficient estimated based on inner surface area (A_i) of the inner tube and U_0 is the overall heat transfer coefficient estimated based on outer surface area (A_0) of the inner tube. U_i and U_0 are given by

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{r_i}{k} ln \frac{r_0}{r_i} + \frac{r_i}{r_0} \frac{1}{h_0}} \quad (5.2)$$

$$U_0 = \frac{1}{\frac{r_0}{r_i} \frac{1}{h_i} + \frac{r_0}{k} ln \frac{r_0}{r_i} + \frac{1}{h_0}} \quad (5.3)$$

FOULING FACTOR

The performance of heat exchangers usually deteriorates with time as a result of accumulation of deposits on heat transfer surfaces. The layer of deposits represents additional resistance to heat transfer and causes the rate of heat transfer in a heat exchanger to decrease. The net effect of these accumulations on heat transfer is represented by fouling factor R_f, which is a measure of the thermal resistance introduced by fouling.

The fouling factor is obviously zero for new heat exchanger and increases with time as the solid deposits build up on the heat exchanger surface. The fouling factor depends on the operating temperature and the velocity of the fluids, as well as the length of service. Fouling increases with increasing temperature and decreasing velocity.

The overall heat transfer coefficient relation given above is valid for clean surfaces and needs to be modified to account for the effects of fouling on both the inner and the outer surfaces of the tube. For a tube-in-tube heat exchanger are an un-finned shell-and-tube heat exchanger, the overall heat exchanger considering fouling resistances can be expressed as

$$U_i = \frac{1}{\frac{1}{h_i} + R_{fi} + \frac{r_i}{k} ln \frac{r_0}{r_i} + \frac{r_i}{r_0} R_{f0} + \frac{r_i}{r_0} \frac{1}{h_0}} \quad (5.4)$$

$$U_0 = \frac{1}{\frac{r_0}{r_i} \frac{1}{h_i} + \frac{r_0}{r_i} R_{fi} + \frac{r_0}{k} ln \frac{r_0}{r_i} + R_{f0} + \frac{1}{h_0}} \quad (5.5)$$

Where $R_{f,i}$ and $R_{f,o}$ are the fouling resistances at the inner and outer surfaces of inner tube.
Representative fouling resistances (thermal resistance due to fouling for a unit surface area) are given in Table 2.

Table 5.2: Fouling resistances for different fluids.

Fluid	$R_f (m^2.^0C/W)$
Distilled water,	
Seawater, river water,	
Boiler feedwater:	
Below 50^0C	0.0001
Above 50^0C	0.0002
Fuel oil	0.0009
Steam (oil-free)	0.0001
Refrigerants (liquid)	0.0002
Refrigerants (vapour)	0.0004
Alcohol vapours	0.0001
Air	0.0004

(Source: Tubular Exchange Manufactures Association)

DESIGN OF HEAT EXCHANGERS

There are two methods of design of heat exchangers, viz.
1. The Log Mean Temperature Difference (LMTD) method and
2. NTU/Effectiveness method.

THE LOG MEAN TEMPERATURE DIFFERENCE (LMTD) METHOD

The Log Mean Temperature Difference (LMTD) is easy to use in heat exchanger analysis when the inlet and the outlet temperatures of the hot and cold fluids are known or can be determined from an energy balance. Once ΔT_{im}, the mass flow rates, and the overall heat transfer coefficient are available, the heat transfer surface area of the heat exchanger can be determined from

$$Q = UA_s \Delta T_{\text{Im}} \tag{5.6}$$

Where U = overall heat-transfer coefficient

A = surface area for heat transfer consider with definition of U

ΔT_m = suitable mean temperature difference across heat exchanger

Consider the double-pipe heat exchanger. The fluid may flow in either parallel flow or counter flow, and the temperature profiles for theses two cases are indicated in Fig. 5.6.

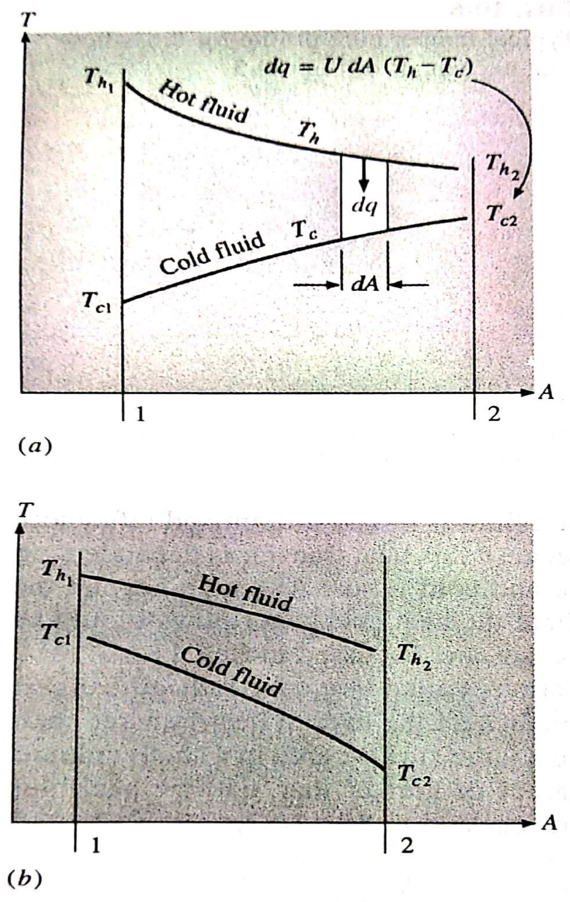


Figure 5.6: Temperature for parallel and counter flow arrangements.

An inspection of Fig. 5.6 shows that the temperature difference between the hot and cold fluids varies between inlet and outlet, and we must determine the average value for use in Eq. (5.6).

Heat transfer rate can also be calculated by energy balance as

$$Q = m_h c_{ph}(\mathrm{T}_{hi} - T_{h0}) = m_c c_{pc}(\mathrm{T}_{c0} - T_{ci}) \tag{5.7}$$

(a) **Parallel-flow Heat Exchanger:**

For the parallel-flow heat exchanger shown in Fig. 5.6(a), the heat transferred through an element of area dA may be written

$$dQ = -m_h c_{ph} dT_h = m_c c_{pc} dT_c \tag{5.8}$$

Where the subscripts h and c designate the hot and cold fluids, respectively. The heat transfer could also be expressed

$$dQ = U(T_h - T_c)dA \tag{5.9}$$

From Eq. (5.8)

$$dT_h = \frac{-dQ}{m_h c_{ph}} \tag{5.10}$$

$$dT_c = \frac{dQ}{m_c c_{pc}} \tag{5.11}$$

Where m represents the mass-flow rate and c_p is the specific heat of the fluid. Thus

$$dT_h - dT_c = d(T_h - T_c) = -dQ\left(\frac{1}{m_h c_{ph}} + \frac{1}{m_c c_{pc}}\right) \tag{5.12}$$

Solving for dQ from Eq. (5.9) and substituting into Eq. (5.12) gives

$$\frac{d(T_h - T_c)}{T_h - T_c} = -U\left(\frac{1}{m_h c_{ph}} + \frac{1}{m_c c_{pc}}\right)dA \tag{5.13}$$

This differential equation may now be integrated between conditions i and o as indicated in Fig. 5.6. The result is

$$\mathrm{In}\,\frac{T_{h0} - T_{c0}}{T_{hi} - T_{ci}} = -UA\left(\frac{1}{m_h c_{ph}} + \frac{1}{m_c c_{pc}}\right) \tag{5.14}$$

Returning to Eq. (5.7), the products $m_c c_{pc}$ and $m_h c_{ph}$ may be expressed in terms of the total heat transfer Q and the overall temperature differences of the hot and cold fluids. Thus

$$m_h c_{ph} = \frac{Q}{T_{hi} - T_{h0}} \tag{5.15}$$

$$m_c c_{pc} = \frac{Q}{T_{c0} - T_{ci}} \tag{5.16}$$

Substituting theses relations into Eq. (5.14) gives

$$Q = UA\frac{(T_{h0} - T_{c0}) - (T_{hi} - T_{ci})}{In\left[(T_{h0} - T_{c0})/(T_{hi} - T_{ci})\right]} \tag{5.17}$$

Comparing Eq. (5.17) with Eq. (5.6), we find that the mean temperature difference is the grouping of terms in the brackets. Thus

$$\Delta T_m = \frac{(T_{h0} - T_{c0}) - (T_{hi} - T_{ci})}{In\left[(T_{h0} - T_{c0})/(T_{hi} - T_{ci})\right]} \tag{5.18}$$

This temperature difference is called the log mean temperature difference (LMTD).

(b) **Counter-flow Heat Exchanger:**

For the counter-flow heat exchanger shown in Fig. 5.6(b), the heat transferred through an element of area dA may be written

$$dQ = -m_h c_{ph} dT_h = -m_c c_{pc} dT_c \tag{5.19}$$

Where the subscripts h and c designate the hot and cold fluids, respectively. The heat transfer could also be expressed

$$dQ = U(T_h - T_c)dA \tag{5.20}$$

From Eq. (5.19)

$$dT_h = \frac{-dQ}{m_h c_{ph}} \tag{5.21}$$

$$dT_c = \frac{-dQ}{m_c c_{pc}} \tag{5.22}$$

Where m represents the mass-flow rate and c_p is the specific heat of the fluid. Thus

$$dT_h - dT_c = d(T_h - T_c) = -dQ\left(\frac{1}{m_h c_{ph}} - \frac{1}{m_c c_{pc}}\right) \tag{5.23}$$

Solving for dQ from Eq. (5.20) and substituting into Eq. (5.23) gives

$$\frac{d(T_h - T_c)}{T_h - T_c} = -U\left(\frac{1}{m_h c_{ph}} - \frac{1}{m_c c_{pc}}\right)dA \tag{5.24}$$

This differential equation may now be integrated between conditions i and o as indicated in Fig. 5.6. The result is

$$\text{In}\frac{T_{h0} - T_{c0}}{T_{hi} - T_{ci}} = -UA\left(\frac{1}{m_h c_{ph}} - \frac{1}{m_c c_{pc}}\right) \tag{5.25}$$

Returning to Eq. (5.7), the products $m_c c_{pc}$ and $m_h c_{ph}$ may be expressed in terms of the total heat transfer Q and the overall temperature differences of the hot and cold fluids. Thus

$$m_h c_{ph} = \frac{Q}{T_{hi} - T_{h0}} \tag{5.26}$$

$$m_c c_{pc} = \frac{Q}{T_{c0} - T_{ci}} \tag{5.27}$$

Substituting theses relations into Eq. (5.14) gives

$$Q = UA\frac{(T_{h0} - T_{ci}) - (T_{hi} - T_{c0})}{In\left[(T_{h0} - T_{ci})/(T_{hi} - T_{c0})\right]} \tag{5.28}$$

Comparing Eq. (5.17) with Eq. (5.6), we find that the mean temperature difference is the grouping of terms in the brackets. Thus

$$\Delta T_m = \frac{(T_{h0} - T_{ci}) - (T_{hi} - T_{c0})}{In\left[(T_{h0} - T_{ci})/(T_{hi} - T_{c0})\right]} \tag{5.29}$$

If a heat exchanger other than the double-pipe type is used, the heat transfer is calculated by using a correction factor applied to the LMTD for a counter-flow double-pipe arrangement with the same hot and cold fluid temperatures. The heat transfer equation then takes the form

$$q = UAF(LMTD)_{counter-flow} \tag{5.30}$$

When a phase change is involved, as in condensation or boiling (evaporation), the fluid normally remains at essentially constant temperature and hance, F = 1.0 for boiling or condensation.

EFFECTIVENESS –NTU METHOD

The LMTD approach to heat-exchanger analysis is useful when the inlet and outlet temperatures are known or are easily determined. The LMTD is then easily calculated, and the heat flow, surface area, or overall heat-transfer coefficient may be determined.

When the inlet or exit temperatures are to be evaluated for a given heat exchanger, the analysis frequently involves an iterative procedure because of the logarithmic function in the LMTD. In these cases the analysis is performed more easily by utilizing a method based on the effectiveness of the heat exchanger in transferring a given amount of heat.

The effectiveness method also offers many advantages for analysis of problems in which a comparison between various types of heat exchangers must be made for purposes of selecting the type best suited to accomplish a particular heat-transfer objective.

We define the heat-exchanger effectiveness as

$$Effectiveness = \varepsilon = \frac{actual\ heat\ transfer}{\max imum\ possible\ heat\ transfer} \tag{5.31}$$

The actual heat transfer may be computed by calculating either the energy lost by the hot fluid or the energy gained by the cold fluid.

$$Q = m_h c_h (T_{h1} - T_{h2}) = m_c c_c (T_{c2} - T_{c2}) \tag{5.32}$$

To determine the maximum possible heat transfer for the exchanger, we first recognize that this maximum value could be attained if one of the fluids were to undergo a temperature change equal to the maximum temperature difference present in the exchanger, which is the difference in the entering temperature for the hot and cold fluids. The fluid which might undergo this maximum temperature difference is the one having the minimum value of mc because the energy balance requires that the energy received by one fluid be equal to that given up the other fluid; if we let the fluid with the larger value of mc go through the maximum temperature difference, this would require that the other fluid undergo a temperature difference greater than the maximum, and this is impossible. So, maximum possible heat transfer is expressed as

$$Q_{\max} = (mc_p)_{\min}(T_{hi} - T_{ci}) \tag{5.34}$$

The minimum fluid may be either the hot or cold fluid, depending on the mass-flow rates and specific heats.

If minimum fluid is hot fluid, then the effectiveness of a heat exchanger can be expressed as

$$\varepsilon = \frac{m_h c_{ph}(T_{h_i} - T_{h_o})}{m_h c_{ph}(T_{h_i} - T_{c_i})} = \frac{T_{h_i} - T_{h_o}}{T_{h_i} - T_{c_i}} \tag{5.35}$$

If minimum fluid is cold fluid, then the effectiveness of a heat exchanger can be expressed as

$$\varepsilon = \frac{m_c c_{pc}(T_{co} - T_{ci})}{m_c c_{pc}(T_{hi} - T_{ci})} = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}} \qquad (5.35)$$

In a general way, the effectiveness is expressed as

$$\varepsilon = \frac{\Delta T\,(\min imum\ \ fluid)}{Maximum\ temperature\ difference\ in\ heat\ exchanger} \tag{5.36}$$

(a) Parallel flow heat exchanger

We may derive an expression for the effectiveness of parallel flow heat exchanger as given below:

Rewriting Eq. (5.14), we have

$$In\frac{T_{ho}-T_{co}}{T_{hi}-T_{ci}}=-UA\left(\frac{1}{m_h c_{ph}}+\frac{1}{m_c c_{pc}}\right)=\frac{-UA}{m_c c_{pc}}\left(1+\frac{m_c c_{pc}}{m_h c_{ph}}\right) \tag{5.37}$$

$$\text{Or}\quad \frac{T_{ho}-T_{co}}{T_{hi}-T_{ci}}=\exp\left[\frac{-UA}{m_c c_{pc}}\left(1+\frac{m_c c_{pc}}{m_h c_{ph}}\right)\right] \tag{5.38}$$

If the cold fluid is the minimum fluid,

$$\varepsilon=\frac{T_{c0}-T_{ci}}{T_{hi}-T_{ci}} \tag{5.39}$$

Rewriting the temperature ratio in Eq. (5.38) gives

$$\frac{T_{h0}-T_{c0}}{T_{hi}-T_{ci}}=\frac{T_{hi}+m_c c_{pc}/m_h c_{ph})(T_{ci}-T_{co})-T_{c0}}{T_{hi}-T_{ci}} \tag{5.40}$$

when the substitution

$$T_{h0}=T_{hi}+\frac{m_c c_{pc}}{m_c c_{ph}}(T_{ci}-T_{c0}) \tag{5.41}$$

is made from Eq. (5.7).

Equation (5.40) may now be rewritten as

$$\frac{\left(T_{hi}-T_{ci}\right)+(m_c c_{pc}/m_h c_{ph})(T_{ci}-T_{c0})-(T_{ci}-T_{c0})}{T_{hi}-T_{ci}}=1-\left(1+\frac{m_c c_{pc}}{m_h c_{ph}}\right)\varepsilon \tag{5.42}$$

Inserting this relation back in Eq.(5.38) gives for the effectiveness

$$\varepsilon=\frac{1-\exp\left[\left(-UA/C_{\min}\right)\left(1+C_{\min}/C_{\max}\right)\right]}{1+C_{\min}/C_{\max}} \tag{5.43}$$

It may be shown that the same expression results for the effectiveness when the hot fluid is the minimum fluid, except that $m_c c_c$ and $m_h c_h$ are interchanged. As a consequence, the effectiveness is usually written

$$\varepsilon=\frac{1-\exp\left[\left(-UA/C_{\min}\right)\left(1+C_{\min}/C_{\max}\right)\right]}{1+C_{\min}/C_{\max}} \tag{5.44}$$

Where C = mc is defined as the heat capacity rate.

(b) Counter flow heat exchanger

A similar analysis may be applied to the counter flow case, and the following relation for effectiveness results:

$$\varepsilon=\frac{1-\exp\left[\left(-UA/C_{\min}\right)\left(1+C_{\min}/C_{\max}\right)\right]}{1-\left(C_{\min}/C_{\max}\right)\exp\left[\left(-UA/C_{\min}\right)\left(1-C_{\min}/C_{\max}\right)\right]} \tag{5.45}$$

The grouping of terms UA/C_{min} is called the number of transfer units (NTU) since it is indicative of the size of the heat exchanger.

(c) Boilers and Condensers

We noted earlier that in a boiling or condensation process the fluid temperature stays essentially constant, or the fluid acts as if it had infinite specific heat. In these cases $C_{min}/C_{max} \rightarrow 0$ and all the heat-exchanger effectiveness relations approach a single simple equation,

$$\varepsilon=1-e^{-NTU} \tag{5.46}$$

Table 5.3: Heat-exchanger effectiveness relations

$N = NTU = \dfrac{UA}{C_{\min}}, \quad C = \dfrac{C_{\min}}{C_{max}}$	
Flow geometry	Relation
Double pipe:	
Parallel flow	$\varepsilon = \dfrac{1-\exp[-N(1+C)]}{1+C}$
Counter flow	$\varepsilon = \dfrac{1-\exp[-N(1-C)]}{1-C\exp[-N(1-C)]}$
Counter flow, C=1	$\varepsilon = \dfrac{N}{N+1}$
Cross flow:	
Both fluids unmixed	$\varepsilon = 1-\exp\left[\dfrac{\exp(-NCn)-1}{Cn}\right]$ where $n=N^{-0.22}$
Bothe fluids mixed	$\varepsilon = \left[\dfrac{1}{1-\exp[-N]} + \dfrac{C}{1-\exp[-NC)]} - \dfrac{1}{N}\right]^{-1}$
C_{max} mixed, C_{min} unimixed	$\varepsilon = (1/C)\{1-\exp[-C(1-e^{-N})]\}$
C_{max} mixed, C_{min} mixed	$\varepsilon = 1-\exp\{-(1/C)[1-\exp(-NC)]\}$
Shell and tube:	
One shell pass, 2, 4, 6, tube passes	$\varepsilon = 2\left\{1+C+(1+C^2)^{1/2} \times \dfrac{1+\exp[-N(1+C^2)^{1/2}]}{1+\exp[-N(1+C^2)^{1/2}]}\right\}^{-1}$
All exchangers with C=0	$\varepsilon = 1-e^{-N}$

A. Questions at remembering / understanding level:

I) Objective Questions

1. Match the following?

A. Jet condenser.	P. Recuperator
B. Shell and tube condenser	Q. Regenerator
C. Air pre-heater	R. Mixer
D. Car radiator	
E. Cooling tower	
a. A-Q,B-P,C-P,D-R,E-P	b. A-P,B-Q,C-R,D-P,E-R
c. A-R,B-P,C-R,D-P,E-R	d. A-P,B-Q,C-R,D-R,E-P

2. The purpose of baffle plates used in a shell-and-tube heat exchanger is
   (a) To increase the surface area so as to increase the rate of heat transfer.
   (b) To increase the residence time of shell fluid.
   (c) To increase the residence time of tube fluid.
   (d) To increase the LMTD.
3. Which of the following is/are true?
   (a) For a new heat exchanger, the fouling resistance is zero.
   (b) Fouling resistance will increase over a period of operation due to formation of scales on the tube surfaces.
   (c) Frequent cleaning of tube surfaces will reduce the fouling resistance and increases the rate of heat transfer.
   (d) Sea water has higher fouling resistance compare to drinking water.
   P). a, b & c Q).a & c R). a, b, c & d S). c & d
4. Assertion (A): A counter flow heat exchanger has a higher effectiveness compared to that of a parallel flow heat exchanger for the same inlet and outlet temperatures of hot and cold fluids.
   Reasoning (R): In a counter flow heat exchanger, the hot and cold fluids flow in opposite direction.
   a) Both A and R are correct. R is the correct reasoning for A.
   b) Both A and R are correct. R is not the correct reasoning for A.
   c) A is correct but R is wrong.
   d) R is correct but A is wrong.
5. Which of the following statement is correct for a given temperature data?
   a) $(LMTD)_{parallel\ flow} < (LMTD)_{counter\ flow} < (LMTD)_{cross\ flow}$
   b) $(LMTD)_{cross\ flow} < (LMTD)_{parallel\ flow} < (LMTD)_{counter\ flow}$
   c) $(LMTD)_{cross\ flow} < (LMTD)_{counter\ flow} < (LMTD)_{parallel\ flow}$
   d) $(LMTD)_{parallel\ flow} < (LMTD)_{cross\ flow} < (LMTD)_{counter\ flow}$
6. Which of the following statement(s) is/are correct?
   a) The higher the NTU, the greater the effectiveness of a heat exchanger.
   b) The higher the NTU, the smaller the effectiveness of a heat exchanger.
   c) The higher the heat capacity ratio C_{min}/C_{max}, the greater the effectiveness of a heat exchanger.
   d) The higher the heat capacity ratio C_{min}/C_{max}, the smaller the effectiveness of a heat exchanger.
7. Define effectiveness of a heat exchanger.

8. Match the effectiveness of the following heat exchangers:

A. Counter flow, $C_{min}/C_{max} = 1$	P. $\frac{1-\exp[-N(1+C)]}{1+C}$
B. All exchangers with $C_{min}/C_{max} = 0$	Q. $\frac{1-\exp[-N(1-C)]}{1-C\exp[-N(1-C)]}$
C. Parallel flow	R. $\frac{N}{N+1}$
D. Counter flow	S. $1-e^{-N}$

II) Descriptive Questions

1. Classify heat exchangers based on construction and flow direction.
2. What are recuperative, regenerative and direct contact heat exchangers. State their applications.
3. Explain the working of shell and tube heat exchanger with a neat sketch.
4. Derive an expression for the LMTD of a parallel flow heat exchanger starting from fundamental principles.
5. Derive an expression for the LMTD of a counter flow heat exchanger starting from fundamental principles.
6. What is a fouling resistance. How do you take into account the fouling resistance in defining the overall heat transfer coefficient of a tube-in-tube heat exchanger.
7. Derive an expression for the effectiveness of a parallel flow heat exchanger in terms of number of transfer units (NTU) and heat capacity ratio, C_{min}/C_{max}.
8. Derive an expression for the effectiveness of a counter flow heat exchanger in terms of number of transfer units (NTU) and heat capacity ratio, C_{min}/C_{max}.
9. Prove that the effectiveness of a condenser/ boiler is $\varepsilon = 1 - e^{-N}$.

B. Question at applying / analyzing level

I) Multiple choice questions.

1. In a heat exchanger, the hot fluid is cooled from 100^0C to 65^0C while the cold fluid is heated from 25^0C to 55^0C. If the LMTD of the heat exchanger is 36^0C, what is the type of heat exchanger?

   a) parallel flow b) cross flow c) counter flow d) None

   Common data for question 2, 3 and 4.

   Cold water (c_p=4.180 kJ/kg.^{0}C) leading to a shower enters a thin-walled double-pipe counter-flow heat exchanger at 15^0C at a rate of 1.25 kg/s and it is heated to 45^0C by hot water (c_p=4.190 kJ/kg.^{0}C) that enters at 100^0C at a rate of 3 kg/s.

2. What is the rate of heat transfer in kW?

   a) 126.75 b) 136.75 c) 146.75 d) 156.75

3. What is the LMTD of heat exchanger in ^{0}C?

   a) 62.36 b) 66.36 c) 63.36 d) 65.36

4. What is the surface area of the heat exchanger in m^2 if U=250 W/m^2 ^{0}C ?

   a) 9.9 b) 8.9 c) 7.9 d) 10.9

Common data for question 5, 6 and 7.

Water (c_p=4180 J/kg.^{0}C) enters the 2.5-cm-internl-diameter tube of a double-pipe counter-flow heat exchanger at 20^{0}C at a rate of 3 kg/s. It is heated by steam condensing at 120^{0}C (h_{fg}=2203 kJ/kg) in the shell. The overall heat transfer coefficient of the heat exchanger is 1500 W/m^2.^{0}C.

5. The LMTD of heat exchanger, if the water is to be heated to 80^{0}C, is

a) 65.5^{0}C b) 55.5^{0}C c) 75.5^{0}C d) 45.5^{0}C

6. The length of the tube required is

a) 89.12 m b) 97.12 m c) 77.12 m d) 67.12 m

7. The rate of condensation of steam is

a) 16.5 kg/min b) 18.5 kg/min c) 22.5 kg/min d) 20.5 kg/min

8. If the number transfer units, NTU for a condenser is 2.5, the effectiveness of a condenser is ________.

a) 0.918 b) 0.818 c) 0.98 d) 0.88

Common data for questions 9 and 10.

Hot fluid enters a tube-in-tube heat exchanger at 100^{0}C and leaves at 60^{0}C. The heat capacities of hot and cold fluids are equal. The cold fluid enters at 20^{0}C.

9. The maximum effectiveness of heat exchanger is

a) 0.7 b) 0.6 c) 0.5 d) 0.6

10. The number of transfer units of the heat exchanger is

a) 1 b) 1.5 c) 2 d) 2.5

Common data for questions 11, 12 and 13.

Hot oil at 100^{0}C is used to heat air in a shell-and-tube heat exchanger. The oil makes six tube passes and the air makes one shell pass; 2.0 kg/s of air are to be heated from 20 to 80^{0}C. The specific heat of the oil is 2100 J/kg.K, and its flow rate is 3.0 kg/s.

11. The effectiveness of heat exchanger is

a) 0.7 b) 0.65 c) 0.75 d) 0.85

12. The number of transfer units of heat exchanger is

a) 1.99 b) 1.69 c) 2.29 d) 1.79

13. The surface area of heat exchanger required when U=200 W/m^2 K is

a) 18.1 m^2 b) 16.1 m^2 c) 22.1 m^2 d) 20.1 m^2

Common data for questions 14 and 15.

Water at the rate of 68 kg/min is heated from 35 to 75^{0}C by an oil having a specific heat of 1.9 kJ/kg.K. The fluids are used in a counter flow double-pipe heat exchanger, and the oil enters the exchanger at 110^{0}C and leaves at 75^{0}C.

14. The rate of heat transfer in kW is

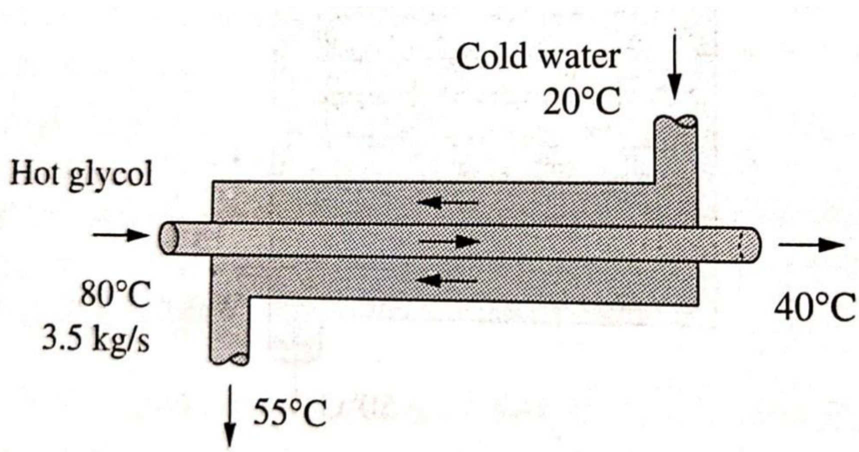
a) 189.5 b) 169.5 c) 209.5 d) 149.4

15. The surface area required if the overall heat-transfer coefficient is 320 W/m^2 .K.

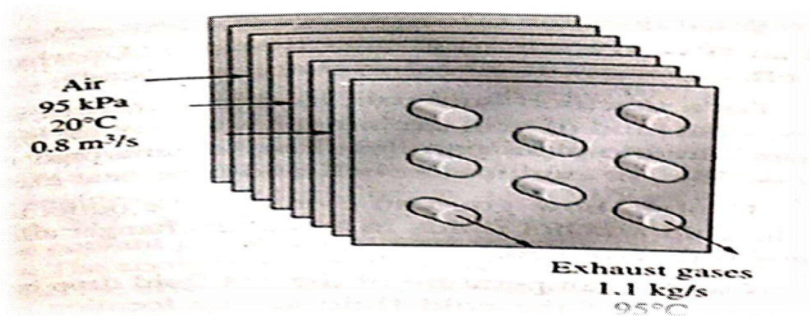
a) 14.82 m^2 b) 15.82 m^2 c) 13.82 m^2 d) 16.82 m^2

II) Problems

1. A double-pipe parallel-flow heat exchanger is used to heat cold tap water with hot water. Hot water (c_p=4.25 kJ/kg.^{0}C) enters the tube at 85^{0}C at a rate of 1.4 kg/s and leaves at 50^{0}C. The heat exchanger is not well insulated, and it is estimated that 3 percent of the heat given up by the hot fluid is lost from the heat exchanger. If the overall heat transfer coefficient and the surface area of the heat exchanger are 1150 W/m^2. ^{0}C and 4 m^2, respectively, determine the rate of heat transfer to the cold water and the log mean temperature difference for this heat exchanger.
2. A shell-and –tube heat exchanger is used for heating 10 kg/s of oil (c_p=2.0 kJ/kg.^{0}K) from 25^{0}C to 46^{0}C. The heat exchanger has 1-shel pass and 6-tube passes. Water enters the shell side at 80^{0}C and leaves at 60^{0}C. The overall heat transfer coefficient is estimated to be 1000 W/m^2.K. Calculate the rate of heat transfer and the heat transfer area.
3. A double-pipe parallel-flow heat exchanger is to heat water (c_p=4180 J/kg.^{0}C) from 25^{0}C to 60^{0}C at a rate of 0.2 kg/s. The heating is to be accomplished by geothermal water (c_p=4310 J/kg.^{0}C) available at 140^{0}C at a mass flow rate of 0.3 kg/s. The inner tube is thin-walled and has a diameter of 0.8 cm. If the overall heat transfer coefficient of the heat exchanger is 550 W/m^2. ^{0}C, determine the length of the tube required to achieve the desired heating.
4. A 1-shell-pass and 8-tube-passes heat exchanger is used to heat glycerin (c_p=2.5 kJ/kg.^{0}C) from 18^{0}C to 60^{0}C by hot water (c_p=4.18 kJ/kg.^{0}C) that enters the thin-walled 1.3-cm-diameter tubes at 80^{0}C and leaves at 50^{0}C. The total length of the tubes in the heat exchanger is 150m. the convection heat transfer coefficient is 23 W/m^2.^{0}C on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (a) before any fouling occurs and (b) after fouling with a fouling factor of 0.00035 m$^{2.0}$C/W on the outer surfaces of the tubes.
5. A double-pipe counter-flow heat exchanger is to cool ethylene glycol (c_p=2560 kJ/kg.^{0}C) flowing at a rate of 3.5 kg/s from 80^{0}C to 40^{0}C by water (c_p=4180 kJ/kg.^{0}C) that enters at 20^{0}C and leaves at 55^{0}C. The overall heat transfer coefficient based on the inner surface area of the tube is 250 W/m$^{2.0}$C. Determine (a) the rate of heat transfer, (b) the mass flow rate of water, and (c) the heat transfer surface area on the inner side of the tube.
6. Engine oil (c_p=2100 kJ/kg.^{0}C) is to be heated from 20^{0}C to 60^{0}C at a rate of 0.3 kg/s in a 2-cm-diameter thin walled copper tube by condensing steam outside at a heat transfer coefficient to of 650 W/m$^{2.0}$C, determine the rate of heat transfer and the length of the tube required to achieve it.

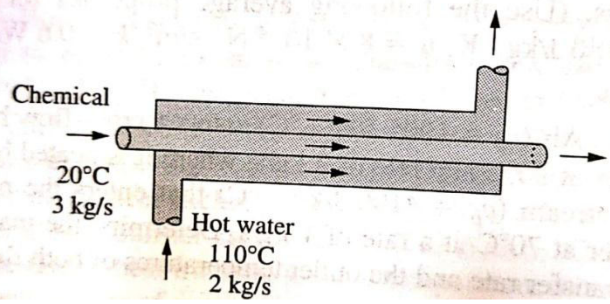


7. Air (c_p=1005 kJ/kg.^{0}C) is to be preheated by hot exhaust gases in a cross-flow heat exchanger before it enters the furnace. Air enters the heat exchanger at 95 kPa and 20^{0}C at a rate of 0.8 m^3/s. The combustion gases (c_p=1001 kJ/kg.^{0}C) enters at 180^{0}C at a rate of 1.1 kg/s and leave at 95^{0}C. The product of the overall heat transfer coefficient and the heat transfer surface area is UA_s=1200 W/^{0}C. Assuming both fluids to be unmixed, determine the rate of heat transfer and the outlet temperature of the air.



8. A shell-and-tube heat exchanger with 2-shell passes and 12-tube passes is used to heat water (c_p=4180 kJ/kg.^{0}C) in the tubes form 20^{0}C to 70^{0}C at a rate of 4.5 kg/s. Heat is supplied by hot oil (c_p=2300 kJ/kg.^{0}C) that enters the shell side at 170^{0}C at a rate of 10 kg/s. For a tube-side overall heat transfer coefficient of 350 W/m^2.^{0}C, determine the heat transfer surface area on the tube side.
9. A shell-and-tube heat exchanger with 2-shel passes and 12-tube passes is used to heat water (c_p=4180 kJ/kg.^{0}C) with ethylene glycol (c_p=2680 kJ/kg.^{0}C). Water enters the tubes at 22^{0}C at a rate of 0.8 kg/s and leaves at 70^{0}C. Ethylene glycol enters the shell at 110^{0}C and leaves at 60^{0}C. If the overall heat transfer coefficient based on the tube side is 280 W/m^2.^{0}C, determine the rate of heat transfer and the heat transfer surface area on the tube side.
10. A thin-walled double-pie parallel-flow heat exchanger is used to heat a chemical whose specific heat is 1800 J/kg.^{0}C with hot water (c_p=4180 kJ/kg.^{0}C). The chemical enters at 20^{0}C at a rate of 3 kg/s, while the water enters at 110^{0}C at a rate of 2 kg/s. The heat transfer surface area of the heat exchanger is 7 m^2 and the overall hat transfer coefficient is 1200 W/m$^{2.0}$C. Determine the outlet temperatures of the chemical and the water.

the chemical and the water.



11. A cross-flow air-to-water heat exchanger with an effectiveness of 0.65 is used to heat water (c_p=4180 kJ/kg.^{0}C) with hot air (c_p=1010 kJ/kg.^{0}C). Water enters the heat exchanger at 20^{0}C at a rate of 4 kg/s. While air enters at 100^{0}C at a rate of 9 kg/s. if the overall het transfer coefficient based on the water side is 260 W/m^2.^{0}C, determine the hat transfer surface area of the heat exchanger on the water side. Assume both fluids are unmixed.
12. Water (c_p=4180 kJ/kg.^{0}C) enters the 2.5-cm-internal-diameter tube of a double-pipe counter-flow heat exchanger at 17^{0}Cat a rate of 1.8 kg/s. Water is heated by steam condensing at 120^{0}C ($h_{fg=2203}$ kJ/kg) in the shell. If the 700 W/m$^{2.0}$C, determine the length of the tube required in order to heat the water to 80^{0}C using (a) the LMTD method and (b) the ε =NTU method.
13. Cold water (c_p=4180 J/kg$^{.0}$C) leading to a shower enters a thin-walled double-pipe counter-flow heat exchanger at 15^{0}C at a rate of 0.25 kg/s and is heated to 45^{0}C by hot water (c_p=4190 J/kg$^{.0}$C) that enters at 100^{0}C at a rate of 3 kg/s. If the overall heat transfer coefficient is 950 W/m$^{2.0}$C, determine the rate of heat transfer and the heat transfer surface area of the heat exchanger using the effectiveness/NTU method.
14. Geothermal water (c_p=4250 j/kg.^{0}C) at 75^{0}C is to be used to heat fresh water (c_p=4180 j/kg.^{0}C) at 17^{0}C at a rate of 1.2 kg/s in a double-pipe counter-flow heat exchanger. The heat transfer surface area is 25 m^2, the overall heat transfer coefficient is 480 W/m$^{2.0}$C, and the mass flow rate of geothermal water is larger than that of fresh water. If the effectiveness of the heat exchanger is desired to be 0.823. determine the mass flow rate of geothermal water and the outlet temperature of both fluids.

C. Questions at evaluating / creating level:

1. Water flows through a shower head steadily at a rate of 8 kg/min. The water heated in an electric water heater from 15^{0}C to 45^{0}C. In an attempt to conserve energy, it is proposed to pass the drained warm water at a temperature of 38^{0}C through a heat exchanger to preheat the incoming cold water. Design a heat exchanger that is suitable for this task, and discuss the potential savings in energy and money for your area.
2. Design a hydro cooling unit that can cool fruits and vegetables from 30^{0}C to 5^{0}C at a rate of 20,000 kg/h under the following conditions:

   The unit will be of flood type that will cool the products as they are conveyed into the channel filled with water. The products will be dropped into the channel filled with water at on end and picked up at the other end. The channel can be as wide as 3 m and as high as 90 cm. The water is to be circulated and cooled by the evaporator section of a refrigeration system. The refrigerant temperature inside the coils is to be -2^{0}C, and the water temperature is not to drop below 1^{0}C and not to exceed 6^{0}C.

Assuming reasonable values for the average product density, specific heat, and porosity (the fraction of air volume in a box), recommend reasonable values for the quantities related to the thermal aspects of the hydro cooler, including (a) how long the fruits and vegetables needs to remain in the channel, (b) the length of the channel, (c) the water velocity through the channel, (d) the velocity of the conveyor and thus the fruits and vegetables through the channel, (e) the refrigeration capacity of the refrigeration system, and (f) the type of heat exchanger for the evaporator and the surface area on the water side.

3. A company owns a refrigeration system whose refrigeration capacity is 200 tons (1 ton of refrigeration =211 kJ/min), and you are to design a forced-air cooling system for fruits whose diameters do not exceed 7 cm under the following conditions:

   The fruits are to be cooled from 28^0C to an average temperature of 8^0C at all times, and the velocity of air approaching the fruits must remain under 2 m/s. The cooling section can be as wide as 3.5 and as high as 2 m.

   Assuming reasonable values for the average fruit density, specific heat, and porosity (the fraction of air volume in a box), recommend reasonable values for the quantities related to the thermal aspects of the forced-air cooling, including (a) how long the fruits need to remain in the cooling section (b) the length of the cooling section, (c) the air velocity approaching the cooling section, (d) the product cooling capacity of the system, in kg. fruit/h, (e) the volume flow rate of air, and (f) the type of heat exchanger for the evaporator and the surface area on the air side.

D. GATE Questions:

1. In a counter flow heat exchanger, for the hot fluid the heat capacity = 2kJ/kg K, mass flow rate = 5 kg/s, inlet temperature =150°C, outlet temperature =100°C. For the cold fluid, heat capacity = 4 kJ/kg K, mass flow rate = 10 kg/s, inlet temperature=20°C. Neglecting heat transfer to the surroundings, the outlet temperature of the cold fluid in °C is [GATE-2003]

   (a) 7.5 (b) 32.5 (c) 45.5 (d) 70.0

2. In a counter flow heat exchanger, hot fluid enters at 60^0c and cold fluid leaves at 30^0C. Mass flow rate of the hot fluid is 1 kg/s and that of the cold fluid is 2 kg/s. Specific heat of the hot fluid is 10 kJ/kgK and that of the cold fluid is 5 kJ/kgK. The Log Mean Temperature difference (LMTD) for the heat exchanger in ^{0}C is [GATE-2007]

   a) 15 b) 30 c) 30 d) 45

3. In a parallel flow heat exchanger operating under steady state, the heat capacity rates (product of specific heat at constant pressure and mass flow rate) of the hot and cold fluid are equal. The hot fluid, flowing at 1kg/s with Cp = 4kJ/kgK, enters the heat exchanger at 102°C while the cold fluid has an inlet temperature of 15°C. The overall heat transfer coefficient for the heat exchanger is estimated to be 1kW/m2K and the corresponding heat transfer surface area is 5m2. Neglect heat transfer between the heat exchanger and the ambient. The heat exchanger is characterized by the following relation: [GATE-2009]

   2e = 1 - exp (-2NTU) .

   The exit temperature (in °C) for the cold fluid is

   (A) 45 (B)55 (C) 65 (D) 75

4. In a condenser of a power plant, the steam condenses at a temperature of 0 60 C. The cooling water enters at 0 30 C and leaves at 0 45 C . The logarithmic mean temperature difference (LMTD) of the condenser is [GATE-2011]
   (A) 0 16.2 C (B) 0 21.6 C (C) 0 30 C (D) 0 37.5 C
5. Water (C_p=4.18kJ/kg.K) at 80^0C enters a counter flow heat exchanger with a mass flow rate of 0.5 kg/s. Air (C_p=1kJ/kg.K) enters at 30^0C with a mass flow rate of 2.09 kg/s. If the effectiveness of the heat exchanger is 0.8, the LMTD (in ^{0}C) is [GATE-2012]
   (A) 40 (B) 20 (C) 10 (D) 5
6. In a heat exchanger, it is observed that $\Delta T_1 = \Delta T_2$, where ΔT_1 is the temperature difference between the two single phase fluid streams at one end and ΔT_2 is the temperature difference at the other end. This heat exchanger is [GATE-2014]
   (A) a condenser (B) an evaporator
   (C) a counter flow heat exchanger (D) a parallel flow heat exchanger
7. 4. In a concentric counter flow heat exchanger, water flows through the inner tube at 25OC and leaves at 42OC. The engine oil enters at 100°C and flows in the annular flow passage. The exit temperature of the engine oil is 50OC. Mass flow rate of water and the engine oil are 1.5 kg/s and 1 kg/s, respectively. The specific heat of water and oil are 4178 J/kg.K and 2130 J/kg.K, respectively. The effectiveness of this heat exchanger is _______ [GATE-2014]
8. A double-pipe counter-flow heat exchanger transfers heat between two water streams. Tube side water at 19 liter/s is heated from 10^0C to 38^0C. Shell side water at 25 liter/s is entering at 46^0C. Assume constant properties of water, density is 1000 kg/m3 and specific heat is 4186 J/kg K. The LMTD (in^{0}C) is ________ [GATE-2014]
9. Consider a parallel-flow heat exchanger with area A_p and a counter-flow heat exchanger with area Ac . In both the heat exchangers, the hot stream flowing at 1 kg/s cools from 80^0C to 50^0C. For the cold stream in both the heat exchangers, the flow rate and the inlet temperature are 2 kg/s and 10^0C, respectively. The hot and cold streams in both the heat exchangers are of the same fluid. Also, both the heat exchangers have the same overall heat transfer coefficient. The ratio A_c/A_p is _______ [GATE-2016]
10. The balanced counter-flow heat exchanger has a surface area of 20 m^2 and overall heat transfer coefficient of 20 W/m^2-K. Air (C_p=1000 J/kg.K) entering at 0.4 kg/s and 280 K is to be preheated by the air leaving the system at 0.4 kg/s and 300 K. The outlet temperature (in K) of the preheated air is [GATE-2016]
    A) 290 B) 300 C) 320 D) 350
11. In a counter-flow heat exchanger water is heated at the rate of 1.5 kg/s from 40C to 80^0C by an oil entering at 120^0C and leaving at 60^0C. The specific heats of water and oil are 4.2 kJ/kg-K and 2 kJ/kg-K, respectively. The overall heat transfer coefficient is 400 W/m^2-K. The required heat transfer surface area (in m^2) is [GATE-2017]
    A) 0.104 B) 0.022 C) 10.4 D) 21.84
12. Steam in the condenser of a thermal power plant is to be condensed at a temperature of 30^0C with cooling water which enters the tubes of the condenser at 14^0C and exits at 22^0C. The total surface area of the tubes is 50 m^2, and the overall heat transfer coefficient is 2000 W/m^2 K. The heat transfer (in MW) to the condenser is ______ (correct to two decimal places). [GATE-2018]
13. Hot and cold fluids enter a parallel flow double tube heat exchanger at 1000C and 150C, respectively. The heat capacity rates of hot and cold fluids are Ch=2000 W/K

and Cc=1200 W/K, respectively. If the outlet temperature of the cold fluid is 450C, the log mean temperature difference (LMTD) of the heat exchangers _______K (round off to two decimal places) [GATE-2019]

E. IES Questions:

1. When $tc1$ and $tc2$ are the temperatures of cold fluid at entry and exit respectively and $th1$ and $th2$ are the temperatures of hot fluid at entry and exit point, and cold fluid has lower heat capacity rate as compared to hot fluid, then effectiveness of the heat exchanger is given by: [IES-1992]

   (a) $\frac{t_{c1}-t_{c2}}{t_{h1}-t_{h2}}$ (b) $\frac{t_{h2}-t_{h1}}{t_{c2}-t_{c1}}$ (c) $\frac{t_{h1}-t_{h2}}{t_{h1}-t_{c2}}$ (d) $\frac{t_{c2}-t_{c1}}{t_{h1}-t_{h1}}$

2. In a parallel flow gas turbine recuperator, the maximum effectiveness is: [IES-1992]

   (a) 100% (b) 75% (c) 50% (b) Between 25% and 45%

3. ε-NTU method is particularly useful in thermal design of heat exchangers when [IES-1993]
   a. The outlet temperature of the fluid streams is not known as apriori
   b. Outlet temperature of the fluid streams is known as a priori
   c. The outlet temperature of the hot fluid streams is known but that of the cold fluid streams is not known as a priori
   d. Inlet temperatures of the fluid streams are known as apriori

4. For evaporators and condensers, for the given conditions, the logarithmic mean temperature difference (LMTD) for parallel flow is: [IES-1993]
   (a) Equal to that for counter flow
   (b) Greater than that for counter flow
   (c) Smaller than that for counter flow
   (d) Very much smaller than that for counter flow

5. In a counter flow heat exchanger, cold fluid enters at 30°C and leaves at 50°C, whereas the enters at 150°C and leaves at l30°C. The mean temperature difference for this case is: [IES-1993]

   (a) Indeterminate (b) 20°C (c) 80°C (d) 100°C

6. A designer chooses the values of fluid flow ranges and specific heats in such a manner that the heat capacities of the two fluids are equal. A hot fluid enters the counter flow heat exchanger at 100°C and leaves at 60°C. The cold fluid enters the heat exchanger at 40°C. The mean temperature difference between the two fluids is:

   (a) (100 +60 + 40)/3°C (b) 60°C (c) 40°C (d) 20°C

7. Given the following data, [IES-1993]

   Inside heat transfer coefficient = 25 W/m^{2}K

   Outside heat transfer coefficient = 25 W/m^{2}K

   Thermal conductivity of bricks (15 cm thick) = 0.15 W/mK,

   The overall heat transfer coefficient (in W/m^{2}K) will be closer to the
   (a) Inverse of heat transfer coefficient
   (b) Heat transfer coefficient
   (c) Thermal conductivity of bricks
   (d) Heat transfer coefficient based on the thermal conductivity of the bricks alone

8. A counter flow heat exchanger is used to heat water from 20°C to 80°C by using hot

exhaust gas entering at 140°C and leaving at 80°C. The log mean temperature difference for the heat exchanger is: [IES-1996]

(a) 80°C (b) 60°C (c) 110°C (d)Not determinable as zero/zero is involved

9. A cross-flow type air-heater has an area of 50 m^2. The overall heat transfer coefficient is 100 W/m^2K and heat capacity of both hot and cold stream is 1000 W/K. The value of NTU is: [IES-1996]

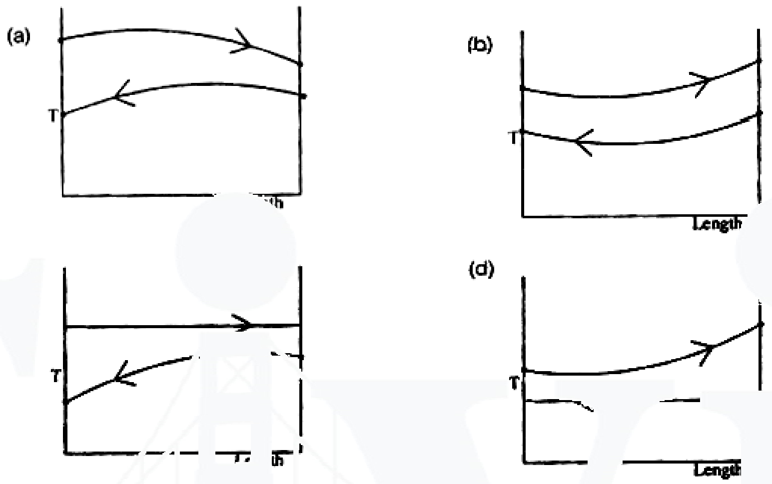
(a) 1000 (b) 500 (c) 5 (d) 0.2

10. A counter flow shell - and - tube exchanger is used to heat water with hot exhaust gases. The water (Cp = 4180 J/kg°C) flows at a rate of 2 kg/s while the exhaust gas (1030 J/kg°C) flows at the rate of 5.25 kg/s. If the heat transfer surface area is 32.5 m^2 and the overall heat transfer coefficient is 200 W/m^2°C, what is the NTU for the heat exchanger? [IES-1996]

(a) 1.2 (b) 2.4 (c) 4.5 (d) 8.6

11. A heat exchanger with heat transfer surface area A and overall heat transfer coefficient U handles two fluids of heat capacities C1, and C2, such that C1 > C2. The NTU of the heat exchanger is given by: [IES-1996]

12. Which one of the following diagrams correctly shows the temperature distribution for a gas-to-gas counter flow heat exchanger?



[IES-1994;1997]

given below the lists: List-I **[IES-1995]**

List-II

List-I	List-II
A. Regenerative heat exchanger	**1.** Water cooling tower
B. Direct contact heat exchanger	**2.** Lungstrom airheater
C. Conduction through a cylindrical wall	**3.** Hyperbolic curve
D. Conduction through a spherical wall	**4.** Logarithmic curve

Codes:	A	B	C	D		A	B	C	D
(a)	1	4	2	3	(b)	3	1	4	2
(c)	2	1	3	4	(d)	2	1	4	3

13. Assertion (A): The LMTD for counter flow is larger than that of parallel flow for a given temperature of inlet and outlet. [IES-1998]
Reason (R): The definition of LMTD is the same for both counter flow and parallel flow.

1. Both A and R are individually true and R is the correct explanation of A
2. Both A and R are individually true but R is not the correct explanation of
3. A is true but R is false 4. A is false but R is true

14. Air can be best heated by steam in a heat exchanger of [IES-2001]
(a) Plate type (b) Double pipe type with fins on steam side
(c) Double pipe type with fins on air side (d) Shell and tube type

15. Which one of the following heat exchangers gives parallel straight line pattern of temperature distribution for both cold and hot fluid? [IES-2001]
(a) Parallel-flow with unequal heat capacities
(b) Counter-flow with equal heat capacities
(c) Parallel-flow with equal heat capacities
(d) Counter-flow with unequal heat capacities

16. For a balanced counter-flow heat exchanger, the temperature profiles of the two fluids are: [IES-2001]
(a) Parallel and non-linear (b) Parallel and linear
(c) Linear but non-parallel (d) Divergent from one another

17. In a counter flow heat exchanger, the product of specific heat and mass flow rate is same for the hot and cold fluids. If NTU is equal to 0.5, then the effectiveness of the heat exchanger is: [IES-2001]
(a) 1.0 (b) 0.5 (c) 0.33 (d) 0.2

18. In a counter-flow heat exchanger, the hot fluid is cooled from 110°C to 80°C by a cold fluid which gets heated from 30°C to 60°C. LMTD for the heat exchanger is: [IES-2001]
(a) 20°C (b) 30°C (c) 50°C (d) 80°C

19. Assertion (A): It is not possible to determine LMTD in a counter flow heat exchanger with equal heat capacity rates of hot and cold fluids. Reason (R): Because the temperature difference is invariant along the length of the heat exchanger. [IES-2002]
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

20. Assertion (A): A counter flow heat exchanger is thermodynamically more efficient than the parallel flow type. [IES-2003]
Reason (R): A counter flow heat exchanger has a lower LMTD for the same temperature conditions.
(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

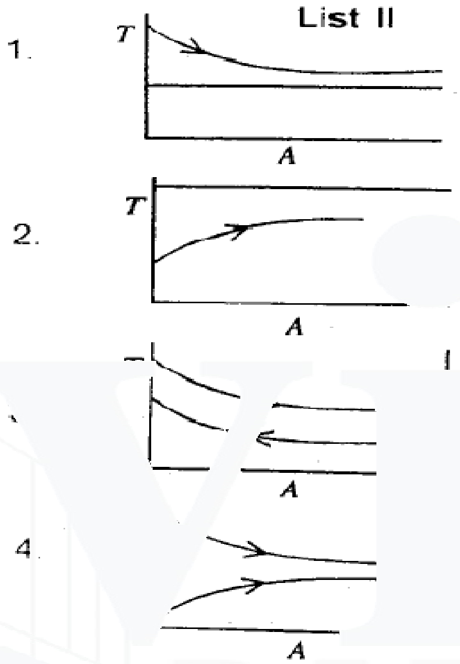
21. Match List-I (Heat exchanger process) with List-II (Temperature area diagram) and select the correct answer: [IES-2004]

A. Counter flow sensible heating

B. Parallel flow sensible heating

C. Evaporating

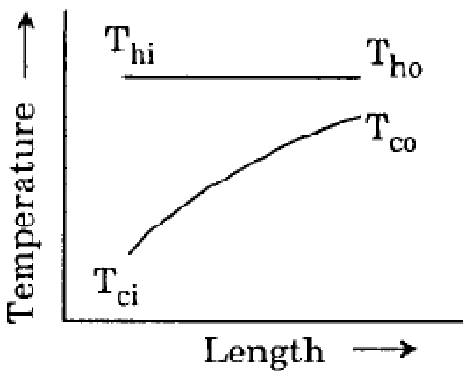
D. Condensing



22. After expansion from a gas turbine, the hot exhaust gases are used to heat the compressed air from a compressor with the help of a cross flow compact heat exchanger of 0.8 effectiveness. What is the number of transfer units of the heat exchanger? [IES-2005]
(a) 2 (b) 4 (c) 8 (d) 16

23. The temperature distribution curve for a heat exchanger as shown in the figure above (with usual notations) refers to which one of the following? [IES-2008]

(a) Tubular parallel flow heat exchanger
(b) Tube in tube counter flow heat exchanger
(c) Boiler
(d) Condenser



24. Match List-I (Application) with List-II (Type of heat exchanger) and select the correct answer using the code given below the lists: [IES-2008]

List-I	List-II
A. Gas to liquid	**1.** Compact
B. Space vehicle	**2.** Shell and Tube
C. Condenser	**3.** Finned tube
D. Air pre-heater	**4.** Regenerative

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	2	4	3	1	(b)	3	1	2	4
(c)	2	1	3	4	(d)	3	4	2	1

25. Match List-I with List-II and select the correct answer [IES-2008]

List-I	List-II
A. Number of transfer units	1. Recuperative type heat exchanger
B. Periodic flow heat exchanger	2. Regenerator type heat exchanger
C. Chemical additive	3. A measure of the heat exchanger size
D. Deposition on heat exchanger surface	4. Prolongs drop-wise condensation
	5. Fouling factor

Codes:	**A**	**B**	**C**	**D**		**A**	**B**	**C**	**D**
(a)	3	2	5	4	(b)	2	1	4	5
(c)	3	2	4	5	(d)	3	1	5	4

26. Consider the following statements: [IES-2008]

In a shell and tube heat exchanger, baffles are provided on the shell side to:

1. Prevent the stagnation of shell side fluid
2. Improve heat transfer
3. Provide support for tubes

Select the correct answer using the codes given below:

27. The 'NTU' (Number of Transfer Units) in a heat exchanger is given by which one of the following? [IES-2008]

(a) $\frac{UA}{C_{\min}}$ (b) $\frac{UA}{C_{\max}}$ (c) $\frac{UA}{E}$ (d) $\frac{C_{\max}}{C_{\min}}$

U = Overall heat transfer coefficient C = Heat capacity
E = Effectiveness A = Heat exchange area

28. Consider the following statements: [IES-2009]

The flow configuration in a heat exchanger, whether counter flow or otherwise, will NOT matter if:

1. A liquid is evaporating 2. A vapour is condensing
3. Mass flow rate of one of the fluids is far greater Of these statements:

(a) 1 and 2 are correct (b) 1 and 3 are correct
(c) 2 and 3 are correct (d) 1, 2 and 3 are correct

29. In a balanced counter flow heat exchanger with M_hC_h M_cC_c the NTU is equal to 1.0. What is the effectiveness of the heat exchanger? [IES-2009]

(a) 0.5 (b) 1.5 (c) 0.33 (d) 0.2

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THERMAL RADIATION

Basic concepts:

Solid and liquid surfaces at all temperatures emit thermal radiation. The rate of emission is a strong function of the temperature level and is proportional to the fourth power of the value on the absolute scale. Thermalradiation is an electromagnetic wave and is similar in nature to other electromagnetic waves. Thus, it does not require any material medium for propagation and one use the attributes of the wavelength or frequency to describe it.

Figure1 shows the wavelength of various types of electromagnetic waves. It is seen that x-rays and gamma rays have short wavelengths, less than 10^{-2} microns(µm).The micron,1 µm=10^{-6}m,is the unit usually used for measuring wavelength .Microwaves and radio waves have long wavelength ,larger than 10^{2} microns.

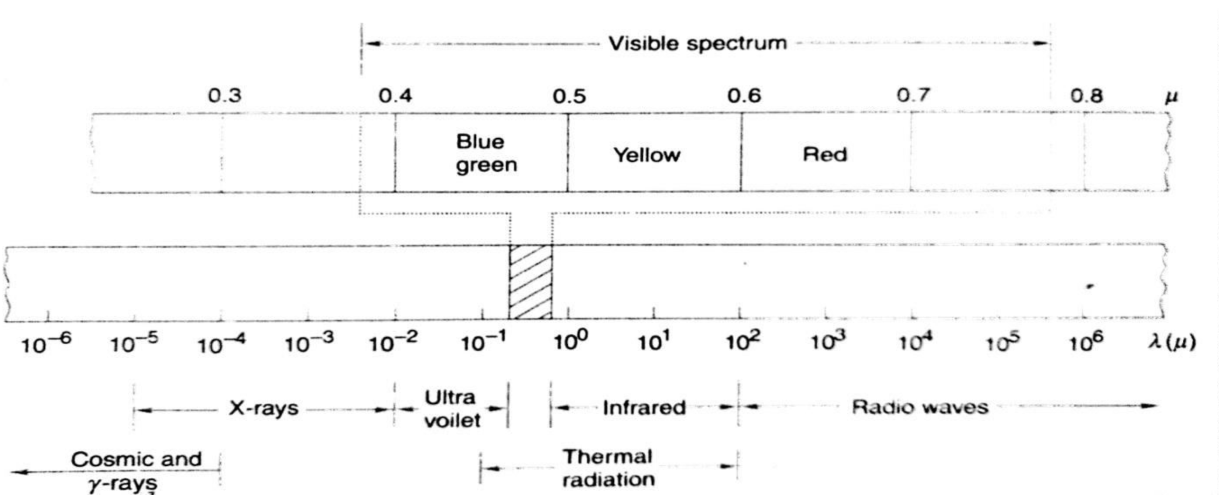


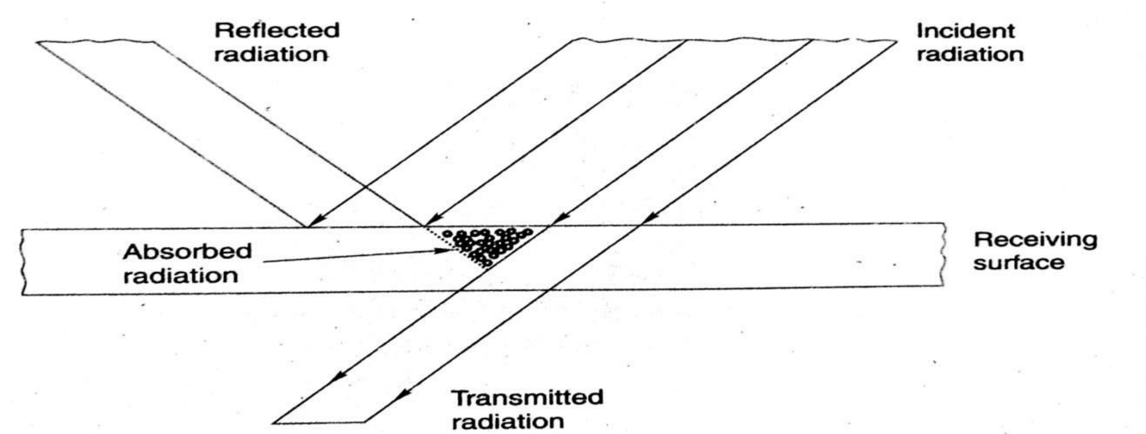
Fig1 .Wavelengths of electromagnetic waves.

Thermal radiationemitted from the surface of the body is a continuous spectrum of all wavelengths from zero to ∞.The typical dependence on the wavelength is showed in Fig 3.2 in which the monochromatic or spectral emission rate per unit area (W/m^2 µm) is plotted against the wavelength. The area under the graph is measure of the emission per unit area (W/m^2).it is seen that most of the energy lies in the range from 0.1 to 10µm for temperatures exceeding 1000K .Thus thermal radiation generally occupies the intermediate wavelength region between the two extremes of X-rays and gamma rays on one side and microwaves and radio waves on the other. Visible light lies in the range from 0.4 to 0.7 µm thermal radiation overlaps the visible light range. The electric filament lamp is an example of a device utilizing this fact

In addition to emitting the surface of a body also has the capsid of absorbing all or a part of the radiation emitted by surrounding surfaces and falling on it .consider a solid with a vacuum a round it placed in an enclosure whose walls are maintained at lower temperature than that of the body . The presence of the vacuum prevents any energy transfer by convection. However radiative energy exchange does occur. The surface of the body emits radiation by virtue of its temperature level. It also intercepts and absorbs radiation emitted by the walls of the enclosure. Since the rate at which it emits radiation is greater than the rate at which it absorbs radiation, the body cools down to the temperature of the surroundings walls and ultimately reaches a state of thermal equilibrium with them

In this state, the radiation energy exchange continues, the rates of emission and absorption at the surface of the solid body being equal

$$Q_\rho + Q_\tau + Q_\alpha = Q \text{ or } \frac{Q_\rho}{Q} + \frac{Q_\tau}{Q} + \frac{Q_\alpha}{Q} = 1$$



The wavelength dependence of thermal radiation is one a factor which has already been mentioned. The other factor is its directional nature. A surfaceemits radiations in all directions encompassed by a hemisphere. The amount (intensity) emitted in each directions depends on the nature of the surface. Both the wavelength dependence and directional nature have to be considered while analyzing thermal radiation problems

In this chapter we shall be concerned only with situations involving radiations exchange between surfaces, in which space between the surfaces is considered to be a vacuum or occupied by a gas which does not participate in the radioactive exchange in any way. Thus the gas neither absorbs any radiation passing form one surface to another, nor does it emit any radiation by virtue of its temperature level. Radiation heatexchange problems involving gases which themselves absorb and emit radiations. However their analysis is more involved and will not be discussed in this book.

Emission characteristics and laws of black-body radiation

We shall now define a number of terms concerned with theemission characteristics of surfaces and then state the associated laws. It is useful to introduce first the concept of ideal surfaces called a black body or black surface. A black surface is one which absorbs all radiation falling on regardless of its wavelength or direction .For a given a temperature and wavelength. It emits the maxima amount of energy. It thus serves as a standard on which to base the emission characteristics.

Definition 1: the radiant flux emitted from the surface of a body is called the total hemispherical emissive power and will be denoted by the symbol e (units W/m^2). The total hemispherical emissive power of black surfacewill be denoted by is. The adjectives

$$e = \epsilon \sigma T^4$$

Where the constant $\sigma = 5.670 \text{ X } 10^{-8} \, W/m^2K^4$. aboveequation is the Stefan-Boltzmann law and the constant σ is called the Stefan-Boltzmann constant.
Definition 2 : the total hemispherical emissivity of a surface is the ratio of the total hemispherical emissive power of the surface to the total hemispherical emissive power of a black surface at the same temperature it will be denoted by the symbol Є. Thus,

$$Є = \frac{e}{e_b}$$

It follows from the definition that the total hemispherical emissivity of a black surface is unity.

Definition-3: The monochromatic hemispherical emissive power of a surface at a wavelength λ is the radiant flux emitted from the surface per unit wavelength about the wavelength(λ).Alternatively, it can be defined as the quantity which when integrated over all wavelengths yields total hemispherical emissive power. It will be denoted by the symbol e_λ(units: W/m^2 µm). The monochromatic hemispherical emissive power of a black surface will be denoted by e_λ .The adjectives monochromatic and hemispherical indicate that the quantity being defined is for a particular wavelength summed over all directions.From the definition it follows that

$$e_\lambda = \frac{d_e}{d\lambda} \text{ or } d_e = e_\lambda \, d\lambda$$

Integrating over all wavelengths, we get

$$e = \int_0^\infty e_\lambda \, d\lambda$$

Similarly fora black surface,

$$e_b = \int_0^\infty e_{b\lambda} d\lambda$$

Definition-4 : The monochromatic hemispherical emissivity of a surface is the ratio of the monochromatic hemispherical emissive power of the surface to the monochromatic hemispherical emissive power of a black surface at the same temperature and wavelength. Thus

$$\epsilon_\lambda = \frac{e_\lambda}{e_{b\lambda}}$$

It is easily seen that the emissivity and monochromatic emissivity of a surface are related as follows:

$$\epsilon = \frac{1}{e_b} \int_0^\infty \epsilon_\lambda e_{b\lambda} d\lambda$$

Typically the monochromatic emissivity of a surface will vary in some irregular fashion between 0 and 1.

However, for purposes of analysis, we often idealize a surface by assuming that the value is a constant. Such a surface is called a *gray surface*.

The symbol H (units: W/m^2). The quantity H is a summation of radiation of all wavelengths coming from all directions.

Definition 7: thetotal hemisphere absorptivity of a surface is the fraction of the total hemisphere irradiation absorbed by it. It will be denoted by the symbol α. Thus

$$\alpha = H_a/H$$

Where H_a is the absorbed flux. For a black surface, $\alpha = 1$.

Definition 8: The *monochromatic hemisphere irradiation* at a wavelength λ is the radiant flux incident on a surface per unit wavelength about the wavelength λ. alternatively, it can be defined as the quantity which when integrated overall wavelengths yields the total hemispherical irradiation. It will be denoted by the symbol H_λ (units/m^2 μm). From the definition

$$H_\lambda = dH/d\lambda$$

Or $dH = H_\lambda d\lambda$

Integrating overall wavelengths, we get

$$H = \int_0^\infty H_\lambda \, d\lambda$$

Definition9: The *monochromatic hemispherical absorptivity* of a surface is the fraction of the monochromatic hemispherical irradiation absorbed by it. It will be denoted by the symbol α_λ. Thus

$$\alpha_\lambda = H_{a\lambda}/H_\lambda$$

Where $H_{a\lambda}$ is the absorbed flux at wavelength λ. For black surface, $\alpha_\lambda = 1$.

From the definitions, it follows that α and α_λ are related by the equation

$$\alpha = 1/H \int_0^\infty \alpha_\lambda H_\lambda \, d\lambda$$

In an analogous manner to definitions 7 and 9, we can also define terms like *total hemispherical reflectivity (ρ), monochromatic hemispherical reflectivity (ρ_λ), total hemispherical transmissivity (Ŧ)* and *monochromatic hemispherical transmissivity ($Ŧ_\lambda$).*Equations similar to equations (3.3.1), (3.3.4) and (3.3.5) can be written with the subscripts r and t being used to indicate the reflected and transmitted quantities. As in section 3.2, we shall often refer to α, ρ and Ŧ as the absorptivity, reflectivity and transmissivity respectively. Similarly α_λ ,ρ_λ and $Ŧ_\lambda$ will be referred to as the monochromatic absorptivity, monochromatic reflectivity and monochromatic transmissivity respectively and monochromatic transmissivity respectively.

From the definitions it follows that

$$\alpha + \rho + Ŧ = 1$$

And $\alpha_\lambda + \rho_\lambda + Ŧ_\lambda = 1$

In case the body is opaque and does not allow any radiation to be transmitted through it, then

$$\alpha + \rho = 1$$

And $\alpha_\lambda + \rho_\lambda = 1$

For a non-black surface having an emissivityϵ,it follows that

$e = \epsilon\sigma T^4$ (3.2.10)

It is of historic interest to note that both Wien's law and the Stefan-Boltzmann law were independently developed before Planck's law and consequently were not originally derived in the manner shown above.

In many applications, it is necessary to determine the flux emitted by a black body at a temperature T over the wavelength interval 0 toλ. This is given by

$$e_{b(0-\lambda)} = \int_0^\lambda e_{b\lambda} \, d\lambda$$

We define $D_{0-\lambda}$ as the ratio of the radiant flux emitted over the wavelength interval 0 to λ to the radiant flux emitted over the entire interval of 0 to∞. Thus

$$D_{0-\lambda} = \frac{\int_0^\lambda e_{b\lambda}\, d\lambda}{\int_0^\infty e_{b\lambda}\, d\lambda} = \frac{\int_0^\lambda e_{b\lambda}\, d\lambda}{\sigma T^4}$$

Substituting the expression for Planck's law and re-arranging terms, we get

$$\mathrm{D}_{0-\lambda} = \int_0^\lambda \frac{2\pi C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]} \frac{1}{\sigma T^4} d\lambda$$

$$= \int_0^\lambda \frac{2\pi C_1}{\sigma(\lambda T)^5[\exp(C_2/\lambda T) - 1]} T d\lambda$$

Changing the variable of integration from λ to λT, we have

$$D_{0-\lambda} = \int_0^{\lambda T} \frac{2\pi C_1}{\sigma(\lambda T)^5[\exp(C_2/\lambda T) - 1]} d(\lambda T)$$

The integration required in equation (3.2.12) cannot be solved analytically and has been done numerically. The results are presented in table 3.1 where the fraction $D_{0-\lambda}$ ranging from 0 to 1 is tabulated as a function of parameters λT. Note the advantage of introducing the combination parameter λT. The integration need not be done for different values of λ and different values of T. thus the effort involved in doing the integration is reduced and the result are also presented in a compact from

Solid Angle and Radiation Intensity

We now turn our tension to the directional nature of thermal radiation and define two terms – solid angle and radiation intensity.

Consider the radiation emitted from the differential area dA_1 towards the area A_2 in below fig. it is obvious that although radiation streams out from dA_1 in all directions encompassed by a hemisphere, only that part which falls within the solid angle subtended by A_2 at dA_1 will reach A_2.

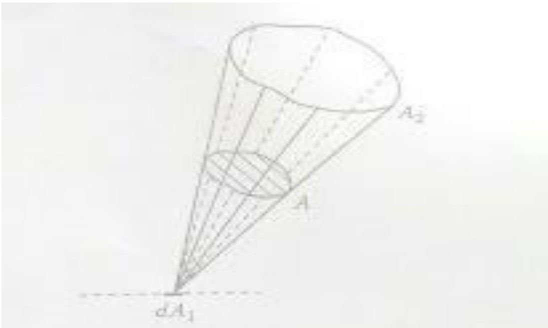


Fig. 3.9 *Definition of solid angle*

Definition 10. The solid angle subtended by the area A_2 at the differential area dA_1 is numerically equal to the area A of the portion of the surface of a sphere of unit radius, with centre at dA_1, which is cut by a conical surface, with vertex at dA_1 passing through the perimeter of A_2. The solid angle is measured is steradians (sr) and denoted by the symbol ω

As an example, consider the hemispherical surface A_2 and the differential area dA_1 shown in Fig.3.10. In this case,the solid angle subtended by A_2 at dA_1 is 2π even if dA_1 is located anywhere on the flat circular face B.

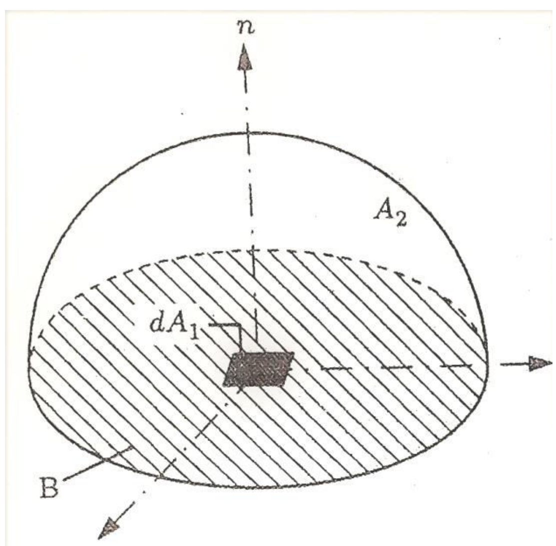


Fig. ***Solid angle subtended by a hemispherical surface A_2 at a differential area dA_1 at its centre***

Consider next the solid angle subtended by a differential area dAthrough which the radiation emitted from dA_1 passes. $dA$is assumed to be normal to the line joining the two differential areas. The length of this line is r and its direction is specified in terms of the zenith and azimuth angles,$\psi$and , of a spherical coordinate system as shown in Fig.3.11. From the definition of a solid angle, it is clear that the differential solid angle dω subtended by dA when viewed from a point on dA_1 is given by

$d\omega = \frac{dA}{r^2}$ (3.4.1)

From Fig.3.11

$dA = r^2 \sin\psi d\psi\, d\phi$

Therefore, we have the useful relationship

$d\omega = \sin\psi\, d\psi\, d\phi$ (3.4.2)

Definition 11: The total intensity of radiation emitted by a surface in a given direction is equal to the radiant flux passing in the specified direction per unit solid angle. It will be denoted by the symbol i (units: $W/m^2 sr$). The adjective total indicates that the quantity being defined is a summation of radiation over all wavelengths from 0 to ∞. Thus from the definition

$i = \frac{de}{d\omega}$

It follows that $e = \int i\, d\omega$ (3.4.3)

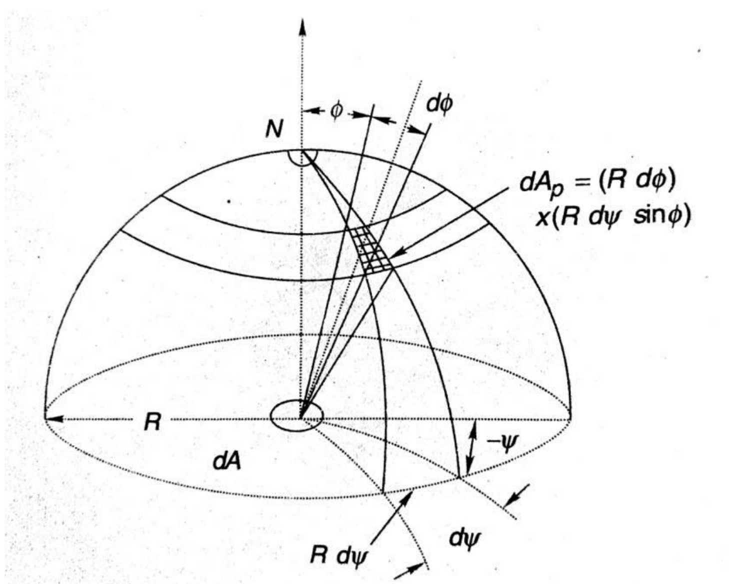


Fig *Differential solid angle in a spherical coordinate system*

Where the integration is carried out over all directions encompassed by a hemisphere.

Substituting from equation (3.4.2) and putting appropriate limits on ψ and ϕ, we have

$e = \int_0^{2\pi} \int_0^{\pi/2} i \sin\psi d\psi\, d\phi$ (3.4.4)

For many real surfaces, the variation of total radiation intensity with direction is given approximately by the relation

i= $i_n \cos\psi$(3.4.5)

Where i_n = intensity in the normal direction to the surface. Equation (3.4.5) is called Lambert's law and a surface emitting radiation in this manner is referred to as a diffuse surface. Black surfaces obey Lambert's law exactly. Substituting Lambert's law into equation (3.4.4), we can find a relation between emissive power and normal intensity. We get

$$e = \int_0^{2\pi} \int_0^{\pi/2} i_n \cos\psi \sin\psi d\psi\, d\phi$$

$$= 2\pi i_n \int_0^{\pi/2} \cos\psi \sin\psi d\psi$$

$$= \pi i_n \quad (3.4.6)$$

Thus for a diffuse surface, the emissive power is π times the normal intensity of radiation.

Laws of black body -radiation:

We shall now consider the laws of black- body radiation.These are planck's law,wien's law and Stefan-Boltzmann law.

Planck's law:

From electromagnetic considerations, planck showed that the monochromatic(spectral) emmisve power of a black surface is given by

$$e_{b\lambda} = \frac{2\pi C_1}{\lambda^5[exp(C_2/\lambda T) - 1]}$$

Where C_1 and C_2 are constants whose values are found from experimental data to be $0.596X10^{-6}$Wm^2 and 0.014387 m K respectively ,λ is the wavelength and T, the absolute temperature (in k) of the black surface.

For non-black surface having a monochromatic emissivity ϵ_λ,it follows that

$$e_{b\lambda} = \frac{2\pi C_1}{\lambda^5[exp(C_2/\lambda T) - 1]}$$

Typical curves obtained by plotting equation (3.2.6) for different values of temperature are in Fig..3.5.It I seen that a particular temperature , the value of $e_{b\lambda}$ increases with wavelength ,passes through a maximum and the decreases asymptotically to zero as $\lambda \longrightarrow \infty$. It is also seen that at the same wavelength as the temperature increases.The truth of the statement made earlier that most of the energy for temperature above 1000K is emitted in the range from 0.1 t0 10μm is also seen from Fig.3.5

Wein's law :

the location of the maximum of the curves plotted in Fig.3.5.can be found readily by differentiating the right –hand side of the equation (3.2.6) with respect to λ and equation it to zeroputting $C_2/\lambda T = \mathcal{Y}$, after simplification we get

$$e^{\mathcal{Y}}(5-\mathcal{Y}) = 5$$

By trail and error.$\mathcal{Y} = 4.965$

Therefore, f λ_m denotes the location of the maximum

$$\lambda_m T = \frac{0.014387}{4965} = 0.00290\ mK$$

above Equation is often called Wien's law.

Stefan-Boltzmann law :

The emissive power of a black surface can be found by integrating the expression for planck's law over all wavelengths. Thus

$$e_b = \int_0^\infty e_b d\lambda = 2\prod C_1 \int_0^\infty \frac{1}{\lambda^5\{\exp(C_2+\lambda T)-1\}} d\lambda$$

$$substituting x = 1/\lambda and dx = (-1/\lambda^2)d\lambda$$

$$e_b = 2\prod C_1 \int_0^\infty x^2\{\exp(C_2 T/T)-1\}^{-1} dx$$

$$= 2\prod C_1 \int_0^\infty x^3\{\exp-(C_2/T)x + \exp-(C_2/T)2x + ...\}$$

$$= 2\prod C_1 \frac{6T^4}{C_2^4}(1+\frac{1}{2^4}+\frac{1}{3^4}+......)$$

$$= 2\prod C_1 \frac{6T^4}{C_2^4}(\frac{\prod^4}{90}) = \sigma T^4$$

Solid angle and radiation intensity:

we now turn our attention to the directional nature of thermal radiation and to define two terms-solid angle and radiation intensity.

Consider the radiation emitted from the differential area dA1 towards the area A2 in fig.3.. it is obvious that although radiation streams out from dA1 in all directions encompassed by a hemisphere ,only that part which falls within the solid angle subtended by A2 at dA1 will reach A2.

Definition 10:the solid angle subtended by the area A2 at the differential area dA1 is numerically equal to the area A of the portion of the surface of a sphere of unit radius, with centre dA1,which is cut out by a conical surface , with vertex at dA1 passing through the perimeter of A2.the solid angle is measured in Steradians (sr) and denoted by the symbol ω.

As an example ,consider the hemispherical surface A2 and the differential area dA1 as shown. In this case ,the solid angle subtended by A2at dA1 is 2π.the solid angle would continue to be 2πeven if dA1 is located anywhere on the flat circular face B.

Consider next the solid angle subtended by a differential area dA through which the radiation emitted from dA1 passes . dA is assumed to be normal to the line joining the two differential areas .the length of this line is r and its direction is specified in terms of the zenith and azimuth angles ,$\varphi and \Psi$, of a spherical co-ordinate system as shown in fig.3.11. from the definition of a solid angle, it is clear that the differential solid angle dω subtended by dA when viewed from a point on dA1, is given by

dω = dA/r2

from fig
dA = r^2sin $\Psi d\Psi d\Phi$

Therefore , we have the useful relationship

$$d\omega = \sin \Psi d\Psi d\Phi$$

Definition 11:

The total intensity of radiation emitted by a surface in a given direction is equal to the radiant flux passing in the specified direction per unit solid angle .it will be denoted by the symbol I (units :W/m^2sr). the adjective total indicates the quantity being defined is a summation of radiation over all wavelengths from 0 to ∞.thus from the definition

$$i=de/d\omega$$

it follows that $e=\int i d\omega$