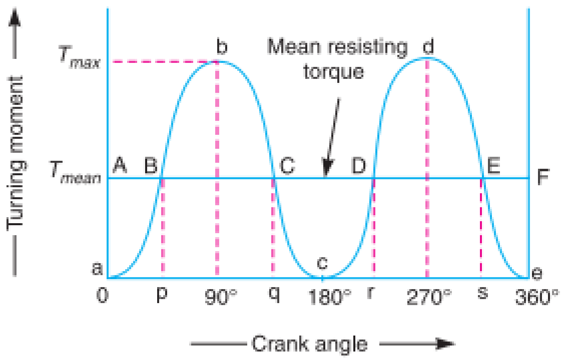
II B.Tech, II Semester – 2018-19 *Mechanical Engineering*

DYNAMICS OF MACHINES

DYNAMIC FORCE ANALYSIS and FLY WHEELS (UNIT- I)

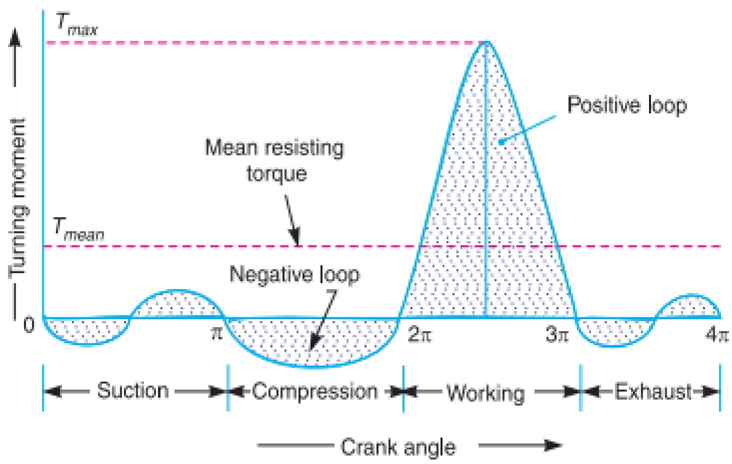
Turning Moment Diagram for a Single Cylinder Double Acting Steam Engine :A turning moment diagram for a single cylinder double acting steam engine is shown in Fig. The vertical ordinate represents the turning moment and the horizontal ordinate represents the crank angle.

$$T = F_P \times r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right)$$



FP = Piston effort, r = Radius of crank, n = Ratio of the connecting rod length and radius of crank, and θ = Angle turned by the crank from inner dead centre.

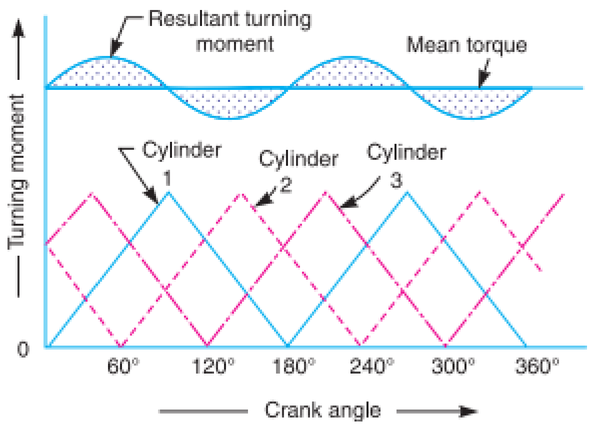
Turning Moment Diagram for a Four Stroke Cycle Internal Combustion Engine A turning moment diagram for a four stroke cycle internal combustion engine is shown in Fig. We know that in a four stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, i.e. 720° (or 4 π radians)



Since the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke, therefore a negative loop is formed as shown in Fig. 16.2. During the compression stroke, the work is done on the gases, therefore a higher negative loop is obtained. During the expansion or working stroke, the fuel burns and the gases expand, therefore a large positive loop is obtained. In this stroke, the work is done by the gases. During exhaust stroke, the work is done on the gases, therefore a negative loop is formed. It may be noted that the effect of the inertia forces on the piston is taken into account in Fig. .

Turning Moment Diagram for a Multi-cylinder Engine

separate turning moment diagram for a compound steam engine having three cylinders and the resultant turning moment diagram is shown in Fig. The resultant turning moment diagram is the sum of the turning moment diagrams for the three cylinders. It may be noted that the first cylinder is the high pressure cylinder, second cylinder is the intermediate cylinder and the third cylinder is the low pressure cylinder. The cranks, in case of three cylinders, are usually placed at 120° to each other.



Determination of Maximum Fluctuation of Energy

A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig. The horizontal line AG represents the mean torque line. Let a1, a3, a5 be the areas above the mean torque line and a2, a4 and a6 be the areas below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.

Let the energy in the flywheel at A = E, then from Fig.

we have Energy at B = E + a1

Energy at C = E + a1– a2

Energy at D = E + a1 – a2 + a3

Energy at E = E + a1 – a2 + a3 – a4

Energy at F = E + a1 – a2 + a3 – a4 + a5

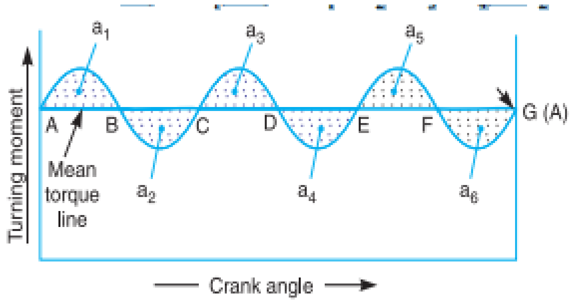
Energy at G = E + a1 – a2 + a3 – a4 + a5 – a6 = Energy at A

(i.e. cycle repeats after G) Let us now suppose that the greatest of these energies is at B and least at E. Therefore, Maximum energy in flywheel = E + a1

Minimum energy in the flywheel = E + a1 – a2 + a3 – a4

∴ Maximum fluctuation of energy,

Δ E = Maximum energy – Minimum energy = (E + a1) – (E + a1 – a2 + a3 – a4) = a2 – a3 + a4



Coefficient of Fluctuation of Energy

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

Flywheel

A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.

In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke. For example, in internal combustion engines, the energy is developed only during expansion or power stroke which is much more than the engine load and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines. The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed. A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases energy, the speed decreases. Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed. In other words, a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.

Coefficient of Fluctuation of Speed

The difference between the maximum and minimum speeds during a cycle is called the maximum fluctuation of speed. The ratio of the maximum fluctuation of speed to the mean speed is called the coefficient of fluctuation of speed.

$$C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2}$$ ...(In terms of linear speeds)

$$m = \frac{1}{C_S} = \frac{N}{N_1 - N_2}$$

Energy Stored in a Flywheel

A flywheel is shown in Fig. that when a flywheel absorbs energy, its speed increases and when it gives up energy, its speed decreases.

Let m = Mass of the flywheel in kg,

k = Radius of gyration of the flywheel in metres,

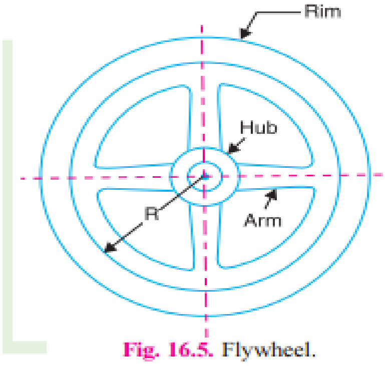


Fig. 16.5. Flywheel.

I = Mass moment of inertia of the flywheel about its axis of rotation in kg-m2 = m.k2,

N1 and N2 = Maximum and minimum speeds during the cycle in r.p.m.,

ω1 and ω2 = Maximum and minimum angular speeds during the cycle in rad/s

N = Mean speed during the cycle in r.p.m. $= \frac{N_1 + N_2}{2}$,

ω = Mean angular speed during the cycle in rad/s $= \frac{\omega_1 + \omega_2}{2}$,

C_S = Coefficient of fluctuation of speed, $= \frac{N_1 - N_2}{N}$ or $\frac{\omega_1 - \omega_2}{\omega}$

We know that the mean kinetic energy of the flywheel,

$$E = \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2$$

As the speed of the flywheel changes from ω1 to ω2,

the maximum fluctuation of energy, ΔE = Maximum K.E. – Minimum K.E

$$= \frac{1}{2} \times I\,(\omega_1)^2 - \frac{1}{2} \times I\,(\omega_2)^2 = \frac{1}{2} \times I\left[(\omega_1)^2 - (\omega_2)^2\right]$$

$$= \frac{1}{2} \times I\,(\omega_1 + \omega_2)(\omega_1 - \omega_2) = I.\omega(\omega_1 - \omega_2)$$

$$= I.\omega^2\left(\frac{\omega_1 - \omega_2}{\omega}\right)$$

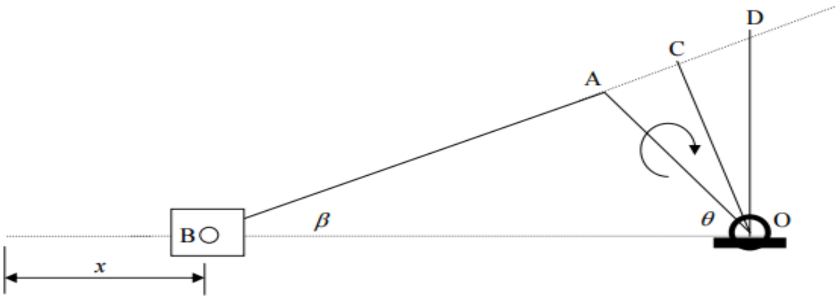
$$= I.\omega^2.C_S = m.k^2.\omega^2.C_S$$

$$= 2.E.C_S \text{ (in N–m or joules)}$$

$$\Delta E = m.R^2.\omega^2.C_S = m.v^2.C_S$$

v = Mean linear velocity (*i.e.* at the mean radius) in m/s = $\omega.R$

Dynamic analysis of Slider Crank Mechanism



x =displacement of piston from inner dead centre

$$= \begin{array}{l} r[(n+1)-(n\cos\beta+\cos\theta)] \\ = r\left[(1-\cos\theta)+\left(n-\sqrt{n^2-\sin^2\theta}\right)\right] \end{array}$$

If the connecting rod is very large is very large, and hence $\sqrt{n^2-\sin^2\theta}$ will approach n. The equation converts to

Velocity of Piston

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}.\frac{d\theta}{dt}$$

$$= r\omega\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right]$$

$$v = r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$$

If sin 2θ/2n can be neglected (when n is quite less) then

$$v = r\omega\sin\theta$$

Acceleration of Piston

$$a = \frac{dv}{dt} = \frac{dv}{d\theta}\cdot\frac{d\theta}{dt}$$
$$= \frac{d}{d\theta}\left[r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)\right]\omega = r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

If n is very large 2a r = ω cosθ which is SHM

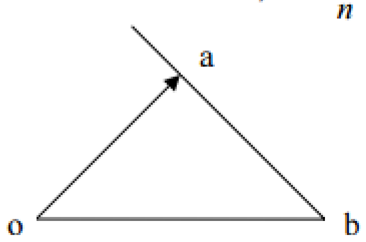
When $\theta = 0^0$ i.e. at IDC $a = r\omega^2\left(1+\frac{1}{n}\right)$

When $\theta = 180^0$ i.e. at ODC $a = r\omega^2\left(-1+\frac{1}{n}\right)$

At $\theta = 180^0$ when the direction of motion is reversed $a = r\omega^2\left(1-\frac{1}{n}\right)$

Angular Velocity and Angular Acceleration of Connecting rod

$$\sin\beta = \frac{\sin\theta}{n}$$



α_c = angular acceleration of the connecting rod

$$= \frac{d\omega_c}{dt} = \frac{d\omega_c}{d\theta}\cdot\frac{d\theta}{dt} = -\omega^2\sin\theta\left[\frac{n^2-1}{\left(n^2-\sin^2\theta\right)^{(3/2)}}\right]$$

Net or Effective force on the Piston

A_1= area of the cover end

A_2 = area of the piston end

P_1= pressure of the cover end

P_2 = pressure of the piston end

m = mass of the reciprocating parts

Force on the piston $F_p = P_1A_1 - P_2A_2$

Inertia force $F_b = ma = mr\omega^2\left(cos\theta + \frac{\cos 2\theta}{n}\right)$

Crank Effort

It is the net effort (force) applied at the crank pin perpendicular to the crank, which gives the required turning moment on the crankshaft.

Ft = crank effort

Fc = force on the connecting rod

$$\text{As } F_t r = F_c r\sin(\theta+\beta)$$

$$\Rightarrow F_t = F_c\sin(\theta+\beta) = \frac{F}{cos\beta}\sin(\theta+\beta)$$

$$\begin{aligned} T &= F_t r \\ &= \frac{F}{\cos\beta}\sin(\theta+\beta)r \\ &= \frac{Fr}{\cos\beta}(\sin\theta\cos\beta + \cos\theta\sin\beta) \\ &= Fr\left(\sin\theta + \cos\theta\sin\beta\frac{1}{\cos\beta}\right) \\ &= Fr\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right) \end{aligned}$$

$$\text{Also} \quad \begin{matrix}\text{Then } m_b + m_d = m \\ \text{and } m_b b = m_d d\end{matrix} \quad r\sin(\theta+\beta) = \text{OD }\cos\beta$$

$$\begin{aligned} T &= F_t r \\ &= \frac{F}{\cos\beta}r\sin(\theta+\beta) \\ &= \frac{F}{\cos\beta}(\text{OD}\cos\beta) \\ &= F(\text{OD}) \end{aligned}$$

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DYNAMICS OF MACHINES

GOVERNORS AND GYROSCOPIC COUPLE

(UNIT –II)

Objective :

- To introduce the concepts of gyroscopic couple and its effect on stability of moving vehicles
- To introduce different types of governors and their characteristics

Outcomes :

Student will be able to

- determine the effect of gyroscopic couple on moving vehicles
- classify governors and determine their characteristics

GOVERNORS:

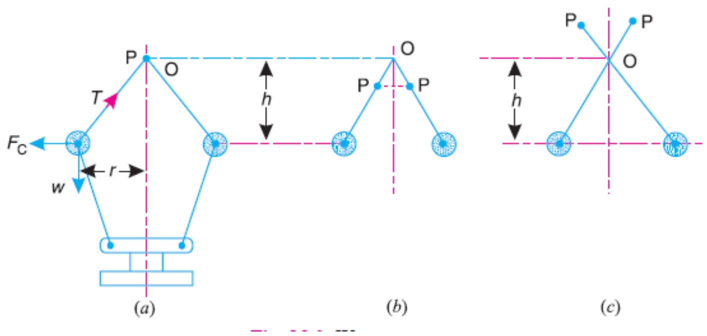
The function of a governor is to regulate the mean speed of an engine, when there are variations in the load *e.g.* when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

The governors are classified into centrifugal and inertia governors. In centrifugal governors , the ball movement is because of the centrifugal effect where as in inertia governors the ball movement is because of the inertia force which arises due to angular acceleration.

The centrifugal governors are again classified in to six types

1. Watt governor
2. Porter governor
3. Proell governor
4. Hartnell governor
5. Hartung governor
6. Wilson Hartnell governor

1. WATT GOVERNOR



m = Mass of the ball in kg,
w = Weight of the ball in Newton's = *m.g*,
T = Tension in the arm in Newton's,
ω = Angular velocity of the arm and ball about the spindle axis in rad/s,
r = Radius of the path of rotation of the ball *i.e.* horizontal distance from the centre of the ball to the spindle axis in metres,

F_C = Centrifugal force acting on the ball in Newton's = $m.\omega^2.r$, and
h = Height of the governor in metres.

It is assumed that the weight of the arms, links and the sleeve are negligible as compared to the weight of the balls. Now, the ball is in equilibrium under the action of
1. the centrifugal force (F_C) acting on the ball, **2.** the tension (T) in the arm, and **3.** the weight (w) of the ball.

Taking moments about point O, we have

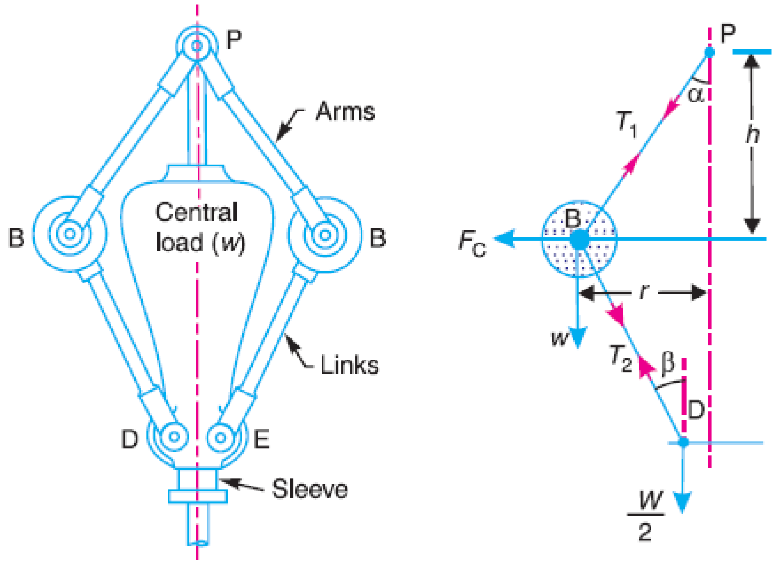
$$F_C \times h = w \times r = m.g.r$$

$$m.\omega^2.r.h = m.g.r \quad \text{or} \quad h = g/\omega^2$$

2. Porter Governor

The Porter governor is a modification of a Watt's governor, with central load attached to the sleeve as shown in Fig. (a). The load moves up and down the central Spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level.

Consider the forces acting on one-half of the governor as shown



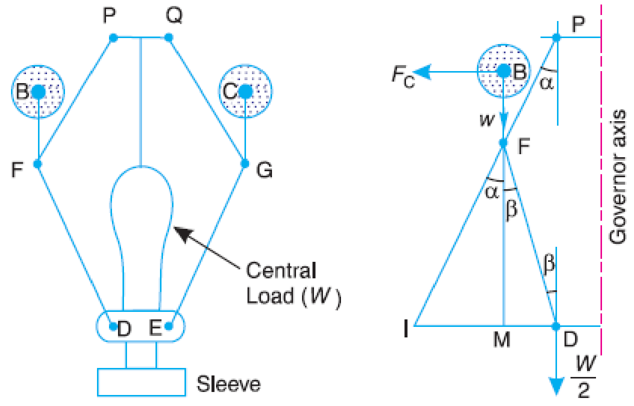
(a) (b)

m = Mass of each ball in kg,
w = Weight of each ball in Newton's = $m.g$,
M = Mass of the central load in kg,
W = Weight of the central load in Newton's = $M.g$,
r = Radius of rotation in metres,
h = Height of governor in metres ,
N = Speed of the balls in r.p.m .,
ω = Angular speed of the balls in rad/s
= $2\pi N/60$ rad/s,
F_C = Centrifugal force acting on the ball in Newton's = $m.\omega^2.r$,
T_1 = Force in the arm in Newton's,
T_2 = Force in the link in Newton's,
α = Angle of inclination of the arm (or upper link) to the vertical, and
β = Angle of inclination of the link

$$h = \frac{m.g + \frac{M.g}{2}(1+q)}{m} \times \frac{1}{\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2}$$

3. Proell Governor

The Proell governor has the balls fixed at *B* and *C* to the extension of the links *DF* and *EG*, as shown in Fig.(*a*). The arms *FP* and *GQ* are pivoted at *P* and *Q* respectively.
Consider the equilibrium of the forces on one-half of the governor as shown in Fig. (*b*).
The instantaneous centre (*I*) lies on the intersection of the line *PF* produced and the line from *D* drawn perpendicular to the spindle axis. The perpendicular *BM* is drawn on *ID*.

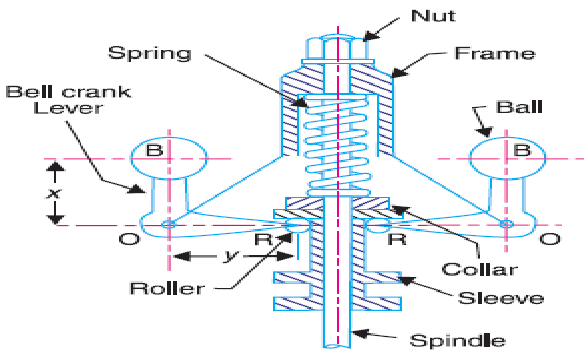


(*a*) (*b*)

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h}$$

4. Hartnell Governor

A Hart nell governor is a spring loaded governor as shown in Fig. It consists of two bell crank levers pivoted at the points *O*,*O* to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm *OB* and a roller at the end of the horizontal arm *OR*. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.

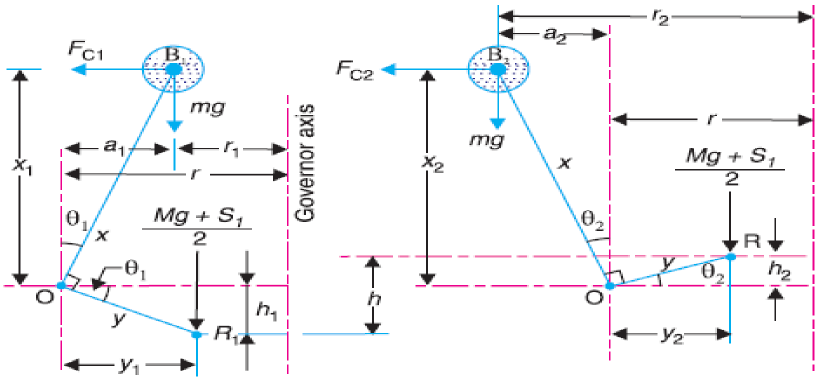


For the minimum position *i.e.* when the radius of rotation changes from *r* to *r*1, as shown in Fig. (*a*), the compression of the spring or the lift of sleeve *h*1 is given by

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x}$$

Similarly, for the maximum position *i.e.* when the radius of rotation changes from *r* to *r*2, as shown in Fig. (*b*), the compression of the spring or lift of sleeve *h*2 is given by

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x}$$



(*a*) Minimum position. (*b*) Maximum position.

Adding equations (*i*) and (*ii*),

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x} \quad \text{or} \quad \frac{h}{y} = \frac{r_2 - r_1}{x}$$

$$\therefore \quad h = (r_2 - r_1)\frac{y}{x}$$

$$F_C = F_{C1} + (F_{C2} - F_{C1})\left(\frac{r - r_1}{r_2 - r_1}\right) = F_{C2} - (F_{C2} - F_{C1})\left(\frac{r_2 - r}{r_2 - r_1}\right)$$

Sensitiveness of Governors :

Consider two governors *A* and *B* running at the same speed. When this speed increases or decreases by a certain amount, the lift of the sleeve of governor *A* is greater than the lift of the sleeve of governor *B*. It is then said that the governor *A* is more sensitive than the governor *B*.

In general, the greater the lift of the sleeve corresponding to a given fractional change in speed, the greater is the sensitiveness of the governor. It may also be stated in another way that for a given lift of the sleeve, the sensitiveness of the governor increases as the speed range decreases. This definition of sensitiveness may be quite satisfactory when the governor is considered as an independent mechanism. But when the governor is fitted to an engine, the practical requirement is simply that the change of equilibrium speed from the full load to the no load position of the sleeve should be as small a fraction as possible of the mean equilibrium speed. The actual displacement of the sleeve is immaterial, provided that it is sufficient to change the energy supplied to the engine by the required amount.

For this reason, the sensitiveness is defined as the **ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.**

Let *N*1 = Minimum equilibrium speed,

*N*2 = Maximum equilibrium speed, and

Sensitiveness of the governor is

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

Isochronous Governors :

A governor is said to be **isochronous** when the equilibrium speed is constant (*i.e.* range of speed is zero) for all radii of rotation of the balls within the working range, Neglecting friction. The isochronism is the stage of infinite sensitivity. Let us consider the case of a Porter governor running at speeds *N*1 and *N*2 r.p.m. We have discussed in Art. That

$$(N_1)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_1}$$

$$(N_2)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_2}$$

$$\frac{M.g + S_1}{M.g + S_2} = \frac{r_1}{r_2}$$

Hunting

A governor is said to be **hunt** if the speed of the engine fluctuates continuously above and below the mean speed. This is caused by a too sensitive governor which changes the fuel supply by a large amount when a small change in the speed of rotation takes place. For example, when the load on the engine increases, the engine speed decreases and, if the governor is very sensitive, the governor sleeve immediately falls to its lowest
position. This will result in the opening of the control valve wide which will supply the fuel to the engine in excess of its requirement so that the engine speed rapidly increases again and the governor sleeve rises to its highest position. Due to this movement of the sleeve, the control valve will cut off the fuel supply to the engine and thus the engine speed begins to fall once again. This cycle is repeated indefinitely. Such a governor may admit either the maximum or the minimum amount of fuel. The effect of this will be to cause wide fluctuations in the engine speed or in other words, the engine will hunt.

Gyroscopic Couple

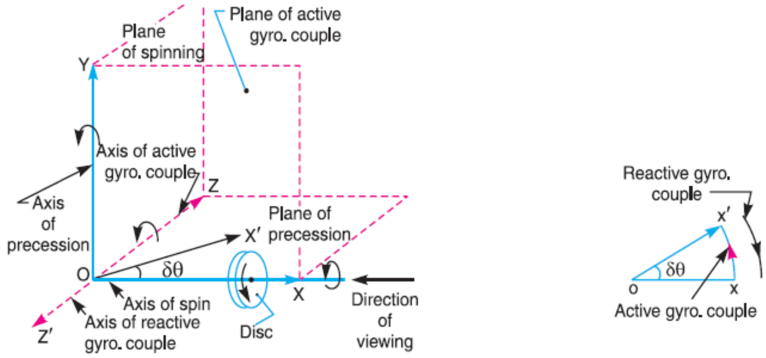
Consider a disc spinning with an angular velocity ω rad/s about the axis of spin *OX*, in Anticlockwise direction when seen from the front, as shown in Fig. (*a*). Since the plane in which the disc is rotating is parallel to the plane *YOZ*, therefore it is called **plane of spinning.** The plane *XOZ* is a horizontal plane and the axis of spin rotates in a plane parallel to the horizontal plane about an axis *OY*. In other words, the axis of spin is said to be rotating or processing about an axis *OY*. In other words, the axis of spin is said to be rotating or processing about an axis *OY* (which is perpendicular to both the axes *OX* and *OZ*) at an angular velocity ωP rap/s. This horizontal plane *XOZ* is called

plane of precession and *OY* is the **axis of precession.**

I = Mass moment of inertia of the disc about OX, and

ω = Angular velocity of the disc.

∴ Angular momentum of the disc = I.ω



$$C = \underset{\delta t \to 0}{\mathrm{Lt}} \; I.\omega \times \frac{\delta\theta}{\delta t} = I.\omega \times \frac{d\theta}{dt} = I.\omega.\omega_P$$

Effect of the Gyroscopic Couple on an Aeroplane

The top and front view of an aeroplane are shown in Fig (*a*). Let engine or propeller rotates in the clockwise direction when seen from the rear or tail end and the aeroplane takes a turn to the left

ω = Angular velocity of the engine in rad/s,

m = Mass of the engine and the propeller in kg,

k = Its radius of gyration in metres,

I = Mass moment of inertia of the engine and the propeller in kg-m2

= *m.k*2,

v = Linear velocity of the aeroplane in m/s,

R = Radius of curvature in metres, and

ωP = Angular velocity of precession *v/R* rad/s

∴ Gyroscopic couple acting on the aeroplane,

C = *I*.ω.ωP

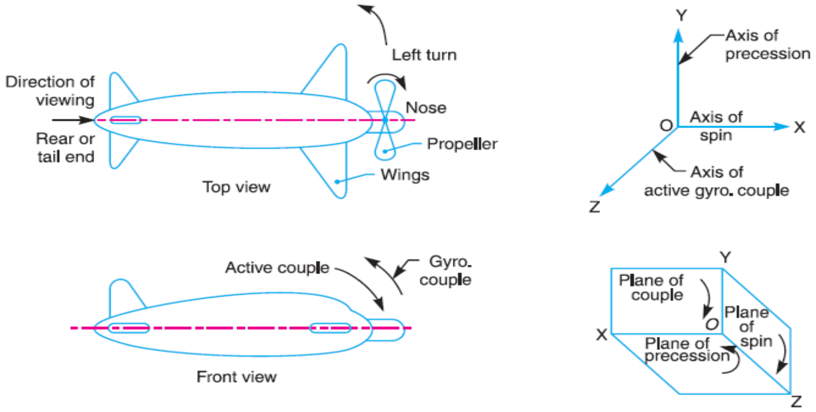
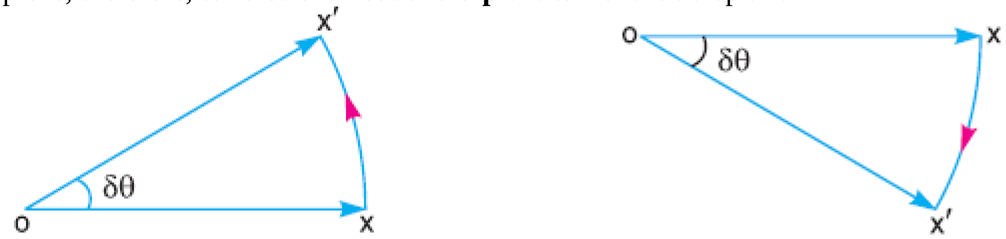


Fig. 14.5. Aeroplane taking a left turn.

Before taking the left turn, the angular momentum vector is represented by *ox*. When it takes left turn, the active gyroscopic couple will change the direction of the angular momentum vector from *ox* to *ox′* as shown in Fig. (*a*). The vector *xx'*, in the limit, represents the change of angular momentum or the active gyroscopic couple and is perpendicular to *ox*. Thus the plane of active gyroscopic couple *XOY* will be perpendicular to *xx'* , *i.e.* vertical in this case, as shown in Fig (*b*). By applying right hand screw rule to vector *xx'*, we find that the direction of active gyroscopic couple is clockwise as shown in the front view of Fig. (*a*). In other words, for left hand turning, the active gyroscopic couple on the aeroplane in the axis *OZ* will be clockwise as shown in Fig. (*b*).The reactive gyroscopic couple (equal in magnitude of active gyroscopic couple) will act in the opposite direction (*i.e.* in the anticlockwise direction) and the effect of this couple is, therefore, to **raise the nose** and **dip the tail** of the aeroplane

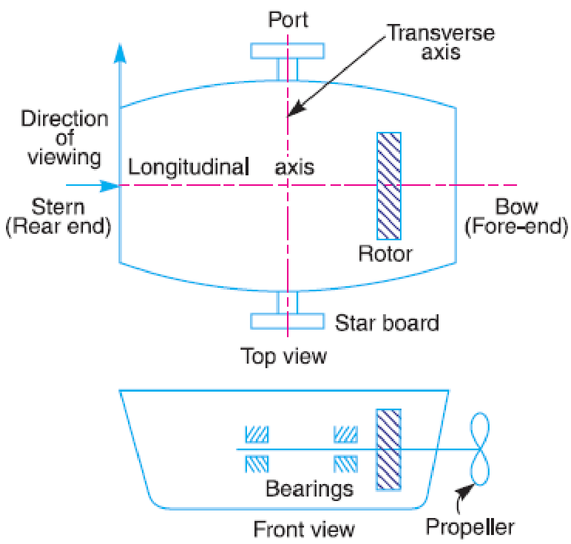


(*a*) Aeroplane taking left turn. (*b*) Aeroplane taking right turn.

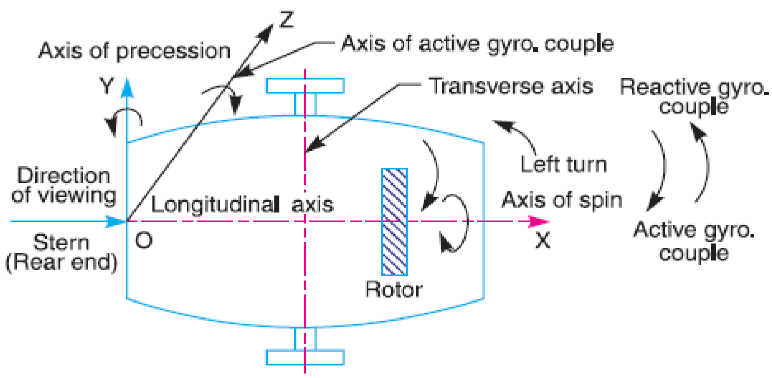
Terms Used in a Naval Ship

The top and front views of a naval ship are shown in Fig 14.7. The fore end of the ship is called **bow** and the rear end is known as **stern** or **aft**. The left hand and right hand sides of the ship, when viewed from the stern are called **port** and **star-board** respectively. We shall now discuss the effect of gyroscopic couple on the naval ship in the following three cases:

1. Steering, **2.** Pitching, and **3.** Rolling.



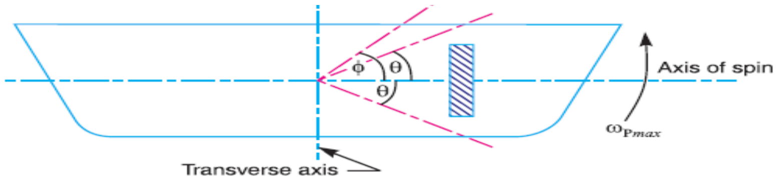
Effect of Gyroscopic Couple on a Naval Ship during Steering



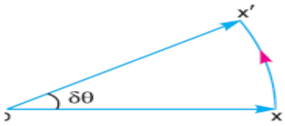
When the rotor of the ship rotates in the clockwise direction when viewed from the stern, it will have its angular momentum vector in the direction *ox* as shown in Fig. (*a*). As the ship steers to the left, the active

gyroscopic couple will change the angular momentum vector from *ox* to *ox′*. The vector *xx′* now represents the active gyroscopic couple and is perpendicular to *ox*. Thus the plane of active gyroscopic couple is perpendicular to *xx′* and its direction in the axis *OZ* for left hand turn is clockwise as shown in Fig. The reactive gyroscopic couple of the same magnitude will act in the
opposite direction (*i.e.* in anticlockwise direction). The ***effect of this reactive gyroscopic couple is to raise the bow and lower the stern.***

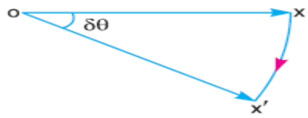
Effect of Gyroscopic Couple on a Naval Ship during Pitching



(*a*) Pitching of a naval ship



(*b*) Pitching upward



(*c*) Pitching downward

∴ Angular displacement of the axis of spin from mean position after time *t* seconds,
$\theta = \varphi \sin \omega 1 . t$
where φ = Amplitude of swing *i.e.* maximum angle turned from the mean
position in radians, and
ω1 = Angular velocity of S.H.M.

$$= \frac{2\pi}{\text{Time period of S.H.M. in seconds}} = \frac{2\pi}{t_p} \text{ rad/s}$$

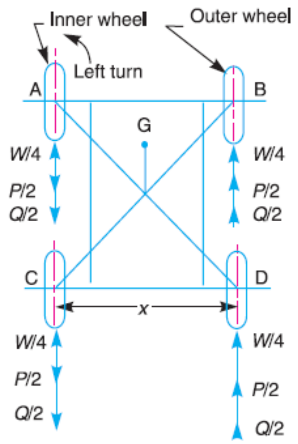
∴ Manimum gyroscopic couple,

$$C_{max} = I.\ \omega.\ \omega_{Pmax}$$

Effect of Gyroscopic Couple on a Naval Ship during Rolling

We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. If, however, the axis of precession becomes parallel to the axis of spin, there will be no effect of the gyroscopic couple acting on the body of the ship. In case of rolling of a ship, the axis of precession (*i.e.* longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the gyroscopic couple acting on the body of a ship.

Stability of a Four Wheel Drive Moving in a Curved Path:
Consider the four wheels *A*, *B*, *C* and *D* of an automobile locomotive taking a turn towards left as shown in Fig. The wheels *A* and *C* are inner wheels, whereas *B* and *D* are outer wheels. The centre of gravity (*C.G.*) of the vehicle lies vertically above the road surface.



Road reaction over each wheel
= *W*/4 = *m.g* /4 Newton's
Let us now consider the effect of
the gyroscopic couple and centrifugal couple on the vehicle

Effect of the gyroscopic couple

∴ Net gyroscopic couple,

$$C = C_W \pm C_E = 4\, I_W.\omega_W.\omega_P \pm I_E.G.\omega_W.\omega_P$$
$$= \omega_W.\omega_P\,(4\, I_W \pm G.I_E)$$

The ***positive*** sign is used when the wheels and rotating parts of the engine rotate in the same direction. If the rotating parts of the engine revolves in opposite direction, then ***negative*** sign is used. Due to the gyroscopic couple, vertical reaction on the road surface will be produced. The reaction will be vertically upwards on the outer wheels and vertically downwards on the inner wheels.
Let the magnitude of this reaction at the two outer or inner wheels be *P* Newton's. Then

$$P/2 = C/2x$$

Effect of the centrifugal couple

Since the vehicle moves along a curved path, therefore centrifugal force will act outwardly at the centre of gravity of the vehicle. The effect of this centrifugal force is also to overturn the vehicle.

$$F_C = \frac{m \times v^2}{R}$$

We know that centrifugal force
Total vertical reaction at each of the outer wheel

$$P_O = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$$

and total vertical reaction at each of the inner wheel,

$$P_I = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2}$$

II B.Tech, II Semester – 2018-19 *Mechanical Engineering*

DYNAMICS OF MACHINES- UNIT-III

BEARINGS , BRAKES , CLUTCHES , DYNAMOMETERS

BEARINGS:

The shafts of ships, steam and water turbines are subjected to axial thrust. In order to take up the axial thrust, they are provided with one or more bearing surfaces at right angle to the axis of the shaft.

A bearing surface provided at the end of a shaft is known as a pivot and that provided at any place along with the length of the shaft with bearing surface of revolution is known as a collar. Pivots are of two forms : flat and conical. The bearing surface provided at the foot of a vertical shaft is called foot step bearing. Due to the axial thrust which is conveyed to the bearings by the rotating shaft, rubbing takes place between the contacting surfaces. This produces friction as well as wearing of the bearing.

Thus, power is lost in over-coming the friction, which is ultimately to be determined in this unit. Obviously, the rate of wearing depends upon the intensity of thrust (pressure) and relative velocity of rotation. Since velocity is proportional to the radius, therefore, pr.$\propto$ Rate of wear

Assumptions Taken

(a) Firstly, the intensity of pressure is uniform over the bearing surface. This assumption only holds good with newly fitted bearings where fit between the two contacting surfaces is assumed to be perfect. After the shaft has run for quite some time the pressure distribution will not remain uniform due to varying wear at different radii.

b) Secondly, the rate of wear is uniform. As given previously, the rate of wear Friction is proportional to pr which means that the pressure will go on increasing radially inward and at the centre where r = 0, the pressure will be infinite which is not true in practical sense. However, this assumption of uniform wear gives better practical results when bearing has become older. The various types of bearings mentioned above will be dealt with separately for each assumption.

Flat Pivot Bearing :

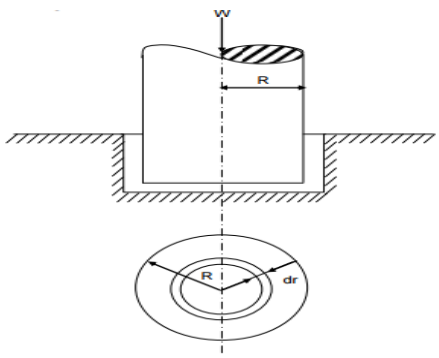


Figure 2.9 : Flat Pivot

Let W = Axial thrust or load on the bearing,

R = External radius of the pivot,

p = Intensity of pressure, and μ = Coefficient of friction between the contacting surfaces.

Consider an elementary ring of the bearing surfaces, at a radius r and of thickness dr .

Axial load on the ring

$$dW = p \times 2\pi r \times dr$$

Total load $\quad W = \int_0^R p \times 2\pi r \times dr$

Frictional force on the ring

$$dF = \mu \times dW = \mu \times p \times 2\pi r \times dr$$

Frictional moment about the axis of rotation

$$dM = dF \times r = \mu \times p \times 2\pi r^2 \times dr$$

Total frictional moment

$$M = \int_0^R dM = \int_0^R \mu \times p \times 2\pi r^2 \times dr$$

Uniform pressure Theory

$$W = p \times 2\pi \int_0^R r \times dr = p \times 2\pi \left[\frac{r^2}{2}\right]_0^R = p \times 2\pi \times \frac{R^2}{2}$$

$\Rightarrow \qquad \boldsymbol{W = p \times \pi R^2}$

And from Eq. (2.9)

$$M = \mu \times p \times 2\pi \int_0^R r^2\, dr$$

$$M = \mu\, p \times 2\pi \times \left[\frac{r^3}{3}\right]_0^R = \frac{2}{3}\mu \times p \times \pi \times R^3$$

But $\qquad p\, \pi\, R^2 = W$

$\Rightarrow \qquad \boldsymbol{M = \frac{2}{3}\mu W R = \mu W \times \frac{2}{3} R}$

Uniform Wear Theory

$$W = \int_0^R 2\pi \times c \times dr = 2\pi R \times c$$

$$\Rightarrow \qquad c = \frac{W}{2\pi R}$$

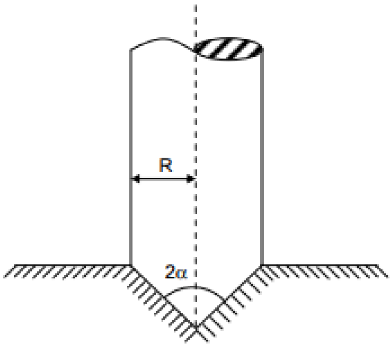
By Eq. (2.9), total frictional moment

$$M = \int_0^R \mu \times p \times 2\pi r^2 \times dr = \int_0^R \mu \times 2\pi \times c \times r \times dr$$

$$= \mu \times 2\pi \times c \times \frac{R^2}{2} = \mu \times 2\pi \times \frac{W}{2\pi R} \times \frac{R^2}{2}$$

$$\therefore \qquad \boldsymbol{M = \mu W \times \frac{R}{2}}$$

Conical Pivot:



Uniform Pressure

$$\boldsymbol{W = p\,\pi\,R_1^2}$$

$$\boldsymbol{M = \frac{\mu W}{\sin\alpha} \times \frac{2}{3} R_1}$$

Uniform Wear

$$W = \int_r^R p \times 2\pi r \times dr$$

$$\boldsymbol{M = \frac{\mu W}{\sin\alpha} \times \frac{(R_1 + R_2)}{2}}$$

Collar Bearing:

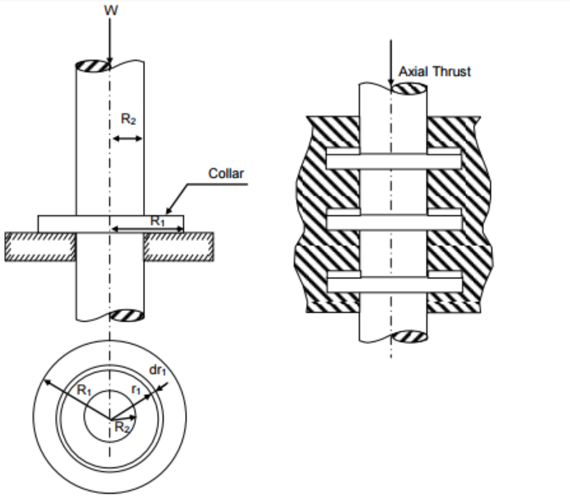


Figure 2.12 : Collar Bearing

Let W = The axial load/thrust,

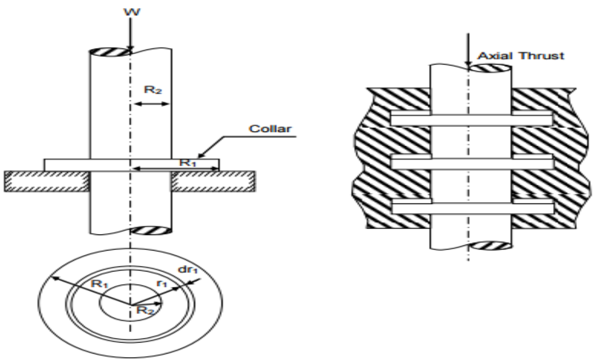
R_1 = External radius of the collar,

and R_2 = Internal radius of the collar.

W = The axial load/thrust,

R_1 = External radius of the collar, and

R_2 = Internal radius of the collar.



Total frictional moment

$$M = \int_{R_2}^{R_1} \mu \times p \times 2\pi r^2 \times dr$$

Uniform Pressure

$$W = p \times \pi (R_1^2 - R_2^2)$$

$$M = \mu \times p \times 2\pi \int_{R_2}^{R_1} r^2 \times dr = \mu \times p \times 2\pi \frac{(R_1^3 - R_2^3)}{3}$$

$$M = \mu W \times \frac{2}{3} \frac{(R_1^3 - R_2^3)}{(R_1^2 - R_2^2)}$$

Uniform Wear

$$M = \mu W \times \frac{(R_1 + R_2)}{2}$$

Conical Collar Bearing :

Let

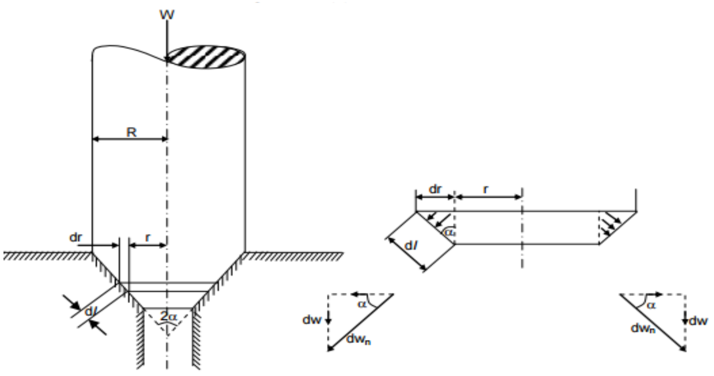
2α= The cone angle,

W = The axial load/thrust,

R_1 = The outer radius of the cone,

R_2 = The inner radius of the cone,

and p = Intensity of pressure



(a) Conical Pivot (b) Enlarged View of the Ring

Total frictional moment

$$M = \int_{R_2}^{R_1} \mu \times p \times 2\pi r^2 \times \frac{dr}{\sin \alpha}$$

Uniform Pressure

$$M = \frac{\mu W}{\sin \alpha} \times \frac{2}{3} \frac{(R_1^3 - R_2^3)}{(R_1^2 - R_2^2)}$$

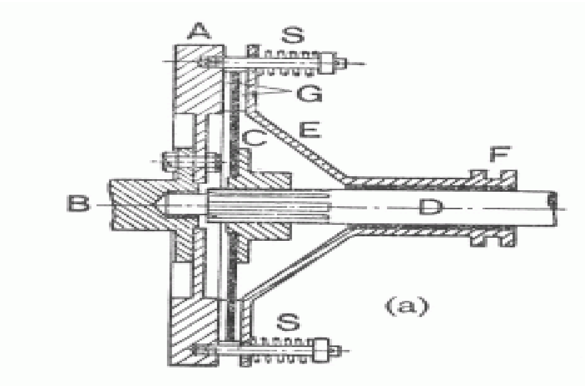
Uniform Wear

$$M = \frac{\mu W}{\sin \alpha} \times \frac{(R_1 + R_2)}{2}$$

Sl. No.	Particular	Frictional Moments : M	
		Uniform Pressure	Uniform Wear
1.	Flat pivot	$\mu W \times \frac{2}{3} R$	$\mu W \times \frac{R}{2}$
2.	Conical pivot (a) Truncated	$\frac{\mu W}{\sin \alpha} \times \frac{2}{3} \frac{(R_1^3 - R_2^3)}{(R_1^2 - R_2^2)}$	$\frac{\mu W}{\sin \alpha} \times \frac{(R_1 + R_2)}{2}$
	(b) Full conical	$\mu W \times \frac{2}{3} R$	$\mu W \times \frac{R}{2}$
3.	Collar	$\mu W \times \frac{2}{3} \frac{(R_1^3 - R_2^3)}{(R_1^2 - R_2^2)}$	$\mu W \times \frac{(R_1 + R_2)}{2}$

CLUTCHES:

Single Plate Clutch:

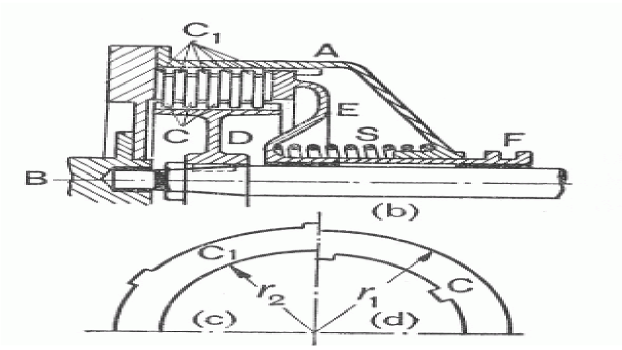


This is shown in the above diagram. The operation is as follows:

- The flywheel A is bolted to a flange on the drive shaft B
- The plate C is fixed to a boss which is free to slide xially along the driven shaft D to which it is splined. It therefore rotates with shaft D.
- Two rings G of special friction material are riveted or bonded to A and E or alternatively to plate C.
- The presser plate E is bushed internally so that it revolves freely on the driven shaft D. It is integral with the withdrawal sleeve F.
- A number of springs are arranged around the clutch (Shown as S) so as to press the two friction surfaces together.

The Clutch operates by moving the withdrawal sleeve to the right. This compresses the Springs S and removes the pressure between the friction surfaces. Hence it is possible to start or stop the driven shaft at will.

Multi Plate Clutch



The operation is very similar to the single plate clutch but the area of frictional surfaces is greatly increased. It should be noted that as the discs are free to slide axially under the spring pressure, each pair of contact surfaces is subjected to the same full axial load.

Analysis

Diagram adapted for flat plate.

The frictional force on the circular element shown on the above diagram:

$$= \mu\, p \,.\, 2\,\pi\, x \,.\, dx \qquad \textbf{(2)}$$

where p is the intensity of normal pressure between the surfaces.

By Moments about the axis, the total friction Torque is given by:

$$\tau = \int_r^R \mu\, p \,.\, 2\,\pi\, x^2 \,.\, dx \qquad \textbf{(3)}$$

The total axial thrust W is given by:

$$W = \int_r^R p \,.\, 2\,\pi\, x \,.\, dx \qquad \textbf{(4)}$$

Equations (**3**) and (**4**) can be integrated once the variation of p with radius is known. Two particular cases are considered here.

Uniform Pressure

$$W = p \,.\, \pi(R^2 - r^2) \qquad \textbf{(5)}$$

and

$$\tau = \frac{2}{3} \,.\, \mu p \,.\, \pi(R^3 - r^3) \qquad \textbf{(6)}$$

Combining equations (**5**) and (**6**)

$$\tau = \frac{2}{3} \, . \, \mu W \times \frac{R^3 - r^3}{R^2 - r^2}$$

(7)

Uniform Wear

In this case px = a constant k and equation (**4**) can be integrated to give:

$$W = 2\,\pi k(R - r)$$

(8)

Similarly,

$$\tau = \mu k \, . \, \pi(R^2 - r^2)$$

(9)

Eliminating k from equation (**9**) by using equation (**8**)

$$\tau = \mu W \times \frac{R + r}{2}$$

(10)

Note that assumption of uniform wear is usually preferred since it results in a lower calculated torque for a given value of W.

It is normal for each plate in a clutch to have two working surfaces (One on each side), and since they are arranged in series the axial load is transmitted equally through each plate. Consequently, if the number of plates on one shaft is n then the torque transmitted as calculated from equations (**7**) or (**10**) must be multiplied by $2n$.

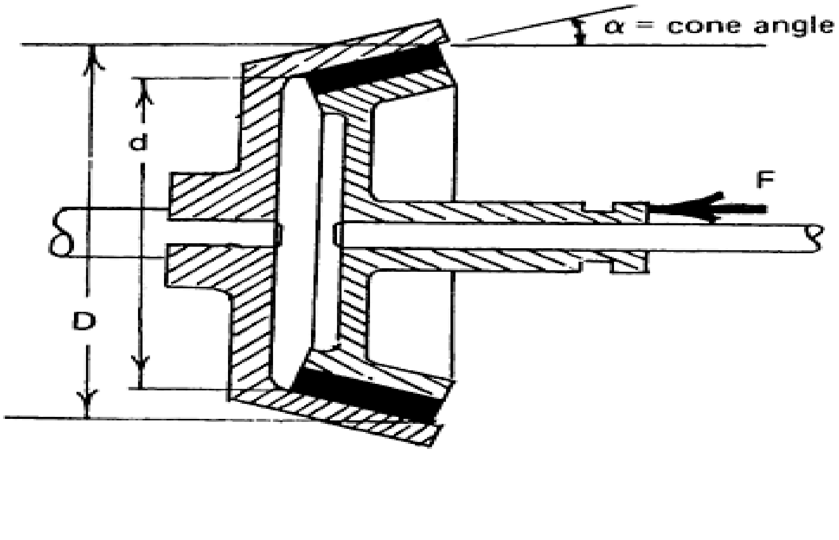
Cone Clutch:

Uniform Wear

$$F = \frac{\pi P_a d}{2}(D - d)$$

$$T = \frac{\pi \mu P_a d}{8 \sin \alpha}(D^2 - d)$$

where P_a = maximum pressure occurring at $d/2$.



Uniform Pressure

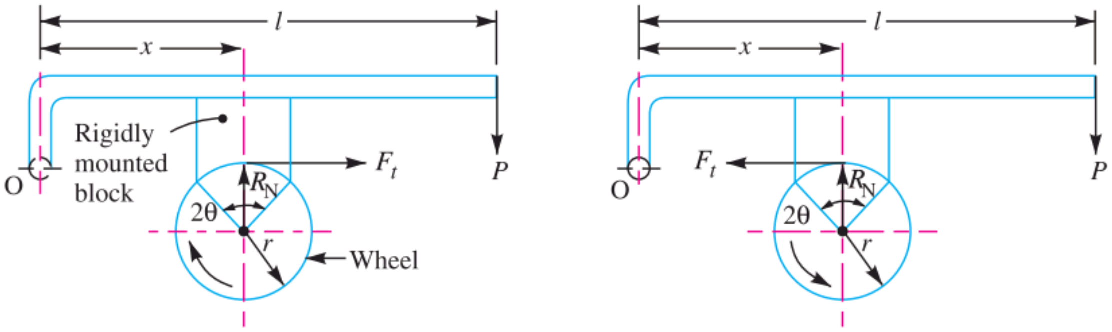
$$F = \frac{\pi P_a}{4}(D^2 - d^2)$$

$$T = \frac{\pi \mu P_a}{12 \sin \alpha}(D^3 - d^3) = \frac{F\mu}{3 \sin \alpha} \times \frac{D^3 - d^3}{D^2 - d^2}$$

BRAKES :

SINGLE BLOCK OR SHOE BRAKE

A single block or shoe brake is shown below. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material than the rim of the wheel. This type of a brake is commonly used on railway trains and tram cars. The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed. The other end of the lever is pivoted on a fixed fulcrum O.



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

P = Force applied at the end of the lever,
RN = Normal force pressing the brake block on the wheel,
r = Radius of the wheel,
2θ = Angle of contact surface of the block,
μ = Coefficient of friction, and
Ft = Tangential braking force or the frictional force acting at the contact surface of the block and the wheel.

If the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel,

Ft= μ.RN

and the braking torque,TB = Ft.r = μ RN. r

Let us now consider the following three cases :

Case I : When the line of action of tangential braking force (F) passes through the fulcrum O of
the lever, and the brake wheel rotates clockwise as shown in Fig. 25.1 (a), then for

equilibrium, taking
moments about the fulcrum O, we have,

$$R_N \times x = P \times l \qquad \text{or} \qquad R_N = \frac{P \times l}{x}$$

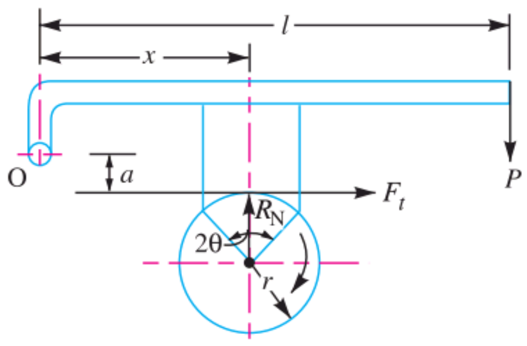
∴ Braking torque,

$$T_B = \mu.R_N.r = \mu \times \frac{P.l}{x} \times r = \frac{\mu.P.l.r}{x}$$

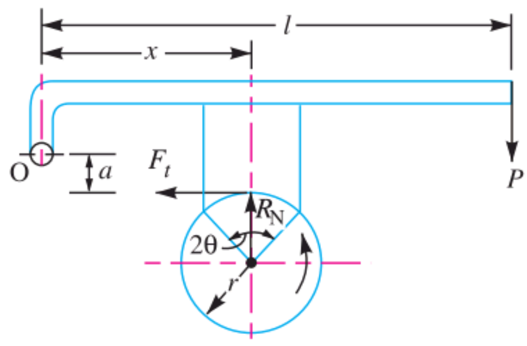
It may be noted that when the brake wheel rotates anticlockwise, then the braking torque is same, i.e.

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x}$$

Case 2 : When the line of action of the tangential braking force (F) passes through a distance 'a' below the fulcrum O, and the brake wheel rotates clockwise, then for equilibrium, taking moments about the fulcrum O,



(*a*) Clockwise rotation of brake wheel.



(*b*) Anticlockwise rotation of brake wheel.

Hence,

$$R_N \times x + F_t \times a = P.l$$

$$R_N \times x + \mu\, R_N \times a = P.l \qquad \text{or} \qquad R_N = \frac{P.l}{x + \mu.a}$$

and braking torque,

$$T_B = \mu\, R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$$

When the brake wheel rotates anticlockwise, then for equilibrium,

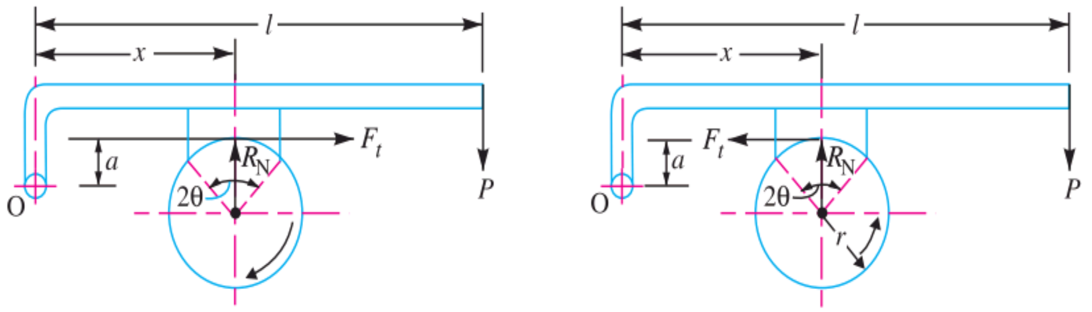
$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a$$

$$R_N (x - \mu.a) = P.l \quad \text{or} \quad R_N = \frac{P.l}{x - \mu.a}$$

and braking torque,

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

Case3 : When the line of action of the tangential braking force passes through a distance 'a' above the fulcrum, and the brake wheel rotates clockwise then for equilibrium, taking moments about the fulcrum O, we have



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a$$

$$R_N (x - \mu.a) = P.l \quad \text{or} \quad R_N = \frac{P.l}{x - \mu.a}$$

and braking torque,

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

When the brake wheel rotates anticlockwise, then for equilibrium, taking moments about the fulcrum O, we have

$$R_N \times x + F_t \times a = P.l$$

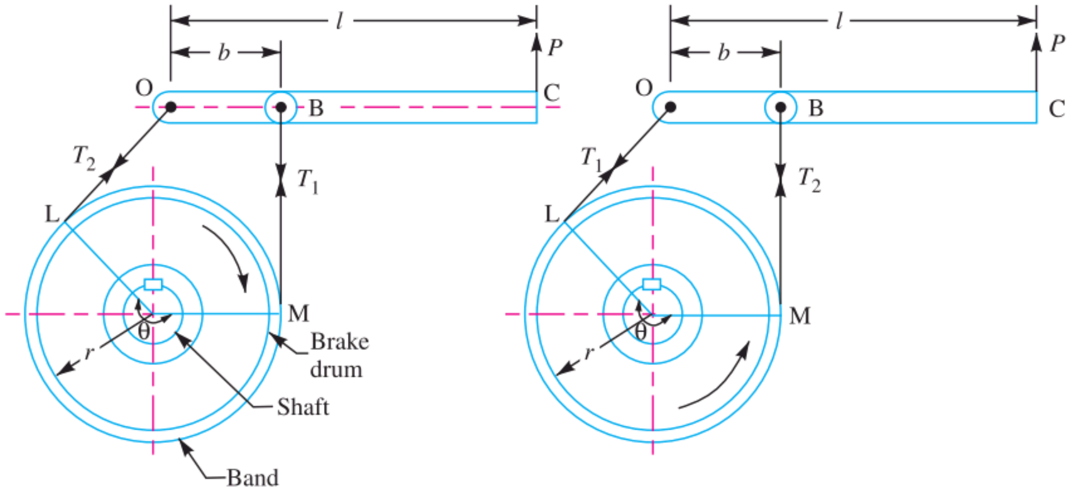
$$R_N \times x + \mu.R_N \times a = P.l \quad \text{or} \quad R_N = \frac{P.l}{x + \mu.a}$$

and braking torque,

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$$

SIMPLE BAND BRAKE

A band brake consists of a flexible band of leather, one or more ropes, or a steel lined with friction material, which embraces a part of the circumference of the drum. A band brake, as, is called a simple band brake in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance b from the fulcrum. When a force P is applied to the lever at C, the lever turns about the fulcrum pin O and tightens the band on the drum and hence the brakes are applied. The friction between the band and the drum provides the braking force. The force P on the lever at C may be determined as discussed below :



(*a*) Clockwise rotation of drum. (*b*) Anticlockwise rotation of drum.

Simple Band Brake

Let

T_1 = Tension in the tight side of the band,

T_2 = Tension in the slack side of the band,

θ = Angle of lap (or embrace) of the band on the drum,

μ = Coefficient of friction between the band and the drum,

r = Radius of the drum,

t = Thickness of the band, and

re = Effective radius of the drum = r + t /2.

We know that limiting ratio of the tensions is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad 2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

and braking force on the drum = T1 - T2

∴ Braking torque on the drum,

TB =(T1 - T2).r

(.......Neglecting the thickness = (T1- T2).re

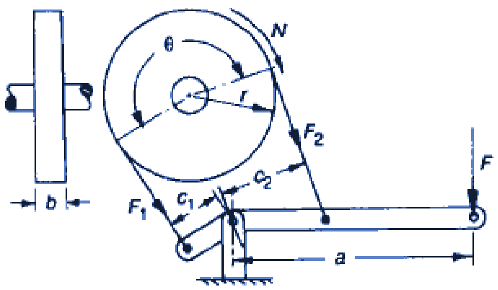
(.......Considering the thickness of the band)

Now considering the equilibrium of the lever OBC. It may be noted that when the drum rotates in the clockwise direction , the end of the band attached to the fulcrum O will be slack with tension T2 and end of the band attached to B will be tight with tension T1. On the other hand, when the drum rotates in the anticlockwise direction, the tensions in the band will reverse, i.e. the end of the band attached to the fulcrum O will be tight with tension T1 and the end of the band attached to B will be slack with tension T2 . Now taking moments about the fulcrum O, we have

$P.l = T_1.b$ ...(for clockwise rotation of the drum)

$P.l = T_2.b$ ...(for anticlockwise rotation of the drum)

DIFERENTIAL BAND BRAKE

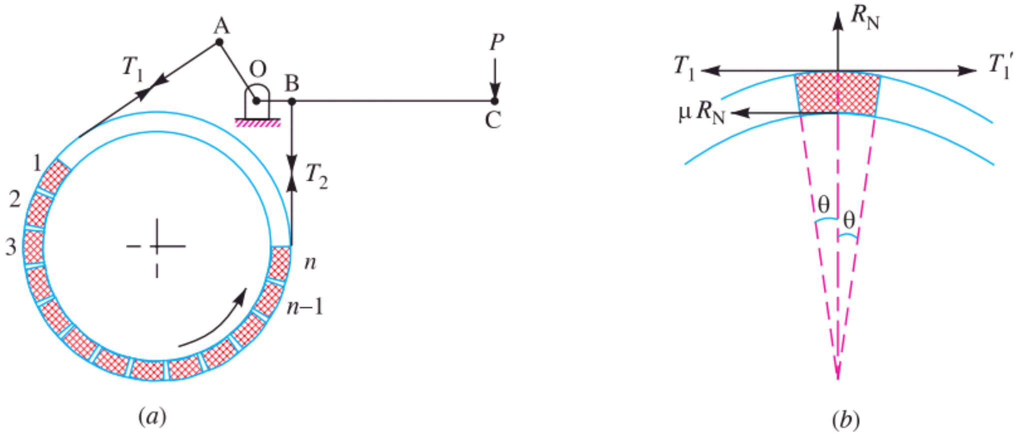


Force required calculate by

$$F = \frac{(F_2 c_2 - F_1 c_1)}{a} = F_2(c_2 - c_1 e^{\mu\theta})$$

BAND AND BLOCK BRAKE

The band brake may be lined with blocks of wood or other material. The friction between the blocks and the drum provides braking action. Let there are 'n' number of blocks, each subtending an angle 2 θ at the centre and the drum rotates in anticlockwise direction.



Band and block brake.

Consider one of the blocks (say first block). This is in equilibrium under the action of the following forces :

1. Tension in the tight side (T1),
2. Tension in the slack side (T1') or tension in the band between the first and second block,

3. Normal reaction of the drum on the block (RN), and
4. The force of friction (μ.RN).

Resolving the forces radially, we have

$$(T_1 + T_1') \sin\theta = R_N$$

Resolving the forces tangentially, we have

$$(T_1 - T_1') \cos\theta = \mu . R_N$$

Dividing above equations

$$\frac{(T_1 - T_1') \cos\theta}{(T_1 + T_1') \sin\theta} = \frac{\mu . R_N}{R_N}$$

$$(T_1 - T_1') = \mu \tan\theta \, (T_1 + T_1')$$

$$\frac{T_1}{T_1'} = \frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}$$

Similarly it can be proved for each of the blocks that

$$\frac{T_1'}{T_2'} = \frac{T_2'}{T_3'} = \frac{T_3'}{T_4'} = \ldots = \frac{T_{n-1}}{T_2} = \frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}$$

$$\frac{T_1}{T_2} = \frac{T_1}{T_1'} \times \frac{T_1'}{T_2'} \times \frac{T_2'}{T_3'} \times \ldots \times \frac{T_{n-1}}{T_2} = \left(\frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}\right)^n$$

Braking torque on the drum of effective radius r_e,

$$T_B = (T_1 - T_2)\, r_e$$
$$= (T_1 - T_2)\, r$$

Dynamometer:

A dynamometer is a brake but in addition it has a device to measure the frictional resistance. Knowing the frictional resistance, we may obtain the torque transmitted and hence the power of the engine.

Types of Dynamometers:

Following are the two types of dynamometers, used for measuring the brake power of an engine. **1.** Absorption dynamometers, and **2.** Transmission dynamometers.

In the **absorption dynamometers,** the entire energy or power produced by the is absorbed by the friction resistances of the brake and is transformed into heat, during the process of measurement. But in the **transmission dynamometers,** the energy is not wasted in friction but is used for doing work. The energy or power produced by the engine is transmitted through the dynamometer to some other machines where the power developed is suitably measured.

Classification of Absorption Dynamometers:

1) Prony Brake Dynamometer

A simplest form of an absorption type dynamometer is a Prony brake dynamometer, as shown in It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to be measured. The blocks are clamped by means of two bolts and nuts, as shown in Fig. A helical spring is provided between the nut and the upper block to adjust the pressure on the pulley to control its speed. The upper block has a long lever attached to it and carries a weight *W* at its outer end. A counter weight is placed at the other end of the lever which balances the brake when unloaded. Two stops S, S are provided to limit the motion of the lever.

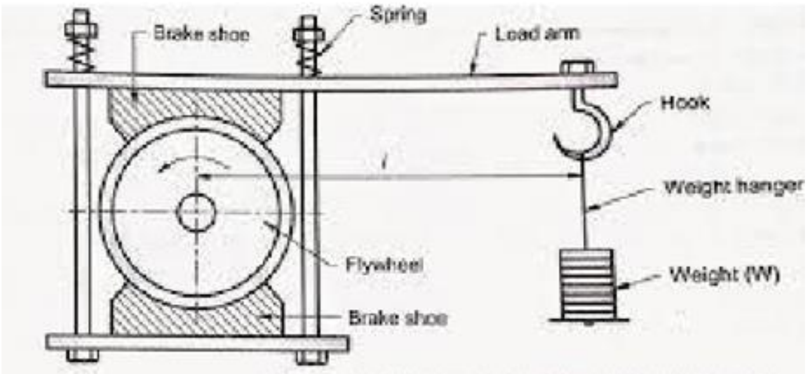
When the brake is to be put in operation, the long end of the lever is loaded with suitableweights *W* and the nuts are tightened until the engine shaft runs at a constant speed and the lever is in horizontal position. Under these conditions, the moment due to the weight *W* must balance the moment of the frictional resistance between the blocks and the pulley.

Frictional torque Wl = Mgl

Power of the machine under test

=MNk

Where k is a constant for a particular brake.



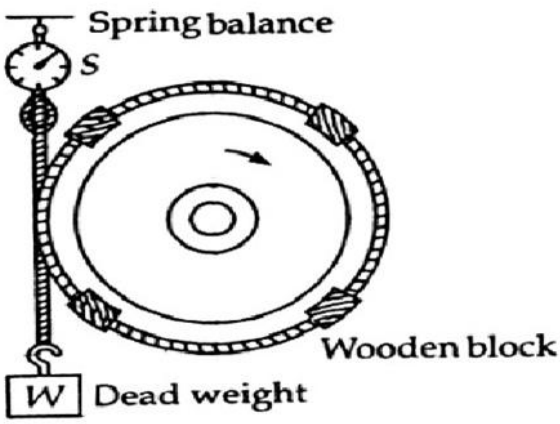
2)Rope Brake Dynamometer

It is another form of absorption type dynamometer which is most commonly used for measuring the brake power of the engine. It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight as shown in Fig. In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel. In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine

Power of the machine = $T\omega$

$= (F \times r)\ \omega$

$= (Mg - s)r$



3)Epicyclic-train Dynamometer

An epicyclic-train dynamometer, as shown in Fig. Consists of a simple epicyclic train of gears, *i.e.* a spur gear, an annular gear (a gear having internal teeth) and a pinion. The spur gear is keyed to the engine shaft (*i.e.* driving shaft) and rotates in anticlockwise direction. The annular gear is also keyed to the driving shaft and rotates in clockwise direction. The pinion or the intermediate gear meshes with both the spur and annular gears. The pinion revolves freely on a lever which is pivoted to the common axis of the driving and driven shafts. A weight w is placed at the smaller end of the lever in order to keep it in position. A little consideration will show that if the friction of the pin on which the pinion rotates is neglected, then the tangential effort P exerted by the spur gear on the pinion and the tangential reaction of the annular gear on the pinion are equal.

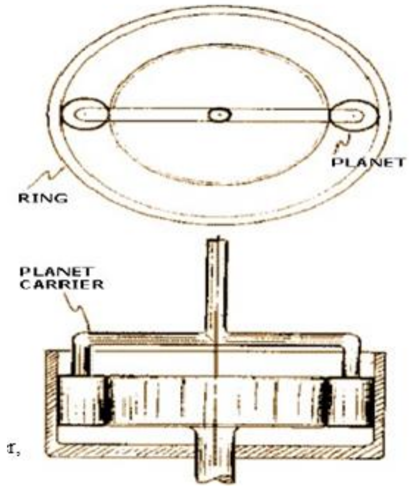
Since these efforts act in the upward direction as shown, therefore total upward force on the lever acting through the axis of the pinion is 2*P*. This force tends to rotate the lever about its fulcrum and it is balanced by a dead weight *W* at the end of the lever. The stops *S*, *S* are provided to control the movement of the lever.

For equilibrium of the lever,

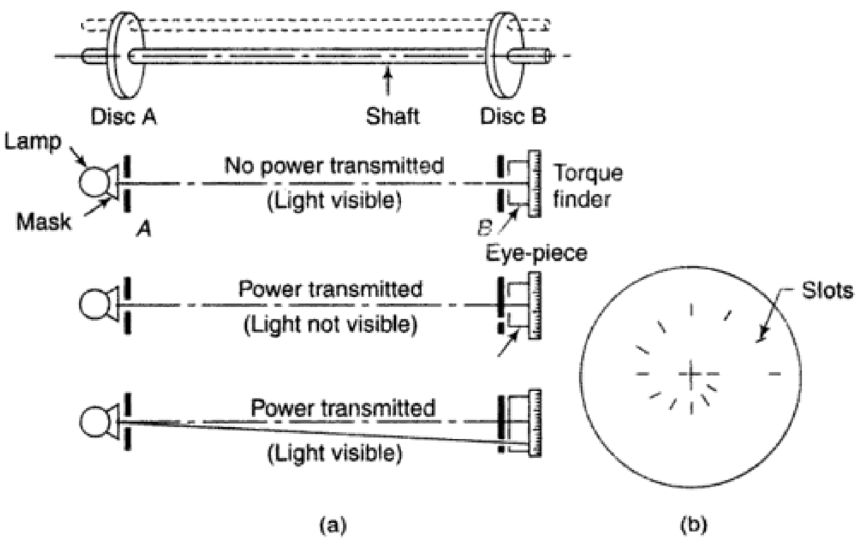
2F.a = Wl or

And torque transmitted= F.r where r is the radius of the driving wheel

Thus power, P = T.ω



4. Bevis-Gibson Flash Light Torsion Dynamometer



It depends upon the fact that the light travels in a straight line through air of uniform density and the velocity of light is infinite. It consists of two discs *A* and *B* fixed on a shaft at a convenient distance apart, Each disc has a small radial slot and these two slots are in the same line when no power is transmitted and there is no torque on the shaft. A bright electric lamp *L*, behind the disc *A*, is fixed on the bearing of the shaft.

This lamp is masked having a slot directly opposite to the slot of disc *A*. At every revolution of the shaft, a flash of light is projected through the slot in the disc *A* towards the disc *B* in a direction parallel to the shaft. An eye piece *E* is fitted behind the disc *B* on the shaft bearing and is capable of slight circumferential adjustment. When the shaft does not transmit any torque (*i.e.* at rest), a flash of light may be seen after every revolution of the shaft, as the positions of the slit do not change relative to one another as shown in Fig. Now when the torque is transmitted, the shaft twists and the slot in the disc *B* changes its position, though the slots in *L*, *A* and *E* are still in line. Due to this, the light does not reach to the eye piece as shown in Fig. If the eye piece is now moved round by an amount equal to the lag of disc *B*, then the slot in the eye piece will be opposite to the slot in disc *B* as shown in Fig. and hence the eye piece receives flash of light. The eye piece is moved by operating a micrometer spindle and by means of scale and vernier, the angle of twist may be measured upto1/100th of a degree. The torsion meter discussed above gives the angle of twist of the shaft, when the uniform torque is transmitted during each revolution as in case of turbine shaft. But when the torque varies during each revolution as in reciprocating engines, it is necessary to measure the angle of twist at several different angular positions. For this, the discs *A* and *B* are perforated with slots arranged in the form of spiral as shown in Fig. The lamp and the eye piece must be moved radially so as to bring them into line with each corresponding pair of slots in the discs.

II B.Tech, II Semester – 2018-19 *Mechanical Engineering*

DYNAMICS OF MACHINES
UNIT – IV
(BALANCING OF ROTATING MASSES)

Learning Material

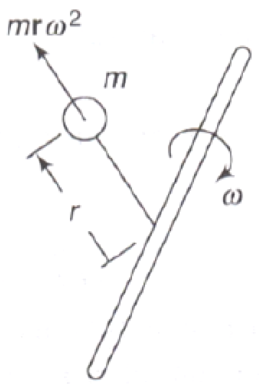
Objective :
To introduce the concepts of balancing of rotating masses
Outcomes :
Students will be able to

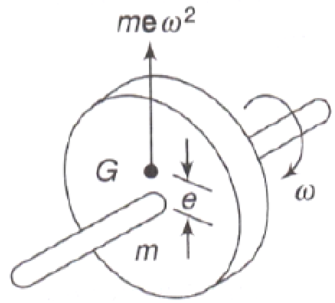
- Determine the unbalanced forces and couples arised because of rotating masses in a system
- Determine the balancing masses magnitudes along with their angular positions to make completely balanced system with rotating masses .

Learning Material

1.1 Introduction

Often an unbalance of forces is produced in rotary or reciprocating machinery due to the inertia forces associated with the moving masses. Balancing is the process of designing or modifying machinery so that the unbalance is reduced to an acceptable level and if possible is eliminated entirely.





Static Balancing:

- A system of rotating masses is said to be in static balance if the combined mass centre of the system lies on the axis of rotation.
- The net dynamic force acting on the shaft is equal to zero

Dynamic Balancing:

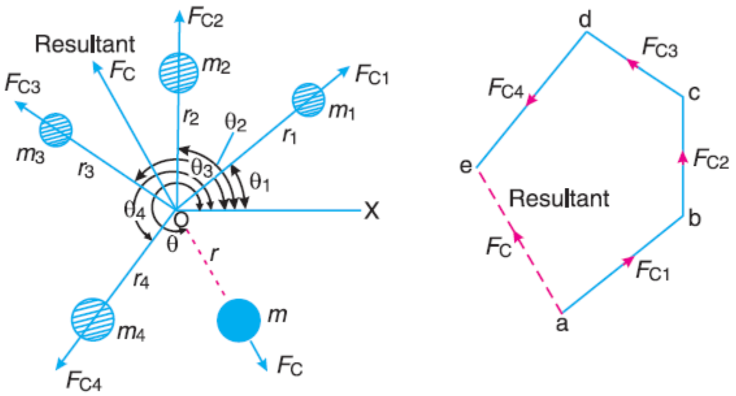
- A system of rotating masses is said to be in dynamic balance if the combined mass centre of the system lies on the axis of rotation and the net couple due to the dynamic forces acting on the shaft is equal to zero.
- The net dynamic force acting on the shaft is equal to zero.
- the algebraic sum of the moments about any point in the plane must be zero.

Balancing of Rotating Masses:

Whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it. In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass. This is done in such a way that the centrifugal forces of both the masses are made to be equal and opposite. The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass is called balancing of rotating masses.

Balancing of Several Masses Rotating in the Same Plane:

Consider any number of masses (say four) of magnitude m_1, m_2, m_3 and m_4 at distances ofr_1, r_2, r_3 and r_4 from the axis of the rotating shaft. Let Θ_1, Θ_2, Θ_3 and Θ_4 be the angles of these masses with the horizontal line OX, Let these masses rotate about an axis through O and perpendicular to the plane of paper, with a constant angular velocity of ω rad/s.



Analytical method:

- Each mass produces a centrifugal force acting radially outwards from the axis of rotation.
- First of all, find out the centrifugal force* (or the product of the mass and its radius of rotation) exerted by each mass on the rotating shaft.
- Resolve the centrifugal forces horizontally and vertically and find their sums

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + \ldots\ldots$$
$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + \ldots\ldots$$

- Magnitude of the resultant centrifugal force

$$F_C = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

- The balancing force is then equal to the resultant force, but in **opposite direction.**
- Now find out the magnitude of the balancing mass, such that

$$F_C = m \cdot r$$

where m = Balancing mass, and
r = Its radius of rotation.

Graphical method:

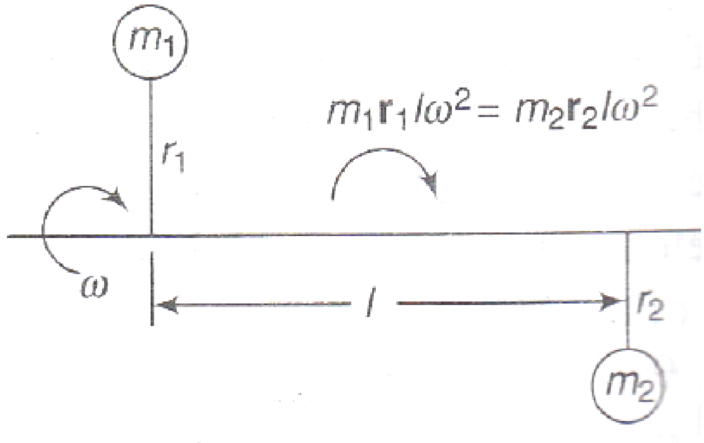
First of all, draw the space diagram with the positions of the several masses Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.

Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that *ab* represents the centrifugal force exerted by the mass m_1 (or $m_1.r_1$) in magnitude and direction to some suitable scale. Similarly, draw *bc, cd* and *de* to represent centrifugal forces of other masses m_2, m_3 and m_4 (or $m_2.r_2$,$m_3.r_3$ and $m_4.r_4$).
Now, as per polygon law of forces, the closing side *ae* represents the resultant force in magnitude and direction,
The balancing force is, then, equal to resultant force, but in opposite direction.

Now find out the magnitude of the balancing mass (m) at a given radius of rotation (r), such that $m.r.\omega^2$ = Resultant centrifugal force
or $m.r$ = Resultant of $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$

Dynamic Balancing:

When several masses rotate in different planes, the centrifugal forces, in addition to being out of balance, also form couples. A system of rotating masses is in dynamic balance when there does not exist any resultant centrifugal force as well as resultant couple.
the products of mr and mrl (instead of $mr\omega^2$ and $mrl\omega^2$),
usually, have been referred as force and couple respectively as it is more convenient to draw force and couple polygons with these quantities.

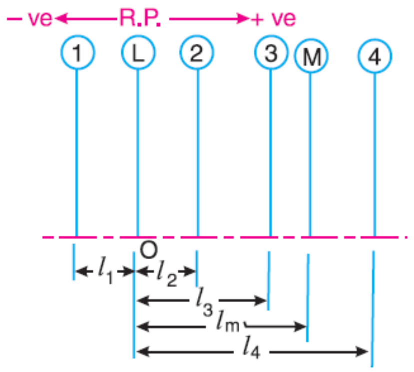


If m_1, and m_2 are two masses revolving diametrically opposite to each other in different planes such that $m_1r_1 = m_2r_2$, the centrifugal forces are balanced, but an unbalanced couple of magnitude m_1r_1l (= m_2r_2l) is introduced. The couple acts in a plane that contains the axis of rotation and the two masses. Thus, the couple is of constant magnitude but variable direction

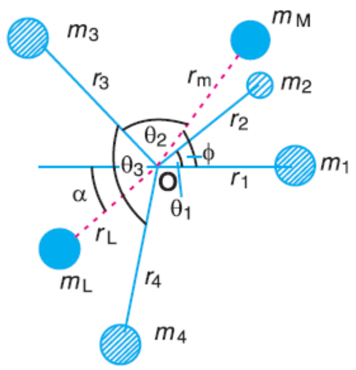
Balancing of Several Masses Rotating in the different Planes:

- Let there be a rotor revolving with a uniform angular velocity ω.
- m_1, $m_2$and m_3 are the masses attached to the rotor at radii r_1, r_2 and $r_3$respectively.
- The masses m_1, m_2 and m_3 rotate in planes1, 2 and 3 respectively.
- Choose a reference plane at *O* so that the distances of the planes 1, 2 and 3 from *O* are l_1, l_2 and l_3 respectively.
- Transference of each unbalanced force to the reference plane introduces the like number of forces and couples.
- The unbalanced forces in the reference plane are $m_1r_1\omega^2$, $m_2r_2\ \omega^2$ and $m_3r_3\ \omega^2$ acting radially outwards.

- The unbalanced couples in the reference plane are $m_1 r_1 l_1 \omega^2$, $m_2 r_2\ l_2 \omega^2$ and $m_3 r_3\ l_3 \omega^2$ which may be represented by vectors parallel to the respective force vectors, i.e., parallel to the respective radii of *m1*, *m2* and *m3*.
- Equate the resultant of unbalanced forces horizontal and vertical components to zero .
- Equate the resultant of unbalanced couples horizontal and vertical components to zero.
- Find the unknown balanced mass magnitude and directions by solving the four equations.



(*a*) Position of planes of the masses.



(*b*) Angular position of the masses.

Plane (1)	*Mass (m)* (2)	*Radius(r)* (3)	*Cent.force* $\div \omega^2$ *(m.r)* (4)	*Distance from Plane L (l)* (5)	*Couple* $\div \omega^2$ *(m.r.l)* (6)
1	m_1	r_1	$m_1.r_1$	$-l_1$	$-m_1.r_1.l_1$
L(R.P.)	m_L	r_L	$m_L.r_L$	0	0
2	m_2	r_2	$m_2.r_2$	l_2	$m_2.r_2.l_2$
3	m_3	r_3	$m_3.r_3$	l_3	$m_3.r_3.l_3$
M	m_M	r_M	$m_M.r_M$	l_M	$m_M.r_M.l_M$
4	m_4	r_4	$m_4.r_4$	l_4	$m_4.r_4.l_4$

II B.Tech, II Semester – 2018-19 *Mechanical Engineering*

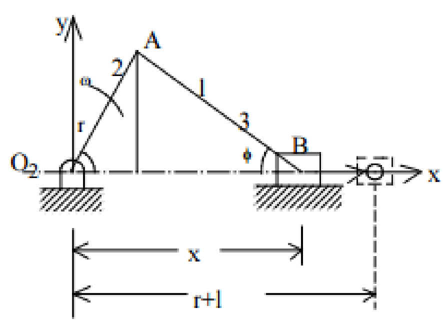
DYNAMICS OF MACHINES
UNIT –V
(BALANCING OF RECIPROCATING MASSES)

Objective :
To introduce the concepts of balancing of reciprocating masses
Outcomes :
Students will be able to

- Determine the balancing mass required for partial balancing of reciprocating masses
- Assess the importance of firing order in balancing of multi cylinder inline engines.
- Apply the concepts of direct and inverse crank approach in balancing of radial engines.

Learning Material

The inertia forces acting cannot be avoided and we have to consider these forces in designing of linkages. (These inertia forces increase stresses in members and loads in bearing). Inertia force $\alpha\omega^2$, where ω is the angular velocity of links. For linkages with very high speed, inertia forces (or loading) results in vibrations, noise emission and sometimes machine failure due to fatigue.



The displacement of the piston is given as

$$x = r\cos\theta + l\left[1 - \frac{\sin^2\theta}{n^2}\right]^{1/2} \quad \text{with} \quad n = l/r \qquad (1)$$

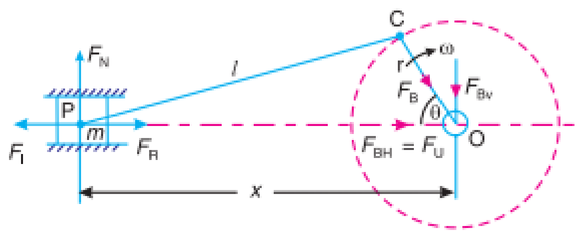
The velocity of the piston can be obtained as

$$\dot{x} = -\omega r\left[\sin\theta + \left(\frac{1}{2n}\right)\frac{\sin 2\theta}{\left[1 - n^{-2}\sin^2\theta\right]^{0.5}}\right] \qquad (2)$$

The acceleration of the piston can be obtained as

$$\ddot{x} = -\omega^2 r\left[\cos\theta + \frac{(n^2-1)\cos 2\theta + \cos^4\theta}{n^3\left[1-n^{-2}\sin^2\theta\right]^{1.5}}\right] \quad (3)$$

Primary and Secondary Unbalanced Forces of Reciprocating Masses



Consider a reciprocating engine mechanism as shown in Fig.

Let m = Mass of the reciprocating parts,

l = Length of the connecting rod PC,

r = Radius of the crank OC,

θ = Angle of inclination of the crank with the line of stroke PO,

ω = Angular speed of the crank,

n = Ratio of length of the connecting rod to the crank radius = l / r.

the acceleration of the reciprocating parts is approximately given by the expression

$$a_R = \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

. ∴ Inertia force due to reciprocating parts or force required to accelerate the reciprocating parts,

$$F_I = F_R = \text{Mass} \times \text{acceleration} = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

The horizontal component of the force exerted on the crank shaft bearing (i.e. FBH) is equal and opposite to inertia force (FI). This force is an unbalanced one and is denoted by FU.

∴ Unbalanced force,

$$F_U = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) = m.\omega^2 \cdot r\cos\theta + m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n} = F_P + F_S$$

primary unbalanced force= $(m \cdot \omega^2 \cdot r \cos\theta)$

secondary unbalanced force = $\left(m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n}\right)$

Partial Balancing of Unbalanced Primary Force in a Reciprocating Engine

∴ Unbalanced force along the line of stroke

$$= m \cdot \omega^2 \cdot r \cos\theta - B \cdot \omega^2 \cdot b \cos\theta$$
$$= m \cdot \omega^2 \cdot r \cos\theta - c \cdot m \cdot \omega^2 \cdot r \cos\theta$$
$$= (1-c) m \cdot \omega^2 \cdot r \cos\theta$$

∴ Resultant unbalanced force at any instant

$$= \sqrt{\left[(1-c) m \cdot \omega^2 \cdot r \cos\theta\right]^2 + \left[c \cdot m \cdot \omega^2 \cdot r \sin\theta\right]^2}$$
$$= m \cdot \omega^2 \cdot r \sqrt{(1-c)^2 \cos^2\theta + c^2 \sin^2\theta}$$

Balancing of Primary Forces of Multi-cylinder In-line Engines

The multi-cylinder engines with the cylinder centre lines in the same plane and on the same side of the centre line of the crankshaft, are known as In-line engines. The following two conditions must be satisfied in order to give the primary balance of the reciprocating parts of a multi-cylinder engine :

1. The algebraic sum of the primary forces must be equal to zero. In other words, the primary force polygon must close ; and

2. The algebraic sum of the couples about any point in the plane of the primary forces must be equal to zero. In other words, the primary couple polygon must close.

We have already discussed, that the primary unbalanced force due to the reciprocating masses is equal to the component, parallel to the line of stroke, of the centrifugal force produced by the equal mass placed at the crankpin and revolving with it. Therefore, in order to give the primary balance of the reciprocating parts of a multi-cylinder engine, it is convenient to imagine the reciprocating masses to be transferred to their respective crankpins and to treat the problem as one of revolving masses.

Balancing of Secondary Forces of Multi-cylinder In-line Engines

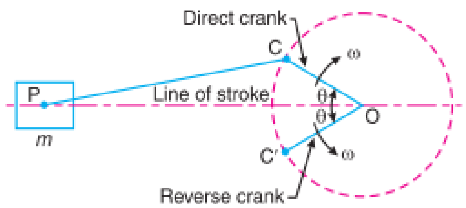
When the connecting rod is not too long (i.e. when the obliquity of the connecting rod is considered), then the secondary disturbing force due to the reciprocating mass arises.

$$F_S = m.\omega^2.r \times \frac{\cos 2\theta}{n}$$

$$F_S = m.(2\omega)^2 \times \frac{r}{4n} \times \cos 2\theta$$

Balancing of Radial Engines(Direct and Reverse Cranks Method)

The method of direct and reverse cranks is used in balancing of radial or V-engines, in which the connecting rods are connected to a common crank. Since the plane of rotation of the various cranks (in radial or V-engines) is same, therefore there is no unbalanced primary or secondary couple.



Consider a reciprocating engine mechanism as shown in Fig. Let the crank OC (known as the direct crank) rotates uniformly at ω radians per second in a clockwise direction. Let at any instant the crank makes an angle θ with the line of stroke OP. The indirect or reverse crank OC′ is the image of the direct crank OC, when seen through the mirror placed at the line of stroke. A little consideration will show that when the direct crank revolves in a clockwise direction, the reverse crank will revolve in the anticlockwise direction. We shall now discuss the primary and secondary forces due to the mass (m) of the reciprocating parts at P.

Considering the primary forces

We have already discussed that primary force is 2 m rω^2cos θ . This force is equal to the component of the centrifugal force along the line of stroke, produced by a mass (m) placed at the crank pin C. Now let us suppose that the mass (m) of the reciprocating parts is divided into two parts, each equal to m / 2.

We know that the centrifugal force acting on the primary direct and reverse crank

$$= \frac{m}{2} \times \omega^2.r$$

∴ Component of the centrifugal force acting on the primary direct crank

$$= \frac{m}{2} \times \omega^2.r\cos\theta \qquad \text{. . . (in the direction from } O \text{ to } P\text{)}$$

and, the component of the centrifugal force acting on the primary reverse crank

$$= \frac{m}{2} \times \omega^2 . r \cos\theta \qquad \text{. . . (in the direction from } O \text{ to } P)$$

∴ Total component of the centrifugal force along the line of stroke

$$= 2 \times \frac{m}{2} \times \omega^2 . r \cos\theta = m.\omega^2 . r \cos\theta = \text{Primary force, } F_P$$

Hence, for primary effects the mass m of the reciprocating parts at P may be replaced by two masses at C and C′ each of magnitude m/2.

Considering secondary forces

We know that the secondary force

$$= m(2\omega)^2 \frac{r}{4n} \times \cos 2\theta = m.\omega^2 r. \times \frac{\cos 2\theta}{n}$$

Balancing of V-engines

Consider a symmetrical two cylinder V-engine as shown in Fig, The common crank OC is driven by two connecting rods PC and QC. The lines of stroke OP and OQ are inclined to the vertical OY, at an angle α as shown in Fig .

Let m = Mass of reciprocating parts per cylinder,

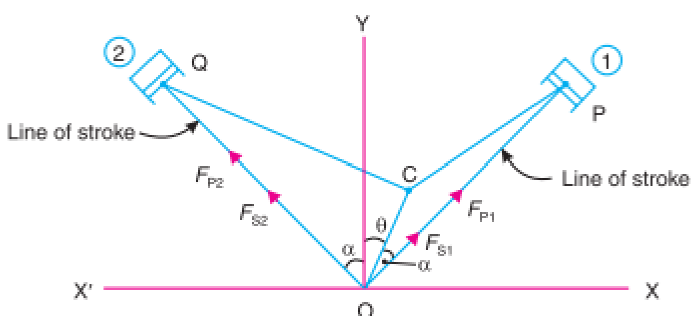
l = Length of connecting rod,

r = Radius of crank,

n = Ratio of length of connecting rod to crank radius = l / r

θ = Inclination of crank to the vertical at any instant,

ω = Angular velocity of crank.



We know that inertia force due to reciprocating parts of cylinder 1, along the line of stroke

$$= m.\omega^2.r\left[\cos(\alpha-\theta)+\frac{\cos 2(\alpha-\theta)}{n}\right]$$

and the inertia force due to reciprocating parts of cylinder 2, along the line of stroke

$$= m.\omega^2.r\left[\cos(\alpha-\theta)+\frac{\cos 2(\alpha+\theta)}{n}\right]$$

Considering primary forces

$\therefore$ Resultant primary force,

$$F_P = \sqrt{(F_{PV})^2+(F_{PH})^2}$$

$$= 2m.\omega^2.r\sqrt{(\cos^2\alpha.\cos\theta)^2+(\sin^2\alpha.\sin\theta)^2}$$

Considering secondary forces

$\therefore$ Resultant secondary force,

$$F_S = \sqrt{(F_{SV})^2+(F_{SH})^2}$$

$$= \frac{2m}{n}\times\omega^2.r\sqrt{(\cos\alpha.\cos 2\alpha.\cos 2\theta)^2+(\sin\alpha.\sin 2\alpha.\sin 2\theta)^2}$$

DYNAMICS OF MACHINES
UNIT 6 - VIBRATIONS

Types of Vibratory Motion

The following types of vibratory motion are important from the subject point of view :

1. Free or natural vibrations. When no external force acts on the body, after giving it an initial displacement, then the body is said to be under **free or natural vibrations.** The frequency ofthe free vibrations is called **free or natural frequency.**

2. Forced vibrations. When the body vibrates under the influence of external force, then the body is said to be under **forced vibrations.** The external force applied to the body is a periodic disturbing force created by unbalance. The vibrations have the same frequency as the applied force.

Note : When the frequency of the external force is same as that of the natural vibrations, resonance takes
place.

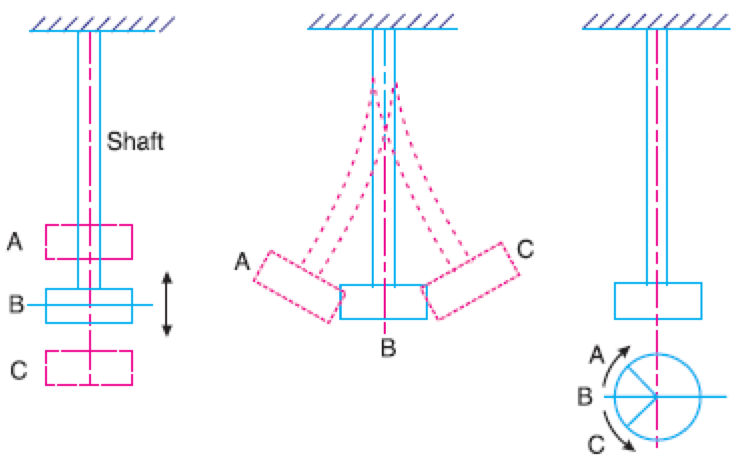
3. Damped vibrations. When there is a reduction in amplitude over every cycle of vibration, the motion is said to be **damped vibration.** This is due to the fact that a certain amount of energy possessed by the vibrating system is always dissipated in overcoming frictional resistances to the motion.

Types of Free Vibrations

The following three types of free vibrations are important from the subject point of view :

1. Longitudinal vibrations, **2.** Transverse vibrations, and **3.** Torsional vibrations.

Consider a weightless constraint (spring or shaft) whose one end is fixed and the other end carrying a heavy disc, as shown in Fig.. This system may execute one of the three above mentioned types of vibrations.



B = Mean position ; *A* and *C* = Extreme positions.

(*a*) Longitudinal vibrations. (*b*) Transverse vibrations. (*c*) Torsional vibrations.

1. Longitudinal vibrations. When the particles of the shaft or disc moves parallel to the axis of the shaft, as shown in Fig, then the vibrations are known as **longitudinal vibrations.** In this case, the shaft is elongated and shortened alternately and thus the tensile and compressive stresses are induced alternately in the shaft.

2. Transverse vibrations. When the particles of the shaft or disc move approximately perpendicular to the axis of the shaft, as shown in Fig., then the vibrations are known as **transverse vibrations.** In this case, the shaft is straight and bent alternately and bending stresses are induced in the shaft.

3. Torsional vibrations*. When the particles of the shaft or disc move in a circle about the axis of the shaft, as shown in Fig., then the vibrations are known as **torsional vibrations.** In this case, the shaft is twisted and untwisted alternately and the torsional shear stresses are induced in the shaft.

Natural Frequency of Free Longitudinal Vibrations:

a) ENERGY METHOD

In this method, the maximum kinetic energy at the mean position is equal to the maximum potential energy (or strain energy) at the extreme position. Assuming the motion executed by the vibration to be simple harmonic, then

$$x = X \sin \omega t$$

Since at the mean position, t = 0, therefore maximum velocity at the mean position,.

$$v = \frac{dx}{dt} = \omega . X$$

Maximum kinetic energy at mean position

$$\frac{1}{2} \times m.v^2 = \frac{1}{2} \times m.\omega^2 .X^2$$

and maximum potential energy at the extreme position

$$\left(\frac{0 + s.X}{2} \right) X = \frac{1}{2} \times s.X^2$$

Equating equations

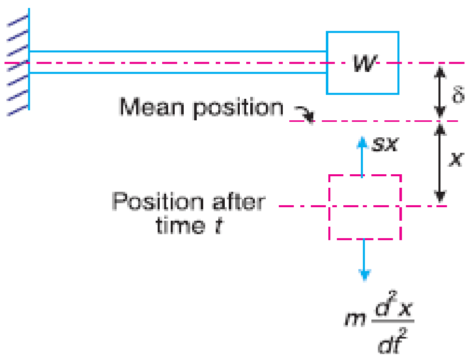
$$\omega^2 = \frac{s}{m} \ ,$$

natural frequency

$$f_n = \frac{1}{t_p} = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{s}{m}}$$

Natural Frequency of Free Transverse Vibrations

Consider a shaft of negligible mass, whose oneend is fixed and the other end carries a body of weight
W, as shown



Restoring force = $-s.x$

accelerating force =

$$m \times \frac{d^2x}{dt^2}$$

Equating equations

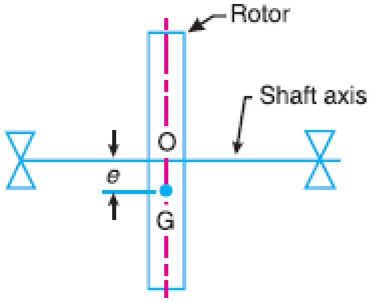
$$m \times \frac{d^2x}{dt^2} = -s.x$$

natural frequency =

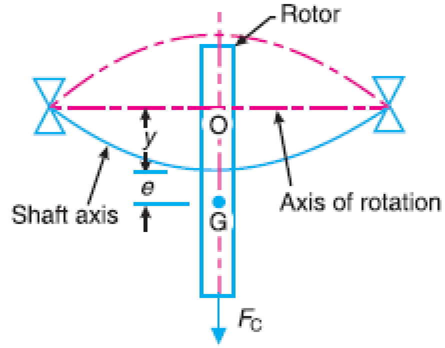
$$f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

Critical or Whirling Speed of a Shaft

A rotating shaft carries different mountings and accessories in the form of gears, pulleys, etc. When the gears or pulleys are put on the shaft, the centre of gravity of the pulley or gear does not coincide with the centre line of the bearings or with the axis of the shaft, when the shaft is tationary. This means that the centre of gravity of the pulley or gear is at a certain distance from the axis of rotation and due to this, the shaft is subjected to centrifugal force. This force will bent the shaft which will further increase the distance of centre of gravity of the pulley or gear from the axis of rotation. This correspondingly increases the value of centrifugal force, which further increases the distance of centre of gravity from the axis of rotation. This effect is cumulative and ultimately the shaft fails. The bending of shaft not only depends upon the value of eccentricity (distance between centre of gravity of the pulley and the axis of rotation) but also depends upon the speed at which the shaft rotates.



(a) When shaft is stationary.



(b) When shaft is rotating.

centrifugal force acting radially outwards =

$$F_C = m.\omega^2 (y + e)$$

resisiting force to deflection is

$$= s.y$$

For the equilibrium position

$$= \quad m.\omega^2.y + m.\omega^2.e = s.y$$

$$y = \frac{m.\omega^2.e}{s - m.\omega^2} = \frac{\omega^2.e}{s/m - \omega^2}$$

$$y = \pm\frac{\omega^2 e}{(\omega_n)^2 - \omega^2} = \frac{\pm e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1} = \frac{\pm e}{\left(\frac{\omega_c}{\omega}\right)^2 - 1}$$

from the above expression that when $\omega_n = \omega_c$ the value of y becomes infinite.Therefore ω_c is the critical speed or whirling speed.

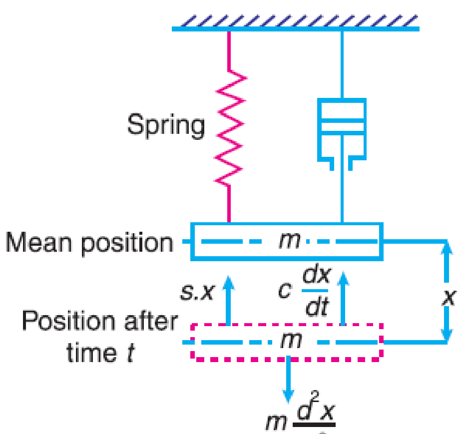
N_c is the critical or whirling speed in r.p.s., then

$$N_c = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{0.4985}{\sqrt{\delta}} \text{ r.p.s.}$$

Hence the critical or whirling speed is the same as the natural frequency of transverse vibration but its unit will be revolutions per second**.**

Damped Vibrations

We have already discussed that the motion of a body is resisted by frictional forces. In vibrating systems, the effect of friction is referred to as damping. The damping provided by fluid resistance is known as viscous damping. We have also discussed that in damped vibrations, the amplitude of the resulting vibration gradually diminishes. This is due to the reason that a certain amount of energy is always dissipated to overcome the frictional resistance. The resistance to the motion of the body is provided partly by the medium in which the vibration takes place and partly by the internal friction, and in some cases partly by a dash pot or other external damping device. Consider a vibrating system, as shown in Fig in which a mass is suspended from one end of the spiral spring and the other end of which is fixed. A damper is provided between the mass and the rigid support.



Damping force or frictional force on the mass acting in **opposite** direction to the motion of the mass

$$c \times \frac{dx}{dt}$$

where C= damping coefficient

Accelerating force on the mass, acting **along** the motion of the mass

$$m \times \frac{d^2x}{dt^2}$$

and spring force on the mass, acting in **opposite** direction to the motion of the mass

$$s.x$$

Therefore the equation of motion becomes

$$m \times \frac{d^2x}{dt^2} = -\left(c \times \frac{dx}{dt} + s.x\right)$$

The most general solution of the differential equation

$$x = C_1 e^{k_1 t} + C_2\, e^{k_2 t}$$

When the roots are real (overdamping)

$$\text{If } \left(\frac{c}{2m}\right)^2 > \frac{s}{m}$$

$$x = C_1 e^{k_1 t} + C_2 e^{k_2 t}$$

When the roots are complex conjugate (underdamping

$$\text{If } \frac{s}{m} > \left(\frac{c}{2m}\right)^2$$

$$x = Ae^{-at} \cos \omega_d .t$$

When the roots are equal (critical damping

$$\text{If } \left(\frac{c}{2m}\right)^2 = \frac{s}{m}$$

$$x = (C_1 + C_2)\, e^{-\frac{c}{2m}t}$$

The critical damping coefficient (cc) may be obtained by substituting cc for c in the condition for critical damping,

$$\left(\frac{c_c}{2m}\right)^2 = \frac{s}{m} \qquad \text{or} \qquad c_c = 2m\sqrt{\frac{s}{m}} = 2m \times \omega_n$$

Damping Factor or Damping Ratio

The ratio of the actual damping coefficient (c) to the critical damping coefficient (cc) is known as damping factor or damping ratio. Mathematically,

$$\frac{c}{c_c} = \frac{c}{2m.\omega_n}$$

Logarithmic Decrement

It is defined as the natural logarithm of the amplitude reduction factor. The amplitude reduction factor is the ratio of any two successive amplitudes on the same side of the mean position Logarithmic decrement

$$\delta = \log_e\left(\frac{x_n}{x_{n+1}}\right) = a.t_p = \frac{2\pi \times c}{\sqrt{(c_c)^2 - c^2}}$$

Forced Vibrations

$$F_x = F\cos\omega.t$$

Maximum displacement or the amplitude of forced vibration,

$$x_{max} = \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}}$$

When damping is negligible, then $c = 0$.

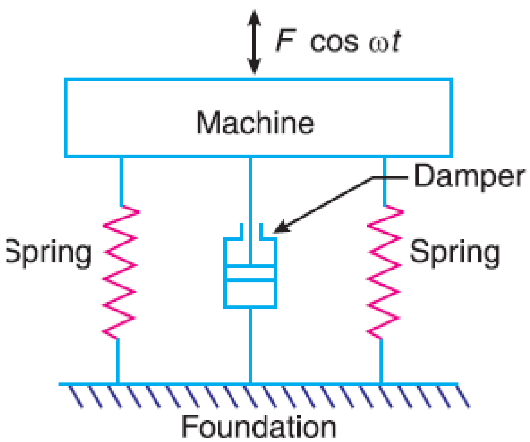
$$\frac{F}{m\left[(\omega_n)^2 - \omega^2\right]}$$

At resonance $\omega = \omega_n$.

$$x_{max} = x_o \times \frac{s}{c.\omega_n}$$

Vibration Isolation and Transmissibility

A little consideration will show that when an unbalanced machine is installed on the foundation, it produces vibration in the foundation. In order to prevent these vibrations or to minimize the transmission of forces to the foundation, the machines are mounted on springs and dampers or on some vibration isolating material, as shown in Fig. The arrangement is assumed to have one degree of freedom, *i.e.* it can move up and down only It may be noted that when a periodic (*i.e.* simple harmonic) disturbing force *F* cos □*t* is applied to a machine



The ratio of the force transmitted (FT) to the force applied (F) is known as the **isolation factor** or **transmissibility ratio** of the spring support.

$$F_T = \sqrt{(s.x_{max})^2 + (c.\omega.x_{max})^2}$$

transmissbility

$$\varepsilon = \frac{F_T}{F} = \frac{x_{max}\sqrt{s^2 + c^2.\omega^2}}{F}$$

When the damper is not provided

$$\varepsilon = \frac{1}{1-(\omega/\omega_n)^2}$$

When $\omega/\omega_n = \sqrt{2}$

then

$\varepsilon = 1$

When $\omega/\omega_n < \sqrt{2}$,

then

$\varepsilon > 1$

When $\omega/\omega_n > \sqrt{2}$

then

$\varepsilon < 1$

.