GUDLAVALLERU ENGINEERING COLLEGE
(An Autonomous Institute with Permanent Affiliation to JNTUK, Kakinada)
Seshadri Rao Knowledge Village, Gudlavalleru – 521 356.

Department of Mechanical Engineering

HANDOUT

on

ELEMENTS OF ELECTRICAL AND ELECTRONICS ENGINEERING

Learning Material

Unit –I

ELECTRICAL CIRCUITS

Objectives

- To introduce the basic concepts of electrical circuits.
- To familiarize the students with the operation of Basic elements.

Syllabus

Basic definitions, Types of elements, Ohm's Law, Resistive networks, Kirchhoff's Laws, Inductive networks, capacitive networks, Series, Parallel circuits and Star-delta and delta-star transformations.

At the end of the chapter student is able to

- Demonstrate the knowledge and understanding of the fundamental principles of electrical engineering.
- Identify the basic elements in given circuit
- Apply the basic principles to solve a given network.

UNIT-1

ELECTRICAL CIRCUITS

- The valance electrons which are loosely attached to the nucleus of an atom are called free electrons.
- The flow of free electrons is called as electric current.
- Time rate of change of charge is called as electric current.

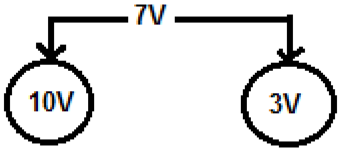
$$\mathrm{i} = \frac{\mathrm{dQ}}{\mathrm{dt}}\ \mathrm{Coulomb}/_{\mathrm{sec}}\ (\mathrm{or})\mathrm{Ampere}$$

If one coulomb charge flows through
one section in one second is called as one Ampere current.

- Voltage is the energy required to move a unit charge through an element.

$$\mathrm{V} = \frac{\mathrm{dW}}{\mathrm{dQ}} \qquad \mathrm{Joule}/_{\mathrm{Coulomb}}\ (\mathrm{or})\mathrm{Volts}$$

- The difference in the potential of two charged bodies is called as potential difference.
  Units: Volt



- Total work done in electric circuit is called as energy (E).
  Units: Joules
- Rate of transfer of energy is called as power (P).

$$\mathrm{P} = \frac{\mathrm{dW}}{\mathrm{dt}}$$

$$\mathrm{P} = \frac{\mathrm{dW}}{\mathrm{dQ}} * \frac{\mathrm{dQ}}{\mathrm{dt}}$$

$$\mathrm{P} = \mathrm{V} * \mathrm{I}$$

"The rate at which work is done in electric circuit is called as power".

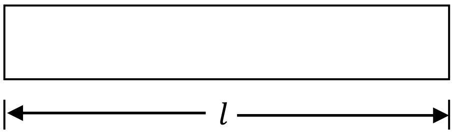
Resistance: (R)

It is a property of a material, which opposes the flow of electric current.

Units: ohm's Ω

Let *'l'* be the length of the material

A be the cross sectional area of material.



Resistance is directly proportional to length of the material,

$$R \propto l \qquad (1.1)$$

As the area of cross section increases, electron can move freely.

∴Resistance is inversely proportional to the area of cross section.

$$\mathrm{R} \propto \frac{1}{\mathrm{A}} \qquad (1.2)$$

From (1) & (2)

$$\mathrm{R} \propto \frac{\mathrm{l}}{\mathrm{A}}$$

$$\mathrm{R} = \frac{\rho \mathrm{l}}{\mathrm{A}}$$

$$\rho = \text{Resistivity (or)Specific Resistance}$$

$$\rho = \frac{\mathrm{RA}}{\mathrm{l}} = \frac{\Omega * \mathrm{m}^2}{\mathrm{m}} = \Omega - \mathrm{m}$$

Reciprocal of resistance is called as conductance. It is denoted by ‘G’

$$\mathrm{G} = \frac{1}{\mathrm{R}}$$

Units: Mho’s (℧)

Open circuit:

- Open circuit is an element where resistance tends to infinity.
- Current doesn’t flow through open circuit.

Short circuit:

- Short circuit is an element when resistance approaches to zero.
- Potential difference in short circuit is zero.

Ohm’s Law:

“Under constant temperature and pressure, current flowing through a conductor is directly proportional to the voltage applied across it”.

$\mathrm{i} \propto \mathrm{V}$

$$\mathrm{i} = \frac{\mathrm{V}}{\mathrm{R}}$$

Where, R=Resistance of conductor

$$\text{Power dissipated by Resistor (P)} = \mathrm{V} * \mathrm{i}$$

$$= \frac{V^2}{R}(\text{or})i^2R$$

$$\text{If V is positive, then } P = \frac{(+V)^2}{R} = \frac{V^2}{R}$$

$$= (+i)^2R = i^2R \qquad (1.3)$$

$$\text{If V is Negative, then } P = (-i)^2R = i^2R$$

$$= \frac{(-V)^2}{R} = \frac{V^2}{R} \qquad (1.4)$$

Conclusion:

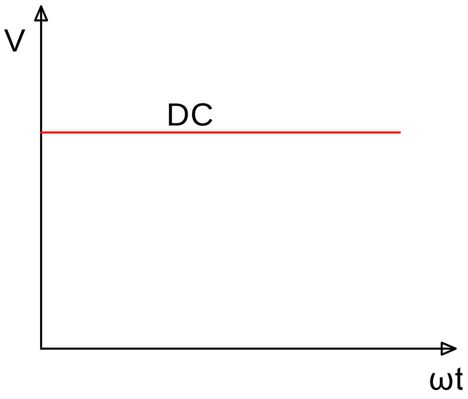
From (1.3) & (1.4) power dissipated by resistor remains same.

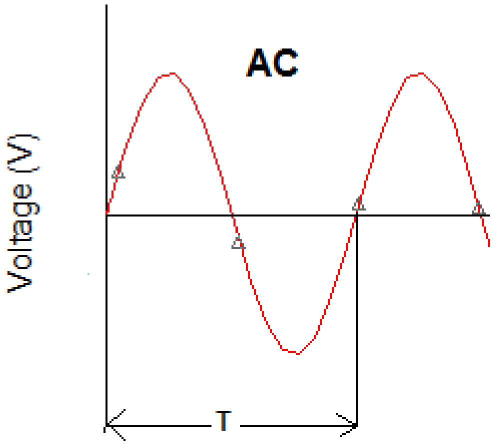
It is independent of direction of applied voltage (or) current.

Type of supplies:

Depends on the nature of the wave form power supplies are classified as

1. Alternating current (AC)
2. Direct current (DC)





DC is a current that remains constant with time.
AC is a current that varies sinusoidally with time.
The minimum time after which the cycle of signal repeats is called as time period (T).

	D.C	A.C
Representation	V = K	V = ASinωt Where ω = 2πf
Time period	∞	T
Frequency	0	$\frac{1}{T}$

- **Faraday's Laws:**
- **First law:**

Whenever conductor experiences the rate of change of flux, emf will be induced in that conductor and if there is a closed path, current will flow in that circuit.

- **Second Law:**

The induced emf (e) is proportional to rate of change of flux.

$$e \propto \frac{d\emptyset}{dt} \rightarrow \text{for one turn}$$

If N turns are there, then

$$e \propto N\frac{d\emptyset}{dt}$$

$$e = -N\frac{d\emptyset}{dt}$$

$$e = -\frac{d(N\emptyset)}{dt}$$

$$e = -\frac{d(\psi)}{dt}$$

Here ψ is flux linkage, where $\psi = N\emptyset$

Here –ve sign indicates that induced emf opposes the current in that conductor which is given by Lenz's Law.

Lenz's Law:

The effect opposes the cause.

Inductance:

"The property of coil that opposes any change in the amount of current flowing through it is called as Inductance".

Flux linkage depends on the amount of current flowing through the coil.

$$\therefore \psi \propto i$$

$$\psi = Li \qquad [L=\text{Inductance of coil}]$$

According to Faraday's Law

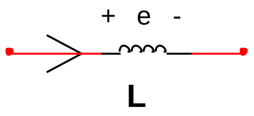
$$e = \frac{d(\psi)}{dt} = \frac{d}{dt}(Li)$$

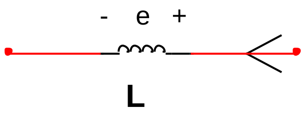
$$e = L\frac{di}{dt}$$

$$\boxed{e = L\frac{di}{dt}}$$

According to Lenz's Law, induced emf should oppose the change in current flow through that coil.

The direction of induced voltage is given by,





Energy stored in the inductance (E) $= \int P\,dt$

$$= \int P\,dt$$

$$= \int vi\,dt$$

$$= \int L\frac{di}{dt}i\,dt$$

$$= \frac{L}{2}\int (2i)\frac{di}{dt}\,dt$$

$$= \frac{L}{2}\int (2i)di$$

$$E = \frac{1}{2}Li^2$$

$$\boxed{E = \frac{1}{2}Li^2}$$

Properties of inductor:

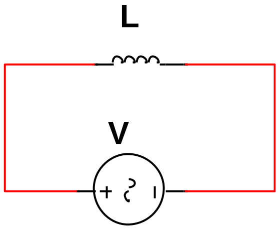
1. Since it does not allow the sudden change in current through it, it is called as current stiff element.
2. It stores the energy in the form of magnetic field.
3. If the applied voltage is positive, it will start charging and if the applied voltage is negative, it will start discharging.

Inductive Reactance (X_L):

Let us consider voltage $V = V_m \sin\omega t$ is applied across the inductor

$$V = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int V.\,dt$$



$$i = \frac{1}{L}\int V_m \sin\omega t\, dt$$

$$i = \frac{V_m}{\omega L}(-\cos\omega t)$$

$$i = \frac{V_m}{\omega L}\left(\sin\left(\omega t - \frac{\pi}{2}\right)\right) \qquad (1.5)$$

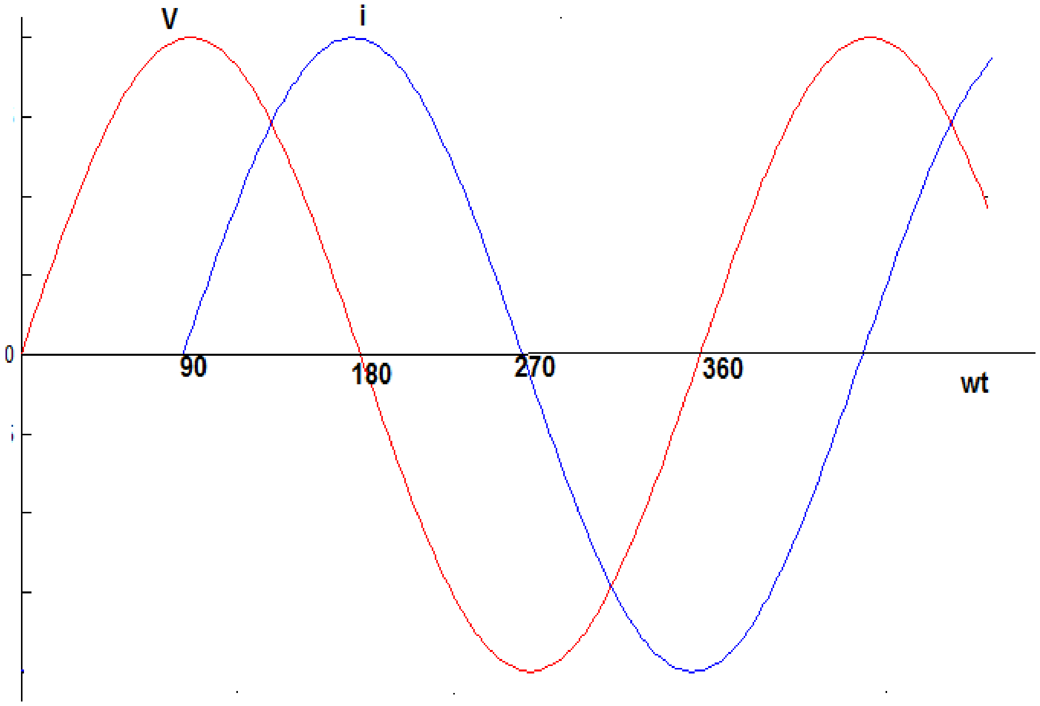
$$i = \frac{V_m}{\omega L}(\sin \omega t)(-j)$$

$$i = \frac{V_m \sin \omega t}{j\omega L}$$

$$i = \frac{V}{j\omega L}$$

$$\boxed{i = \frac{V}{j\omega L}}$$

Here the Inductive Reactance, $X_L = j\omega L$



From equation (1.5), current in pure inductor lags voltage by 90 (or) $\frac{\pi}{2}$ *radians*

Capacitor:

Any two conducting surfaces separated by an insulating material (dielectric) is called as capacitor.

Capacitance:

The ability of a capacitor to start charge is known as its capacitance.

Charge stored in capacitor is proportional to applied voltage.

$$\therefore Q \propto V$$

$$Q = CV$$

$$\text{We know that, } i = \frac{dQ}{dt} = \frac{d}{dt}(CV)$$

$$i = C\frac{dV}{dt}$$

Energy stored in capacitor:

Let us consider 'V' voltage is applied across capacitor. At this instant, 'W' joules of work will be done in transferring 1C of charge from one plate to other

If small charge dq is transferred, then work done is

$$dW = Vdq$$

$$W = \int_0^V CVdq$$

$$W = \frac{1}{2}CV^2$$

$$W = \frac{1}{2}C\left[\frac{q}{C}\right]^2$$

$$W = \frac{1}{2}\frac{q^2}{C}$$

Reactance offered by capacitor (X_C):

Let us consider ($V = V_m \sin \omega t$) V volts applied across capacitor.

$$i = C\frac{dV}{dt}$$

$$i = C\frac{d(V_m \sin \omega t)}{dt}$$

$$i = CV_m\frac{d(\sin \omega t)}{dt}$$

$$i = CV_m\omega \cos\omega t$$

$$i = V_m\omega C \, j \sin\omega t$$

$$i = \frac{V_m \sin\omega t}{\frac{1}{j\omega C}}$$

$$\boxed{X_C = \frac{1}{j\omega C}}$$

Where, $\omega = 2\pi f$

Properties of capacitor:

- It doesn't allow the sudden change in voltage. It is called as voltage stiff element.
- It stores energy in the form of electrostatic field.

For D.C, frequency (f) = 0. i.e $\omega = 0$

$$X_L = j\omega L = j(2\pi f)L = j(2\pi * 0)L = 0$$

For D.C supply opposition offered by inductor is Zero.

i.e. it (Inductor) acts as short circuit.

$$X_C = \frac{1}{j\omega C} = \frac{1}{j(0)C} = \infty$$

For D.C, opposition offered by capacitor is Infinity.

i.e. it (capacitor) acts as open circuit.

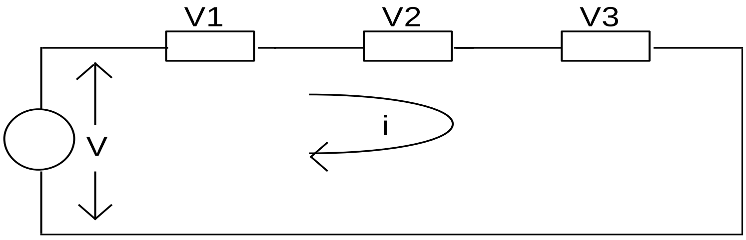
Kirchhoff's voltage Law: (KVL)

This law is related to emf's and voltage drops in a circuit. It stated as "in an electrical circuit, algebraic sum of all the voltages in a closed path is Zero".

$$-V + V_1 + V_2 + V_3 = 0$$

$$V = V_1 + V_2 + V_3$$

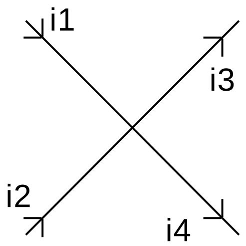
$$\boxed{V = V_1 + V_2 + V_3}$$



- KVL is independent of nature of element.

Kirchhoff's current Law:

This law is related to current at the junction points a circuit. It is stated as "In a circuit, at node at any instant the algebraic sum of current flowing towards a junction in circuit is Zero".



$$i_1 + i_2 - i_3 - i_4 = 0$$

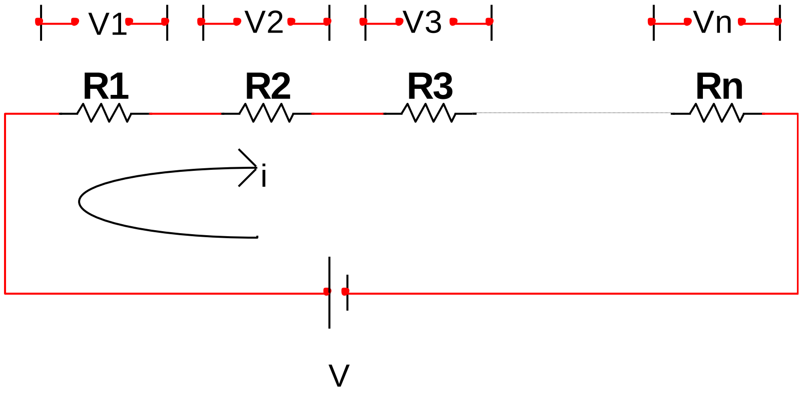
$$\frac{dQ_1}{dt} + \frac{dQ_2}{dt} - \frac{dQ_3}{dt} - \frac{dQ_4}{dt} = 0$$

$$Q_1 + Q_2 - Q_3 - Q_4 = 0$$

- According to law of conservation of energy, the net charge at node is Zero.
- KCL is independent of nature of element.

Series R-circuit:

Let us consider ‘n’ Resistors are connected in series.



Apply KVL

$$-V + V_1 + V_2 + V_3 + \cdots\cdots\cdots + V_n = 0$$

$$-iR_{eq} + iR_1 + iR_2 + iR_3 + \cdots\cdots\cdots + iR_n = 0$$

$$R_{eq} = R_1 + R_2 + R_3 + \cdots\cdots\cdots + R_n$$

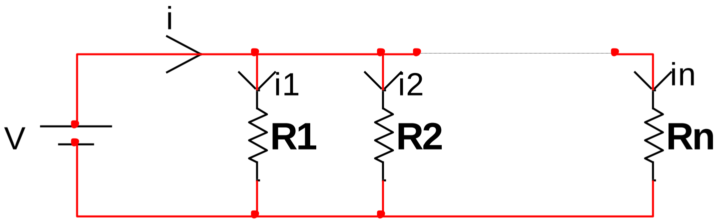
Note: If ‘n’ Resistors are in series, then equivalent Resistance will be greater than $R_1, R_2, R_3 \ldots R_n$.

Parallel circuit:

Apply KCL

$$-i + i_1 + i_2 + i_3 + \cdots\cdots + i_n = 0$$

$$-\frac{V}{R_{eq}} + \frac{V}{R_1} + \frac{V}{R_2} + \cdots\cdots + \frac{V}{R_n} = 0$$



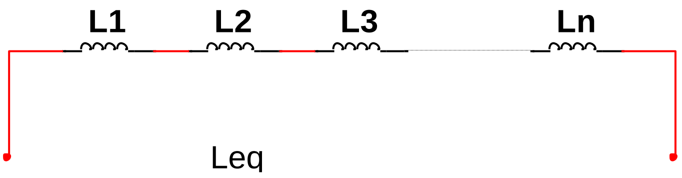
- When ‘n’ Resistances are in parallel, equivalent Resistance is smaller than all Resistances.

NOTE:

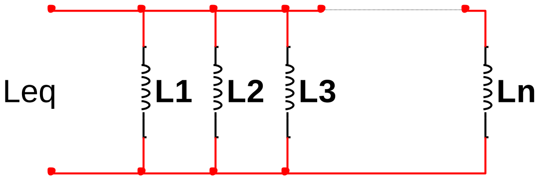
- When ‘n’ Resistances are in series, the current through all the Resistors are same.
- When ‘n’ Resistors are in parallel, then voltage across all resistors is same.

Inductive circuits:

$$L_{eq} = L_1 + L_2 + \cdots\cdots + L_n$$



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots\cdots + \frac{1}{L_n}$$

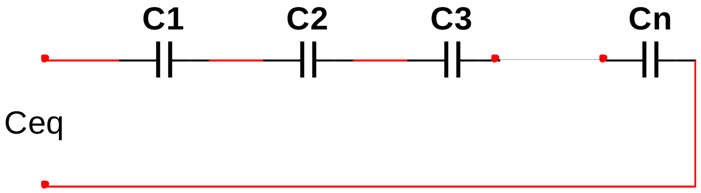


Capacitive circuits:

Series circuit:

Apply KVL

$$V = V_1 + V_2 + V_3 + \cdots\cdots + V_n$$



$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \cdots\cdots + \frac{Q}{C_n}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots\cdots + \frac{1}{C_n}$$

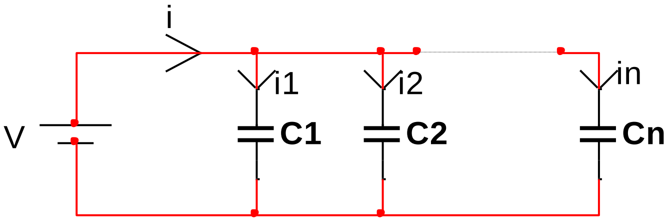
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots\cdots + \frac{1}{C_n}}$$

Parallel circuit:

Apply KCL

$$-i + i_1 + i_2 + i_3 + \cdots\cdots + i_n = 0$$

$$-Q + Q_1 + Q_2 + \cdots\cdots + Q_n = 0$$



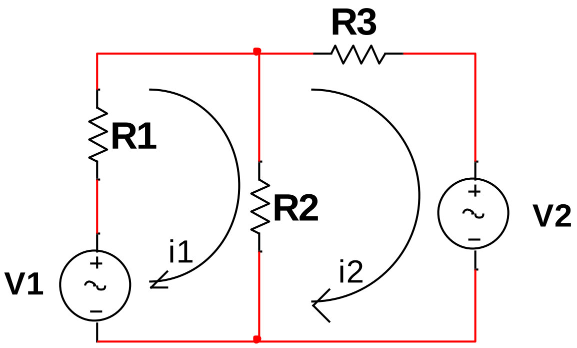
$$-C_{eq}V + C_1V + C_2V + \cdots\cdots + C_nV = 0$$

$$-C_{eq} + C_1 + C_2 + \cdots\cdots + C_n = 0$$

$$\boxed{C_{eq} = C_1 + C_2 + \cdots\cdots + C_n}$$

Mesh analysis (KVL + ohm's Law)

- Identify the Number of Loops/ Meshes.
- Assign the currents in each loop.



- Apply KVL for each mesh and write ohm's law form.
- Solve the equations and obtain mesh currents.

Apply KVL for loop (1)

$$-V_1 + i_1R_1 + (i_1 - i_2)R_2 = 0$$

For loop (2)

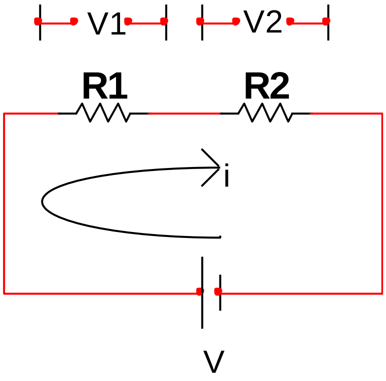
$$V_2 + i_2R_3 + (i_2 - i_1)R_2 = 0$$

Nodal Analysis (KCL + ohm's Law)

- Identify the Number of nodes when current is dividing and assign voltage to nodes.
- Write KCL equation at each node and accept as reference node.
- Write ohm's law form for current in nodal equation & solve the equation.

Voltage division Rule:

It is applicable for series circuit.



$$i = \frac{V}{R_1 + R_2}$$

$$V_1 = iR_1 = \left(\frac{V}{R_1 + R_2}\right)R_1$$

$$V_2 = iR_2 = \left(\frac{V}{R_1 + R_2}\right)R_2$$

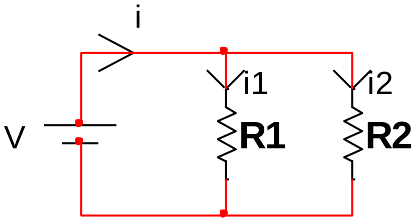
i.e When 'n' Resistors R_1, R_2, R_3… R_n are in series and $V_1, V_2, V_3, \cdots\cdots V_n$ are voltage drops across resistors, then

$$V_1 = \left(\frac{V}{R_1 + R_2 + \cdots\cdots + R_n}\right)R_1$$

$$V_n = \left(\frac{V}{R_1 + R_2 + \cdots\cdots + R_n}\right)R_n$$

$$\boxed{V_n = \left(\frac{V}{R_1 + R_2 + \cdots\cdots + R_n}\right)R_n}$$

Current division Rule:



$$i = i_1 + i_2 + i_3 + \cdots\cdots + i_n$$

$$R_{eq} = \frac{V}{i} = \frac{R_1R_2}{R_1 + R_2}$$

$$i = \frac{V}{R_{eq}} = \frac{V(R_1 + R_2)}{R_1 R_2}$$

$$i_2 = \frac{V}{R_2} = \frac{V(R_1 + R_2)R_1}{R_1 R_2 (R_1 + R_2)}$$

$$i_2 = \frac{V(R_1 + R_2)}{R_1 R_2} * \frac{R_1}{R_1 + R_2}$$

$$i_2 = \frac{i * R_1}{R_1 + R_2}$$

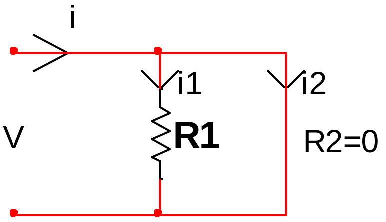
$$i_1 = \frac{i * R_2}{R_1 + R_2}$$

$$\boxed{i_2 = \frac{i * R_1}{R_1 + R_2} \quad \& \quad i_1 = \frac{i * R_2}{R_1 + R_2}}$$

Case (i):

$$i_1 = \frac{i * R_2}{R_1 + R_2} = 0$$

$$i_2 = \frac{i * R_1}{R_1 + R_2} = i$$

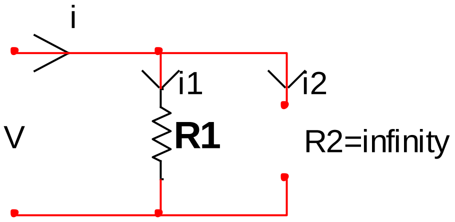


Observation: Current always choose lower Resistance path.

Case (ii)

$$i_1 = \frac{i * R_2}{R_1 + R_2} = i$$

$$i_2 = \frac{i * R_1}{R_1 + R_2} = 0$$



Note: Current will not flow through open circuit.

Classification of elements:

1. **Active & passive:**

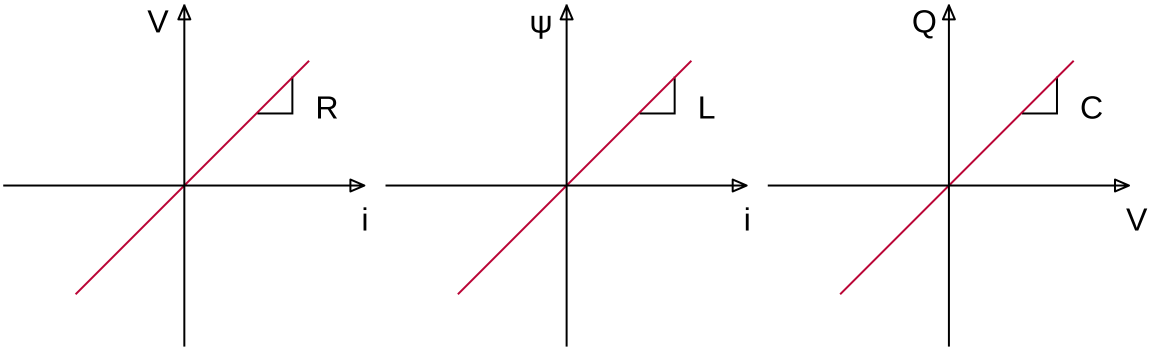
An element is said to be active, if it is able to deliver the energy to outside world for infinite time, otherwise passive. Example for active elements are sources and passive are R, L, C.

Note:

1. Ohm’s Law is not applicable for active elements.
2. If V/I ratio is positive, then it is called as passive element
3. passive elements cannot supply more energy than what it had drawn previously.

2. Linear & Non-linear elements:

If the characteristic of an element is a straight line passing through the origin, it is called as linear element and these characteristics are constant.



Examples:

- Linear elements are R, L, and C.
- Non-Linear elements are Diode, Transistor.

3. Unilateral &Bilateral elements:

If an elements offers same impedance (opposition) for both the directions of flow of current through it is called as bilateral element otherwise it is unilateral element.

Examples:

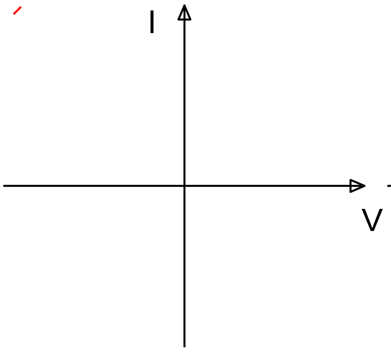
- Bilateral elements are R, L, C.
- Unilateral element is Diode, transistor.

For forward voltage Diode acts as short circuit. i.e. R=0. In reverse Bias it acts as open circuit i.e. R is infinity. So here Diode offers different resistance for different excitation. Therefore, it is called as unilateral element.

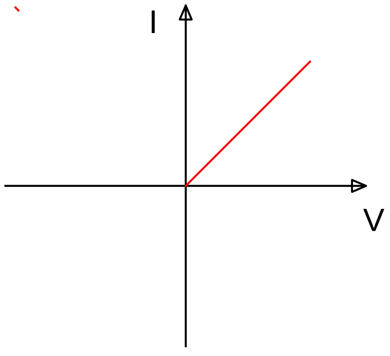
If V/I characteristics are same in all direction, it is called as Bilateral element.

4. Time variant / invariant:

If the element characteristics are independent of time, it is called as time variant, otherwise time variant.



(i)



(ii)

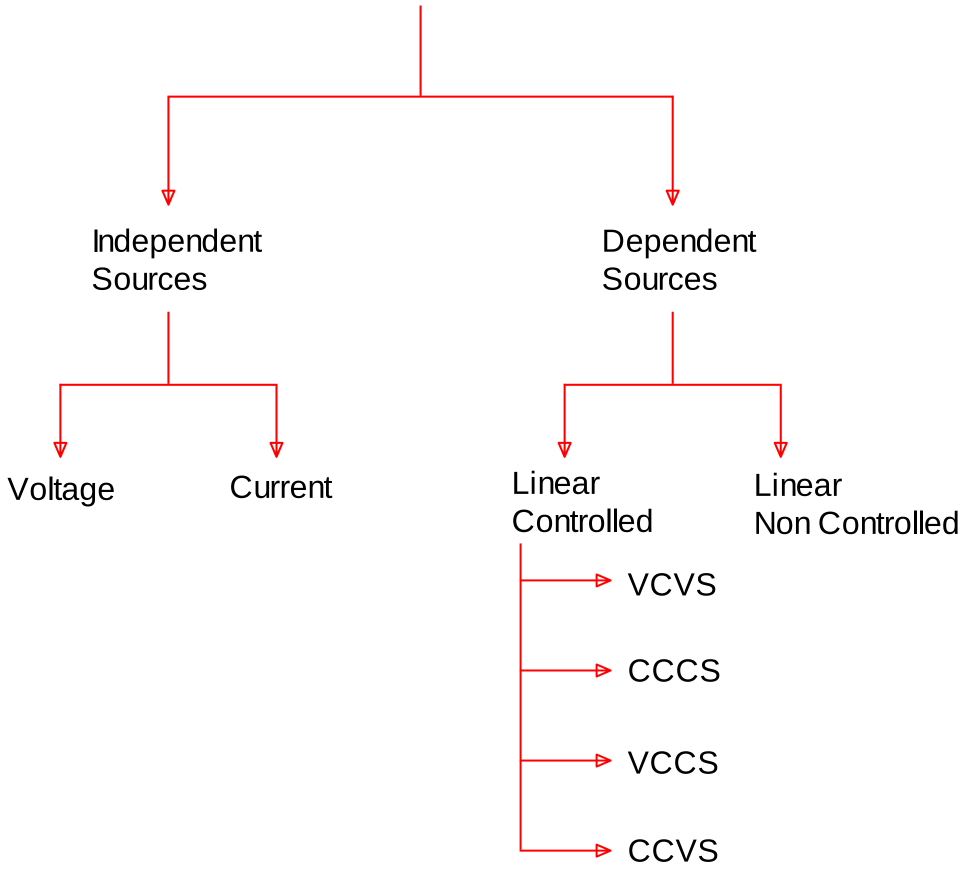
Case (i)

$\frac{V}{I}$ is Positive. ∴ It is passive element, bilateral element, linear element.

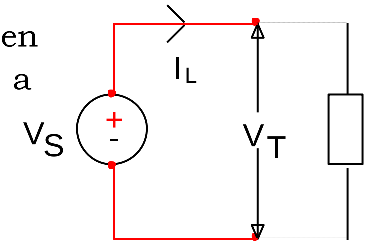
Case (ii)

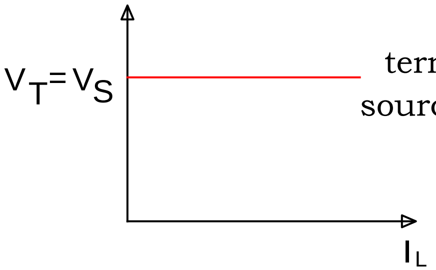
$\frac{V}{I}$ is Positive in one Quadrant and $\frac{V}{I}$ is Negative in other direction. ∴ $\frac{V}{I}$ ratio is not same in both directions. ∴ It is active element, unilateral element, non-linear element.

Classification of Sources



➢ Ideal Voltage Source

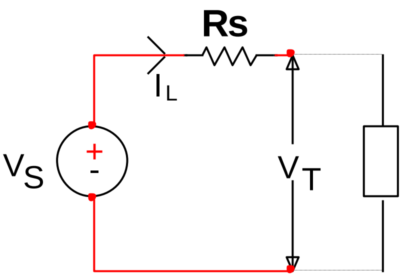


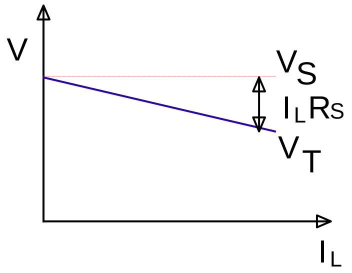


When the terminal voltage of a source is independent of load element, it is called as independent ideal voltage source.

Practical voltage source

It has internal resistance(R_S). Whenever load current increases, the drop across R_S will increase. Therefore, terminal voltage will reduce as load current rises.

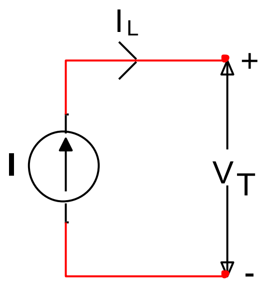


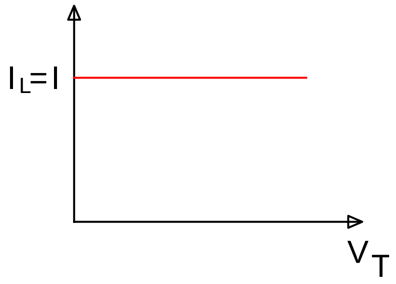


$$V_T = V_S - I_S R_S$$

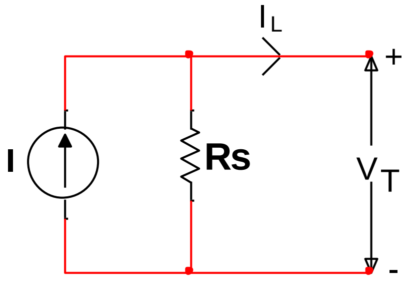
Ideal Current Source

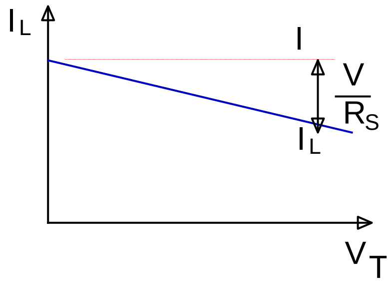
It is a two terminal device which delivers constant current to the network connected across its terminals. i.e. current supplied by the source is independent of its terminal voltage.





Practical Current Source

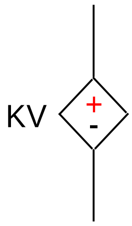




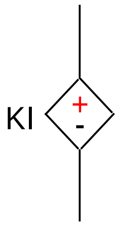
- **Dependent (or) Controlled Source**

  A controlled voltage/ current source is one whose terminal voltage or current is a function of some other voltage or current. These devices have two pairs of terminals. One pair corresponds to the controlling quantity & other pair represents controlled quantity.

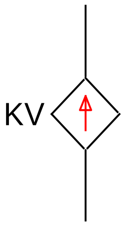
  Controlled quantity is directly proportional to controlling quantity.



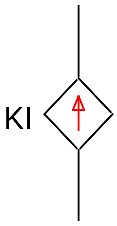
Voltage controlled
Voltage source



Current controlled
Voltage source



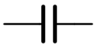
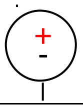
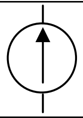
Voltage controlled
Current source



Current controlled
Current source

$$\text{Here}, K = \text{Constant}$$

Basic Symbols

	Symbol/Notations	**Units**
Resistor		
Resistance	Ω	Ohm’s
Inductor		
Inductance	H	Henry
Capacitor		
Capacitance	F	Farad
Voltage Source		
Voltage	V	Volts
Current Source		

Current	A	Amperes

Superposition theorem:

The superposition theorem can be used to find the solution to networks with two or more sources that are not in series or parallel. The most obvious advantage of this method is that it does not require the use of a mathematical technique such as determinants to find the required voltages or currents. Instead, each source is treated independently, and the algebraic sum is found to determine a particular unknown quantity of the network. The superposition theorem states the following:

The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.

- The superposition theorem extends the use of Ohm's Law to circuits with multiple sources.
- In order to apply the superposition theorem to a network, certain conditions must be met:
  - All the components must be linear, meaning that the current is proportional to the applied voltage.
  - All the components must be bilateral, meaning that the current is the same amount for opposite polarities of the source voltage.
  - Passive components may be used.
  - Active components may not be used.
- To consider the effects of each source independently requires that sources be removed and replaced without affecting the final result.
- To remove a voltage source when applying this theorem, the difference in potential between the terminals of the voltage source must be set to zero(short circuit); removing a current source requires that its terminals be opened (open circuit).
- Any internal resistance or conductance associated with the displaced sources is not eliminated but must still be considered
- Figure 1 reviews the various substitutions required when removing an ideal source, and Figure .2 reviews the substitutions with practical sources that have an internal resistance.

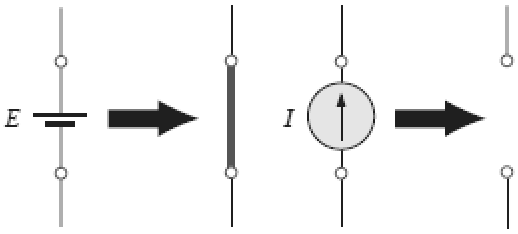


Figure: 1 Removing the effects of ideal sources

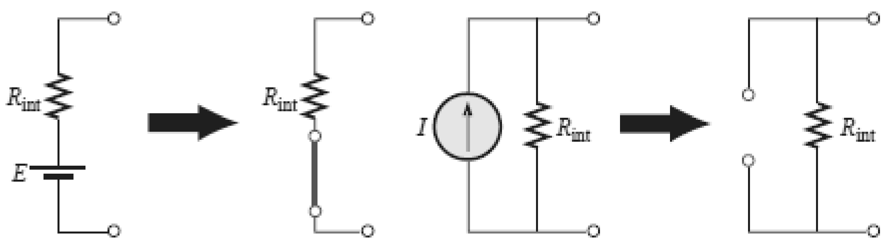


Figure : 2 removing the effects of practical sources

- The total current through any portion of the network is equal to the algebraic sum of the currents produced independently by each source.
- That is, for a two-source network, if the current produced by one source is in one direction, while that produced by the other is in the opposite

direction through the same resistor, the resulting current is the difference of the two and has the direction of the larger. If the individual currents are in the same direction, the resulting current is the sum of two in the direction of either current.

Figure 3:demonstration of the fact that superposition is not applicable to power effects

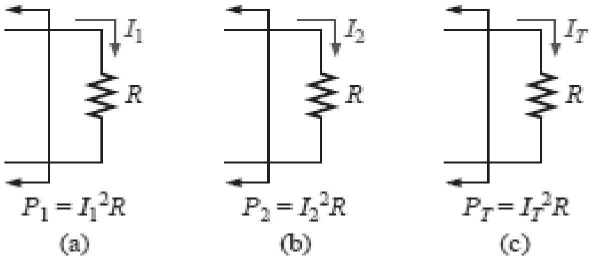


Fig-3

- This rule holds true for the voltage across a portion of a network as determined by polarities, and it can be extended to networks with any number of sources.
- The superposition principle is not applicable to power effects since the power loss in a resistor varies as the square (nonlinear) of the current or voltage. For instance, the current through the resistor R of Fig. 3(a) is I_1 due to one source of a two-source network.
- The current through the same resistor due to the other source is I_2 as shown in Fig. 3(b). Applying the superposition theorem, the total current through the resistor due to both sources is I_T, as shown in Fig. 3(c)with

$$I_T = I_1+I_2$$

The power delivered to the resistor in Fig. 5.3(a) is

$$P_1 = I_1^2R$$

while the power delivered to the same resistor in Fig. 5.3(b) is

$$P_2 = I_2^2R$$

If we assume that the total power delivered in Fig. 5.3(c) can be obtained by simply adding the power delivered due to each source, we find that

$$P_T = P_1 + P_2 = I_1^2R + I_2^2R$$

$$P^2_T = I_1^2 + I_2^2$$

- This final relationship between current levels is incorrect, however, as can be demonstrated by taking the total current determined by the superposition theorem and squaring it as follows:

$$I_T^2 = (I_1+I_2)^2 = I_1^2 + I_2^2 + 2I_1I_2$$

which is certainly different from the expression obtained from the addition of power levels.

In general, therefore, the total power delivered to a resistive element must be determined using the total current through or the total voltage across the element and cannot be determined by a simple sum of the power levels established by each source.

THEVENIN’S THEOREM

Thevenin’s theorem simplifies the process of solving for the unknown values of voltage and current in a network by reducing the network to an equivalent series circuit connected to any pair of network terminals.

Thévenin’s theorem states the following:
Any two-terminal, linear bilateral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series impedance, as shown in Fig.4

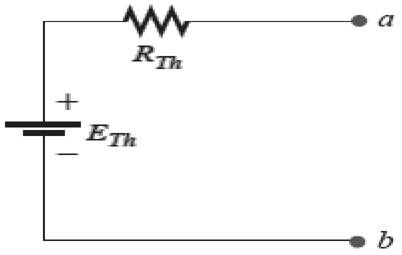


Fig-4 Thévenin equivalent circuit

In a given circuit except load impedance remaining circuit is to be replaced by a single voltage source in series with impedance.

The following sequence of steps will lead to the proper value of Z_{Th} and V_{Th}.

- Remove that portion of the network across which the Thévenin’s equivalent circuit is to be found i.e, the load impedance Z_L is be temporarily removed from the network.
- Mark the terminals of the remaining two-terminal network.
- Calculate Z_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant impedance between the two marked terminals. (If the internal impedance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero).

- Calculate V_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals.

Conclusion:

- Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the impedance Z_L between the terminals of the Thévenin equivalent circuit.

MAXIMUM POWER TRANSFER THEOREM

- The maximum power transfer theorem states the following:

A load will receive maximum power from a linear bilateral dc network when its total resistive value is exactly equal to the Thévenin's resistance of the network as "seen" by the load.

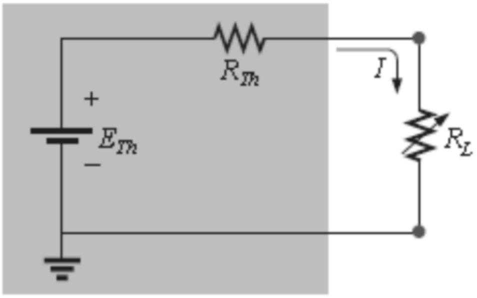


Fig-7

Defining the conditions for maximum power to a load using the Thévenin equivalent circuit.

For the network of Fig. 7, maximum power will be delivered to the load whenR_L=R_{Th}

For the network of Fig 7,

$$I = \frac{E_{Th}}{R_{Th} + R_L}$$

And

$$P_L = I^2 R_L = \left(\frac{E_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

So that

$$P_L = \frac{E_{Th}^2 R_L}{(R_{Th} + R_L)^2}$$

$$I = \frac{E_{Th}}{R_{Th} + R_L} = \frac{E_{Th}}{2R_{Th}}$$

$$P_L = I^2 R_L = \left(\frac{E_{Th}}{2R_{Th}}\right)^2 R_{Th} = \frac{E_{Th}^2 R_{Th}}{4R_{Th}^2}$$

and

$$P_{Lmax} = \frac{E_{Th}^2}{4R_{Th}^2} (watts, \mathrm{W})$$

GUDLAVALLERU ENGINEERING COLLEGE

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Department of Electrical and Electronics Engineering

HANDOUT

on

ELEMENTS OF ELECTRICAL ENGINEERING

UNIT – II

TRANSFORMERS

Objectives:

1. To familiarize the students with the constructional details and working principle of single phase transformers.
2. To familiarize the students with phase diagram and equivalent circuit of single phase transformer.
3. To familiarize the students with the Predetermination of regulation and efficiency of single phase transformer.

Syllabus:

Principle of operation of single phase transformer- Types - Constructional features - Emf equation of Transformer- Equivalent circuit of Single-Phase transformer- Losses & Efficiency- Regulation of Transformer.

Learning Outcomes:

After the completion of this unit, students will be to

1. Explain the various types of a single phase transformer.
2. Draw the equivalent circuit and phasor diagram of single phase transformer.
3. Explain the procedure to conduct OC and SC tests on single phase transformer.
4. Predetermine of efficiency and regulation of single phase transformer.

Learning Material

2.1 Introduction

The transformer is probably one of the most useful electrical devices ever invented. It can change the magnitude of alternating voltage or current from one value to another. This useful property of transformer is mainly responsible for the widespread use of alternating currents rather than direct currents i.e., electric power is generated, transmitted and distributed in the form of alternating current. Transformers have no moving parts, rugged and durable in construction, thus requiring very little attention. They also have a very high efficiency—as high as 99%.

A transformer is a static piece of equipment used either for raising or lowering the voltage of an a.c. supply with a corresponding decrease or increase in current. It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in Fig. (1). The winding connected to the a.c. source is called primary winding (or primary) and the one connected to load is called secondary winding (or secondary). The alternating voltage V_1 whose magnitude is to be changed is applied to the primary. Depending upon the number of turns of the primary (N_1) and secondary (N_2), an alternating e.m.f. E_2 is induced in the secondary. This induced e.m.f. E_2 in the secondary causes a secondary current I_2. Consequently, terminal voltage V_2 will appear across the load. If $V_2 > V_1$, it is called a step up-transformer. On the other hand, if $V_2 < V_1$, it is called a step-down transformer.

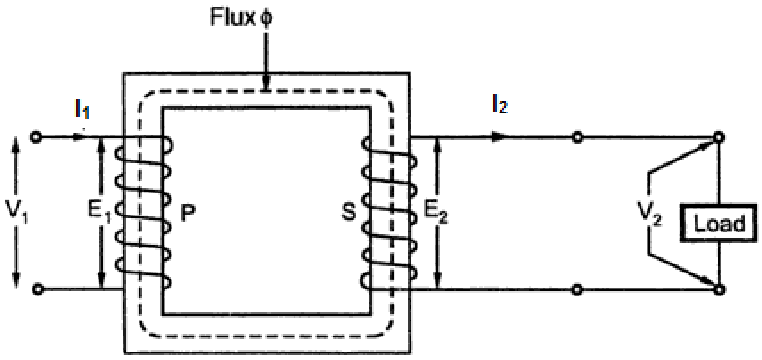


Figure 1

2.2 Working Principle of a Transformer

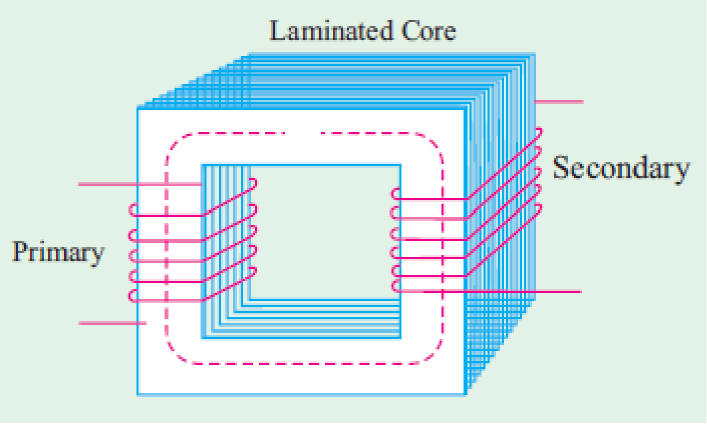


Figure 2

A transformer is a static (or stationary) piece of apparatus by means of which electric power in one circuit is transformed into electric power of the same frequency in another circuit. It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. The physical basis of a transformer is **mutual induction** between two circuits linked by a common magnetic flux. In its simplest form, it consists of two inductive coils which are electrically separated but magnetically linked through a path of low reluctance as shown in Fig. 2. The two coils possess high mutual inductance. If one coil is connected to a source of alternating voltage, an alternating flux is set up in the laminated core, most of which is linked with the other coil in which it produces mutually-induced e.m.f. (according to Faraday's Laws of Electromagnetic Induction **e = MdI/dt**). If the second coil circuit is closed, a current flows in it and so electric energy is transferred (entirely magnetically) from the first coil to the second coil. The first coil, in which electric energy is fed from the a.c. supply mains, is called **primary** winding and the other from which energy is drawn out, is called **secondary** winding. In brief, a transformer is a device that

1. transfers electric power from one circuit to another
2. it does so without a change of frequency
3. it accomplishes this by electromagnetic induction and
4. where the two electric circuits are in mutual inductive influence of each other.

When an alternating voltage V_1 is applied to the primary, an alternating flux $\emptyset$ is set up in the core. This alternating flux links both the windings and induces e.m.f.s E_1 and E_2 in them according to Faraday's laws of electromagnetic induction. The e.m.f. E_1 is termed as primary e.m.f. and e.m.f. E_2 is termed as secondary e.m.f.

The losses that occur in a transformer are:

a) Core losses—eddy current and hysteresis losses
b) Copper losses—in the resistance of the windings

In practice, these losses are very small so that output power is nearly equal to the input primary power. In other words, a transformer has very high efficiency.

(i) Transformer on DC

A transformer cannot be operate on dc supply and never be connected to a dc source. If a rated dc voltage is applied to the primary of a transformer, the flux produce in the transformer core will not vary but remain constant in magnitude and, therefore, no emf will be included in the secondary winding except at the moment of switching on. Thus the transformer is not capable of raising or lowering the dc voltage. Also there will be no self induced emf in the primary winding, which is only possible with varying flux linkage, to oppose the applied voltage and since the resistance of primary winding is quite low, therefore, a heavy current will flow through the primary winding which may result in the burning out of the primary winding. This is reason that dc is never applied to a transformer.

2.3 Transformer Construction

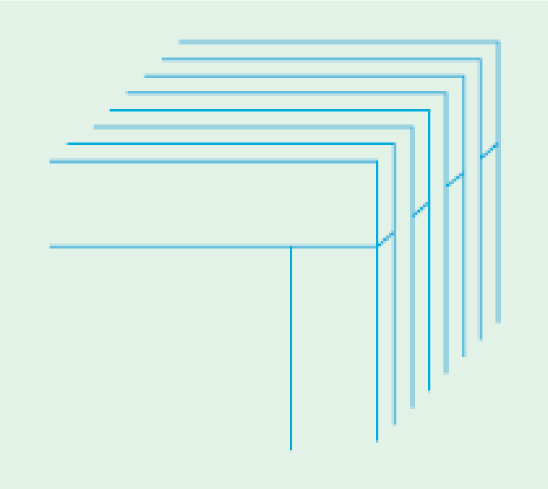


Figure 3

The simple elements of a transformer consist of two coils having mutual inductance and a laminated steel core. The two coils are insulated from each other and the steel core. Other necessary parts are: some suitable container for assembled core and windings; a suitable medium for insulating the core and its windings from its container; suitable bushings (either of porcelain, oil-filled or capacitor-type) for insulating and bringing out the terminals of windings from the tank. In all types of transformers, the core is constructed of transformer sheet steel laminations assembled to provide a continuous magnetic path with a minimum of air-gap included. The steel used is of high silicon content, sometimes heat treated to produce a high permeability and a low hysteresis loss at the usual operating flux densities. The eddy current loss is minimised by laminating the core, the laminations being insulated from each other by a light coat of core-plate varnish or by an oxide layer on the surface. The thickness of laminations varies from 0.35 mm for a frequency of 50 Hz to 0.5 mm for a frequency of 25 Hz. The core laminations (in the form of strips) are joined as shown in Fig. 5.2. It is seen that the joints in the alternate layers are staggered in order to avoid the presence of narrow gaps right through the cross-section of the core. Such staggered joints are said to be 'imbricated'.

2.4 Types of Transformers

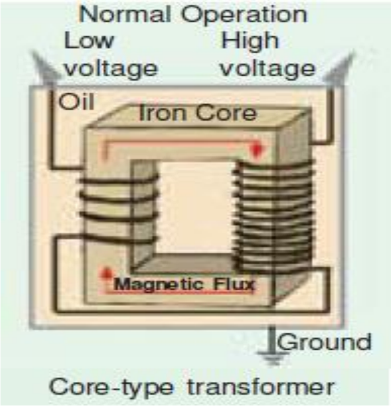


Figure 4

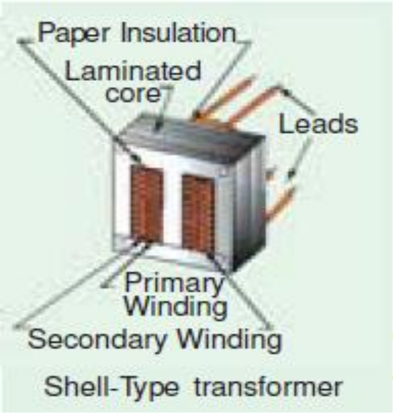


Figure 5

Constructionally, the transformers are of two general types, distinguished from each other merely by the manner in which the primary and secondary coils are placed around the laminated core. The two types are known as

(i) core-type and

(ii) shelltype.

(iii) Another recent development is spiral-core or wound-core type, the trade name being spirakore transformer.

In the so-called core type transformers, the windings surround a considerable part of the core whereas in shell-type transformers, the core surrounds a considerable portion of the windings as shown schematically in Fig. 6.(a) and (b) respectively.

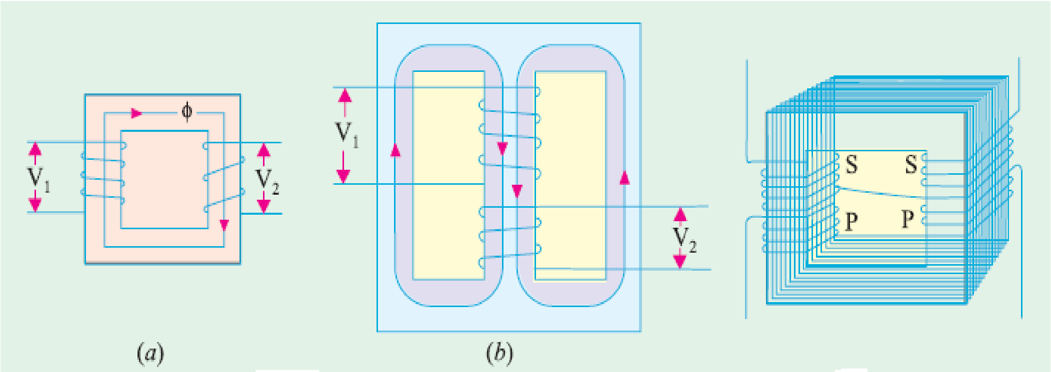


Figure 6

In the simplified diagram for the core type transformers [Fig. 5.4 (*a*)], the primary and secondary winding are shown located on the opposite legs (or limbs) of the core, but in actual construction, these are always interleaved to reduce leakage flux. As shown in Fig. 5.5, half the primary and half the secondary winding have been placed side by side or concentrically on each limb, not primary on one limb (or leg) and the secondary on the other.

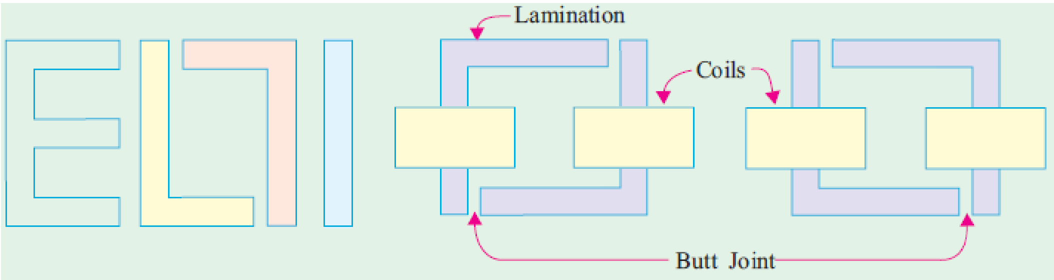


Figure 7

In both core and shell-type transformers, the individual laminations are cut in the form of long strips of L's, E's and I's as shown in Fig. 6. The assembly of the complete core for the two types of transformers is shown in Fig.7 and Fig. 8.

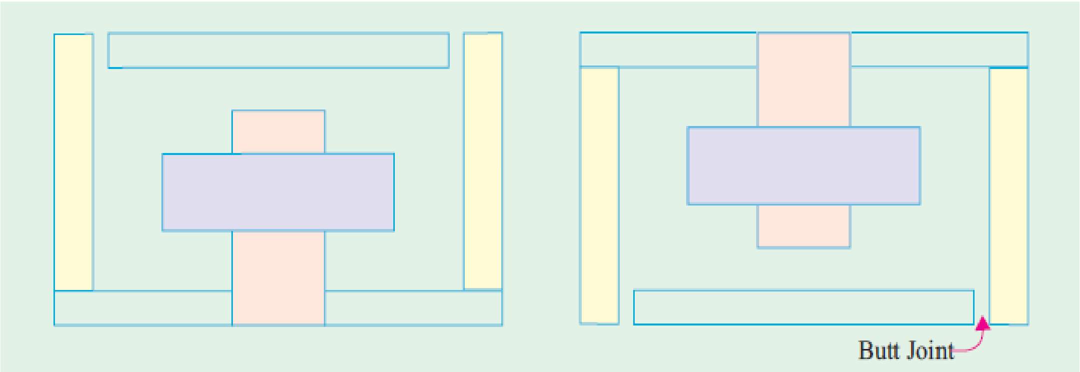


Figure 8

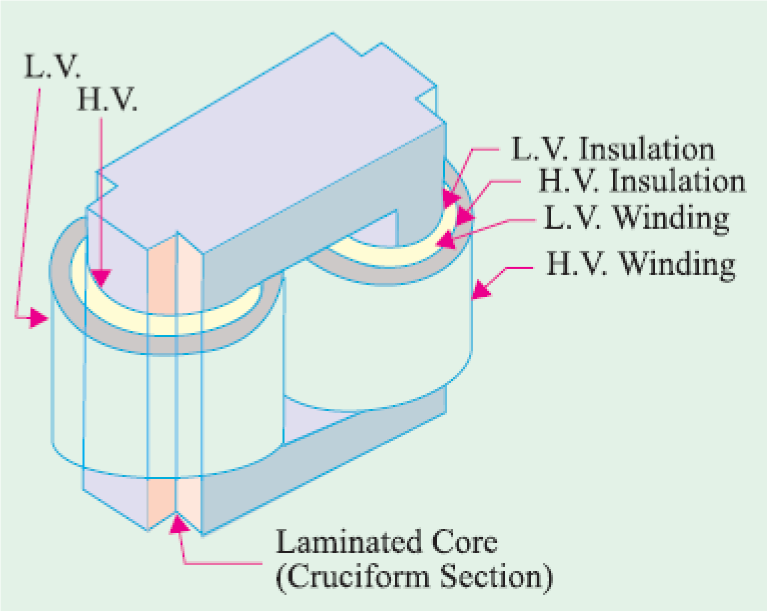
As said above, in order to avoid high reluctance at the joints where the laminations are butted against each other, the alternate layers are stacked differently to eliminate these joints as shown in Fig

(i) Core-type Transformers

Figure 9

The coils used are form-wound and are of the cylindrical type. The general form of these coils may be circular or oval or rectangular. In small size core-type transformers, a simple rectangular core is used with cylindrical coils which are either circular or rectangular in form. But for large-size core-type transformers, round or circular cylindrical coils are used which are so wound as to fit over a cruciform core section as shown in Fig. 5.9(a). The circular cylindrical coils are used in most of the core-type transformers because of their mechanical strength. Such cylindrical coils are wound in helical layers with the different layers insulated from each other by paper, cloth, micarta board or cooling ducts.

Fig. 9(c) shows the general arrangement of these coils with respect to the core. Insulating cylinders of fuller board are used to separate the cylindrical windings from the core and from each other. Since the low voltage (LV) winding is easiest to insulate, it is placed nearest to the core (Fig. 5.9).



Because of laminations and insulation, the net or effective core area is reduced, due allowance for which has to be made (Ex. 5.7). It is found that, in general, the reduction in core sectional area due to the presence of paper, surface oxide etc. is of the order of 10% approximately.

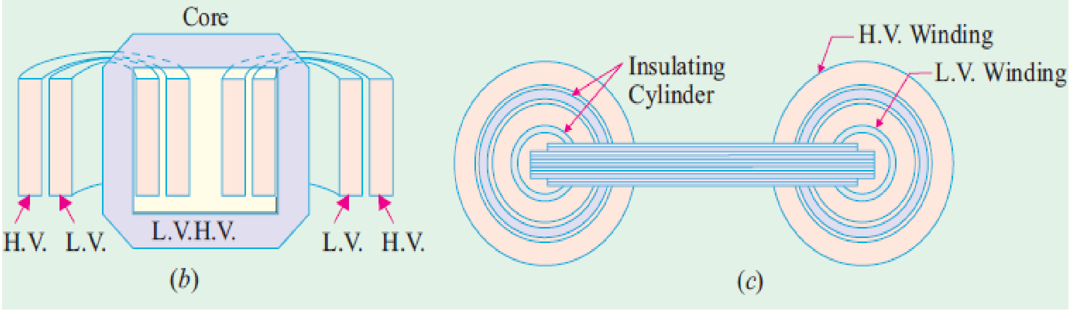


Figure 10

As pointed out above, rectangular cores with rectangular cylindrical coils can be used for small-size core-type transformers as shown in Fig. 10 (a) but for large-sized transformers, it becomes wasteful to use rectangular cylindrical coils and so circular cylindrical coils are preferred. For such purposes, square cores may be used as shown in Fig. 5.10 (b) where circles represent the tubular former carrying the coils. Obviously, a considerable amount of useful space is still wasted.

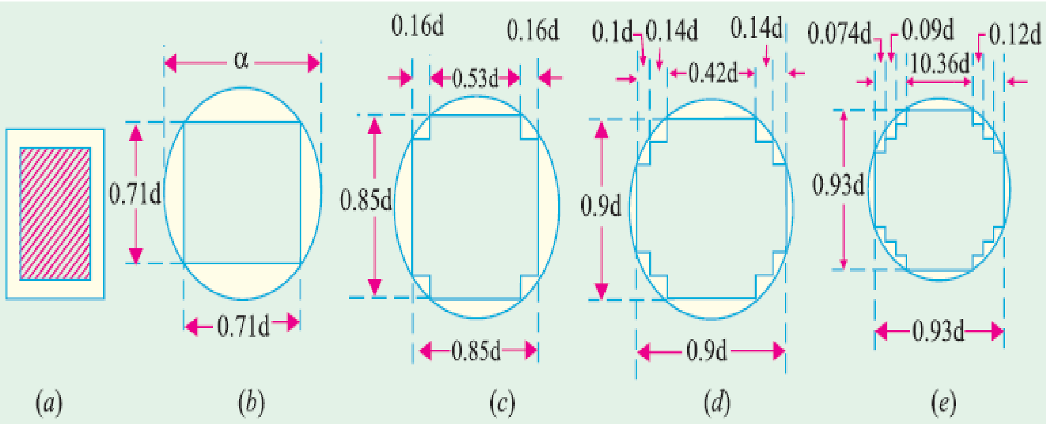


Figure 11

A common improvement on square core is to employ cruciform core as in Fig.11 (c) which demands, at least, two sizes of core strips. For very large transformers, further core-stepping is done as in Fig. 11 (d) where at least three sizes of core plates are necessary. Core-stepping not only gives high space factor but also results in reduced length of the mean turn and the consequent I^2R loss. Three stepped core is the one most commonly used although more steps may be used for very large transformers as in Fig. 5.10 (e). From the geometry of

Fig. 5.10, it can be shown that maximum gross core section for Fig. 5.10 (b) is 0.5 d2 and for Fig. 5.10 (c) it is 0.616 d2 where d is the diameter of the cylindrical coil.

(ii) Shell-type Transformers

In these case also, the coils are form-would but are multi-layer disc type usually wound in the form of pancakes. The different layers of such multi-layer discs are insulated from each other by paper. The complete winding consists of stacked discs with insulation space between the coils–the spaces forming horizontal cooling and insulating ducts. A shell-type transformer may have a simple rectangular form as shown in Fig.11 or it may have distributed form as shown in Fig.12.

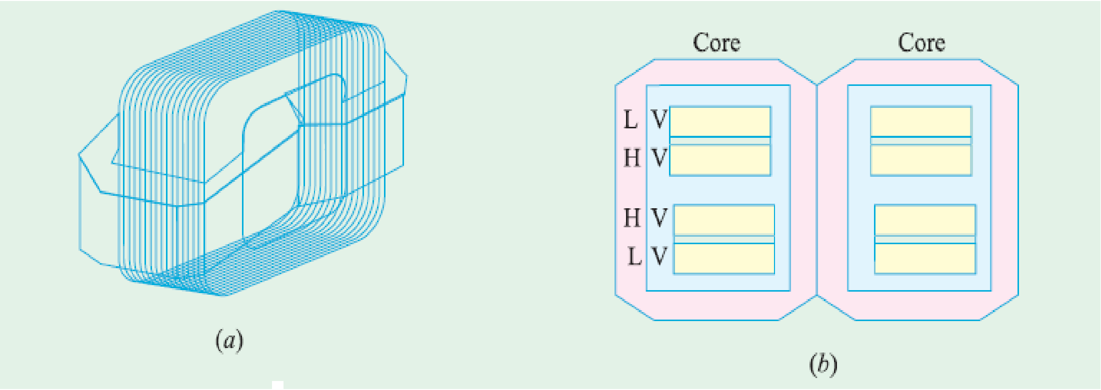


Figure 12

A very commonly-used shell-type transformer is the one known as Berry Transformer–so called after the name of its designer and is cylindrical in form. The transformer core consists of laminations arranged in groups which radiate out from the centre as shown in section in Fig. 5.13.

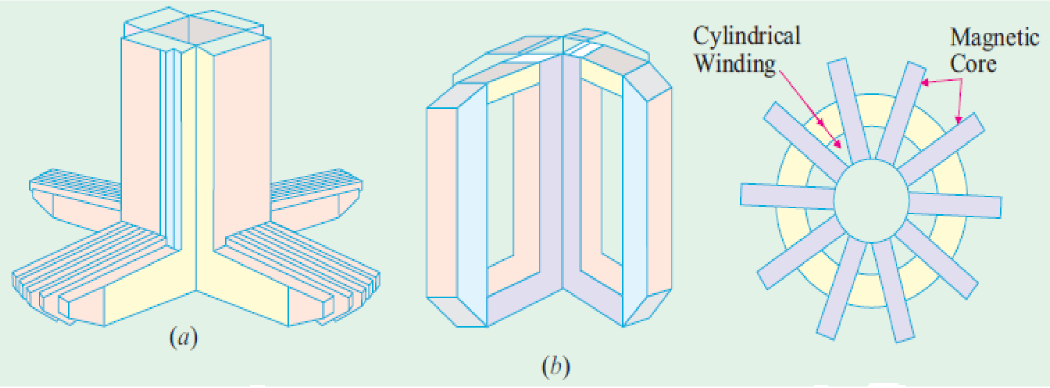


Figure 13

It may be pointed out that cores and coils of transformers must be provided with rigid mechanical bracing in order to prevent movement and possible insulation damage. Good bracing reduces vibration and the objectionable noise–a humming sound–during operation.

The spiral-core transformer employs the newest development in core construction. The core is assembled of a continuous strip or ribbon of transformer steel wound in the form of a circular or elliptical cylinder. Such construction allows the core flux to follow the grain of the iron. Cold-rolled steel of high silicon content enables the designer to use considerably higher operating flux densities with lower loss per kg. The use of higher flux density reduces the weight per kVA. Hence, the advantages of such construction are

i. a relatively more rigid core
ii. lesser weight and size per kVA rating
iii. lower iron losses at higher operating flux densities and
iv. Lower cost of manufacture.

Transformers are generally housed in tightly-fitted sheet-metal; tanks filled with special insulating oil*. This oil has been highly developed and its function is two-fold. By circulation, it not only keeps the coils reasonably cool, but also provides the transformer with additional insulation not obtainable when the transformer is left in the air.

In cases where a smooth tank surface does not provide sufficient cooling area, the sides of the tank are corrugated or provided with radiators mounted on the sides. Good transformer oil should be absolutely free from alkalies, sulphur and particularly from moisture. The presence of even an extremely small percentage of moisture in the oil is highly detrimental from the insulation viewpoint because it lowers the dielectric strength of the oil considerably. The importance of avoiding moisture in the transformer oil is clear from the fact that even an addition of 8 parts of water in 1,000,000 reduces the insulating quality of the oil to a value generally recognized as below standard. Hence, the tanks are sealed air-tight in smaller units. In the case of large-sized transformers where complete air-tight construction is impossible, chambers known as **breathers** are provided to permit the oil inside the tank to expand and contract as its temperature increases or decreases. The atmospheric moisture is entrapped in these breathers and is not allowed to pass on to the oil. Another thing to avoid in the oil is sledging which is simply the decomposition of oil with long and continued use. Sledging is caused principally by exposure to oxygen during heating and results in the formation of large deposits of dark and heavy matter that eventually clogs the cooling ducts in the transformer.

No other feature in the construction of a transformer is given more attention and care than the insulating materials, because the life on the unit almost solely depends on the quality, durability and handling of these materials. All the insulating materials are selected on the basis of their high quality and ability to preserve high quality even after many years of normal use.

All the transformer leads are brought out of their cases through suitable bushings. There are many designs of these, their size and construction depending on the voltage of the leads. For moderate voltages, porcelain bushings are used to insulate the leads as they come out through the tank. In general, they look almost like the insulators used on the transmission lines. In high voltage installations, oil-filled or capacitor type bushings are employed.

The choice of core or shell-type construction is usually determined by cost, because similar characteristics can be obtained with both types. For very high-voltage transformers or for multi winding design, shell type construction is preferred by many manufacturers. In this type, usually the mean length of coil turn is longer than in a comparable core-type design. Both core and shell forms are used and the selection is decided by many factors such as voltage rating, kVA rating, weight, insulation stress, heat distribution etc.

2.5 Concept of ideal transformer

An ideal transformer is one which has

1. Its windings have no ohmic resistance and hence which has no I^2R losses.
2. There is no magnetic leakage and hence which has no core losses. In other words, an ideal transformer consists of two purely inductive coils wound on a loss-free core (or Leakage flux is zero i.e. 100% flux produced by primary links with the secondary).
3. Permeability of core is so high that negligible current is required to establish the flux in it.

Although ideal transformer cannot be physically realized, yet its study provides a very powerful tool in the analysis of a practical transformer. In fact, practical transformers have properties that approach very close to an ideal transformer.

2.6 E.M.F. Equation of a Transformer

Let $N_1 = \text{No. of turns in primary}$

$N_2 = \text{No. of turns in secondary}$

$\emptyset m = \text{Maximum flux in core in webers}$

$= B_m \times A$

$f = \text{Frequency of a.c. input in Hz}$

As shown in Fig. 5.14, flux increases from its zero value to maximum value $\emptyset m$ in one quarter of the cycle i.e. in $1/4$ f second.

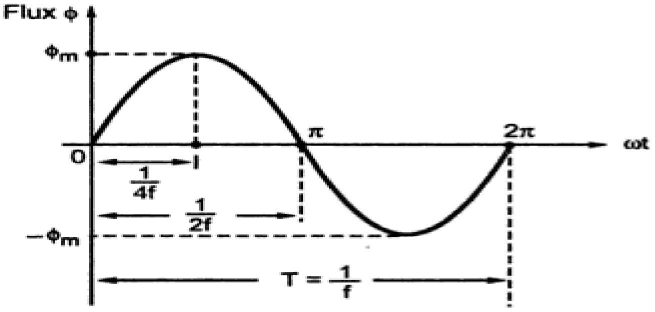


Fig 5.14

$$\therefore \text{ Average rate of change of flux } = \frac{\emptyset m}{1/4f}$$

$$= 4\,f\,\emptyset m \text{ Wb/s or volt}$$

Now, rate of change of flux per turn means induced e.m.f. in volts.

$$\therefore \text{ Average e.m.f./turn } = 4\,f\,\emptyset m \text{ volt}$$

If flux Φ varies sinusoidally, then r.m.s. value of induced e.m.f. is obtained by multiplying the average value with form factor.

$$\text{Form factor } = \frac{\text{r.m.s value}}{\text{average value}} = 1.11$$

$$\therefore \text{ r.m.s. value of e.m.f./ turn} = 1.11 \times 4\,f\,\emptyset m = 4.44\,f\,\emptyset m \text{ volt}$$

Now,

r.m.s. value of the induced e.m.f. in the whole of primary winding = (induced e.m.f/turn) × No. of primary turns

$$E1 = 4.44\,f\,N_1\,\emptyset m = 4.44\,f\,N_1\,B_m A \qquad (1)$$

Similarly, r.m.s. value of the e.m.f. induced in secondary is,

$$E2 = 4.44\,f\,N_2\,\emptyset m = 4.44\,f\,N_2\,B_m A \qquad (2)$$

It is seen from (5.1) and (5.2) that

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44\ f\ \emptyset m$$

It means that e.m.f./turn is the same in both the primary and secondary windings.

In an ideal transformer on no-load, $V_1 = E_1$ and $V_2 = E_2$, where V_2 is the terminal voltage.

Voltage Transformation Ratio (K)

From equations (1) and (2), we get

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = K$$

This constant K is known as voltage transformation ratio.

i. If $N_2 > N_1$ i.e. K > 1, then transformer is called **step-up** transformer.

ii. If $N_2 < N_1$ i.e. K < 1, then transformer is known as **step-down** transformer.

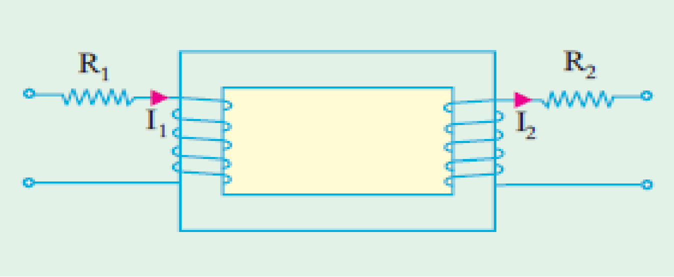
Again, for an ideal transformer, input VA = output VA.

$$V_1 I_1 = V_2 I_2$$

$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence, currents are in the inverse ratio of the (voltage) transformation ratio.

2.7 Equivalent Resistance



In Fig. 5.19 a transformer is shown whose primary and secondary windings have resistances of R_1 and R_2 respectively. The resistances have been shown external to the windings. It would now be shown that the resistances of the two windings can be transferred to any one of the two windings.

The advantage of concentrating both the resistances in one winding is that it makes calculations very simple and easy because one has then to work in one winding only. It will be proved that a resistance of R_2 in secondary is equivalent to $\frac{R_2}{K^2}$ in primary. The value $\frac{R_2}{K^2}$ will be denoted by R_2'– **the equivalent secondary resistance as referred to primary**.

The copper loss in secondary is $I_2^2 R_2$. This loss is supplied by primary which takes a current of I_1. Hence if R_2' is the **equivalent resistance in primary which would have caused the same loss** as R_2 in secondary, then

$$I_1{}^2R_2{}' = I_2{}^2R_2$$

$$R_2{}' = \frac{I_2{}^2}{I_1{}^2}R_2 = \frac{R_2}{K^2}$$

$$R_2{}' = \frac{R_2}{K^2}$$

Similarly, equivalent primary resistance as referred to secondary is

$$R_1{}' = K^2R_1$$

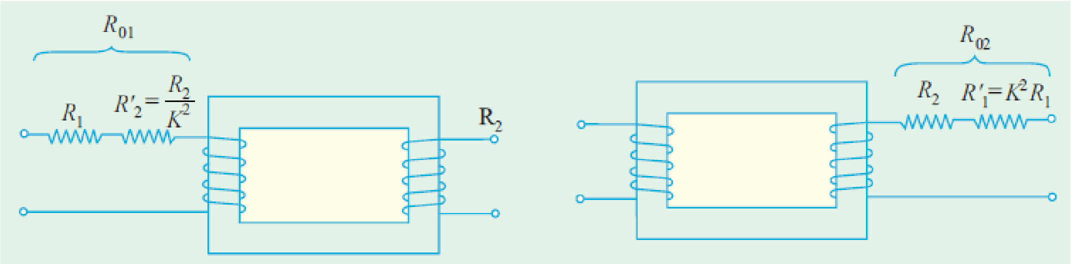
In Fig. 5.20, secondary resistance has been transferred to primary side leaving secondary circuit resistance less. The resistance $R_1 + R_2{}' = R_1 + \frac{R_2}{K^2}$ is known as the **equivalent or effective resistance of the transformer as referred to primary** and may be designated as R_{01}.

$$\therefore R_{01} = R_1 + R_2{}' = R_1 + \frac{R_2}{K^2}$$

Similarly, the **equivalent resistance of the transformer as referred to secondary is**

$$\therefore R_{02} = R_2 + R_1{}' = R_2 + K^2R_1$$

This fact is shown in Fig. 5.21 where all the resistances of the transformer has been concentrated in the secondary winding.



It is to be noted that

1. A resistance of R_1 in primary is equivalent to K^2R_1 in secondary. Hence, it is called **equivalent resistance as referred to secondary** i.e. $R_1{}'$.
2. A resistance of R_2 in secondary is equivalent to $\frac{R_2}{K^2}$ in primary. Hence, it is called the **equivalent secondary resistance as referred to primary** i.e. $R_2{}'$.
3. Total or effective resistance of the transformer as referred to primary is

R_{01} = primary resistance + equivalent secondary resistance as referred to primary

$$R_{01} = R_1 + R_2{}' = R_1 + \frac{R_2}{K^2}$$

4. Similarly, total transformer resistance as referred to secondary is,

R_{02} = secondary resistance + equivalent primary resistance as referred to secondary

$$R_{02} = R_2 + R_1{}' = R_2 + K^2R_1$$

Note: It is important to remember that

a. When shifting any primary resistance to the secondary, **multiply** it by K^2 i.e. $(\text{transformation ratio})^2$.

b. When shifting secondary resistance to the primary, **divide** it by K^2.

c. However, when shifting any voltage from one winding to another only K is used.

(i) Magnetic Leakage

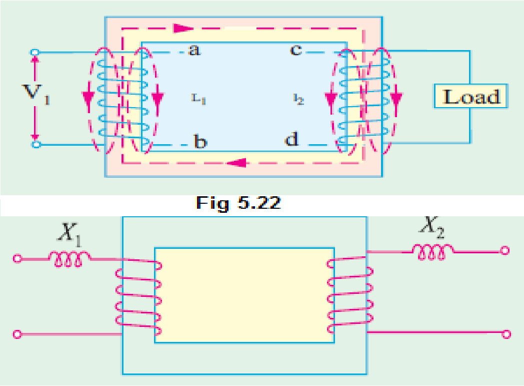


Fig 5.22

In the preceding discussion, it has been assumed that all the flux linked with primary winding also links the secondary winding. But, in practice, it is impossible to realize this condition. It is found, however, that all the flux linked with primary does not link the secondary but part of it i.e. $\emptyset_{L_1}$ completes its 1magnetic circuit by passing through air rather than around the core, as shown in Fig. 5.22. This leakage flux is produced when the m.m.f. due to primary ampere-turns existing between points a and b, acts along the leakage paths.

Hence, this flux is known as **primary leakage flux** and is proportional to the primary ampere-turns alone because the secondary turns do not link the magnetic circuit of $\emptyset_{L_1}$ The flux $\emptyset_{L_1}$ is in time phase with I_1. It induces an e.m.f. E_{L_1} in primary but not in secondary.

Similarly, secondary ampere-turns (or m.m.f.) acting across points c and d set up leakage flux $\emptyset_{L_2}$, which is linked with secondary winding alone (and not with primary turns). This flux $\emptyset_{L_2}$ is in time phase with I_2 and produces a self-induced e.m.f. E_{L_2} in secondary (but not in primary).

At no load and light loads, the primary and secondary ampere-turns are small, hence leakage fluxes are negligible. But when load is increased, both primary and secondary windings carry huge currents. Hence, large m.m.f.'s are set up which, while acting on leakage paths, increase the leakage flux.

As said earlier, the leakage flux linking with each winding produces a self-induced e.m.f. in that winding. Hence, in effect, it is equivalent to a small choker or inductive coil in series with each winding such that voltage drops in each series coil is equal to that produced by leakage flux. In other words, **a transformer with magnetic leakage is equivalent to an ideal transformer with inductive coils connected in both primary and secondary circuits** as shown in Fig. 5.23 such that the internal e.m.f. in each inductive coil is equal to that due to the corresponding leakage flux in the actual transformer.

$$X_1 = \frac{E_{L1}}{I_1} = \frac{2\pi f L_1 I_1}{I_1} = 2\pi f L_1$$

$$X_2 = \frac{E_{L2}}{I_2} = \frac{2\pi f L_2 I_2}{I_2} = 2\pi f L_2$$

The terms X_1 and X_2 are known as primary and secondary leakage reactance's respectively.

Following few points should be kept in mind:

1. The leakage flux links one or the other winding but **not both**, hence it in no way contributes to the transfer of energy from the primary to the secondary winding.
2. The primary voltage V_1 will have to supply reactive drop I_1X_1 in addition to I_1R_1. Similarly E_2 will have to supply I_2R_2 and I_2X_2.
3. In an actual transformer, the primary and secondary windings are not placed on separate legs or limbs as shown in Fig. 5.23 because due to their being widely separated, large primary and secondary leakage fluxes would result. These leakage fluxes are minimized by sectionalizing and interleaving the primary and secondary windings as in Fig. 5.22 or Fig. 5.24.

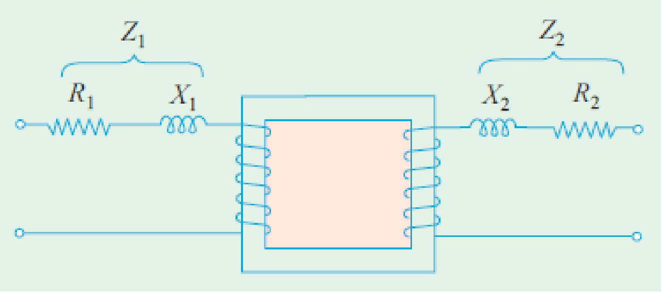
2.8 Transformer with Resistance and Leakage Reactance

In Fig. 5.24 the primary and secondary windings of a transformer with reactances taken out of the windings are shown. The primary impedance is given by

$$Z_1 = \sqrt{{R_1}^2 + {X_1}^2}$$

Similarly, secondary impedance is given by

$$Z_2 = \sqrt{{R_2}^2 + {X_2}^2}$$



The resistance and leakage reactance of each winding is responsible for some voltage drop in each winding. In primary, the leakage reactance drop is I_1X_1 (usually 1 or 2% of V_1).

Hence

$$V_1 = -E_1 + I_1(R_1 + jX_1)$$

Similarly, there are I_2R_2 and I_2X_2 drops in secondary which combine with V_2 to give E_2.

$$E_2 = V_2 + I_2(R_2 + jX_2)$$

The vector diagram for such a transformer for different kinds of loads is shown in Fig. 5.25. In these diagrams, vectors for resistive drops are drawn parallel to current vectors whereas reactive drops are perpendicular to the current vectors. The angle ϕ_1 between V_1 and I_1 gives the power factor angle of the transformer.

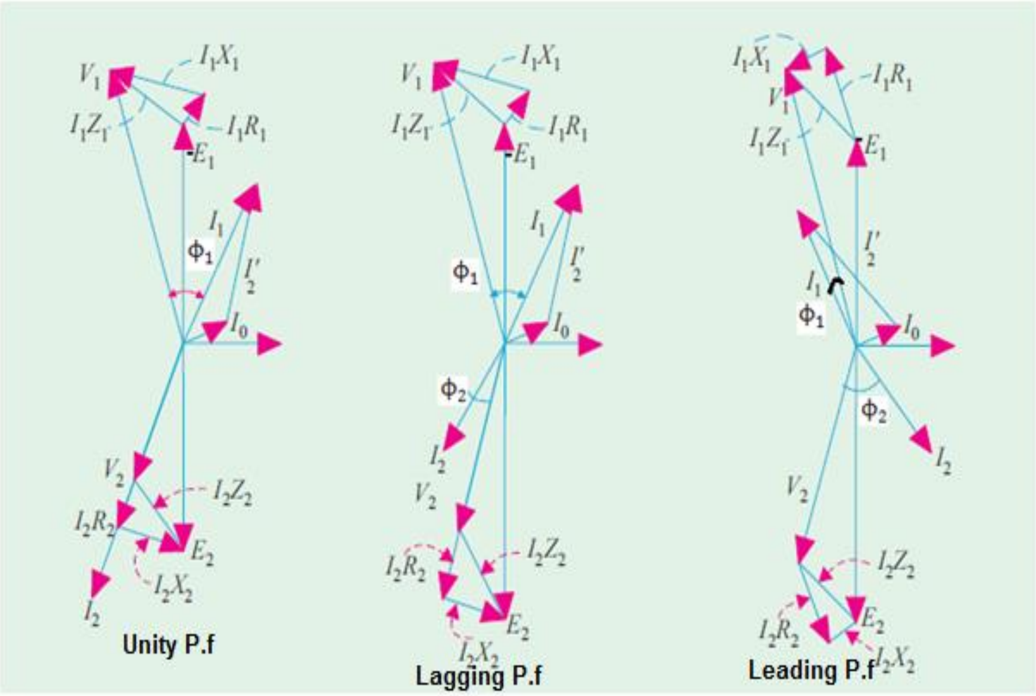
It may be noted that leakage reactances can also be transferred from one winding to the other in the same way as resistance.

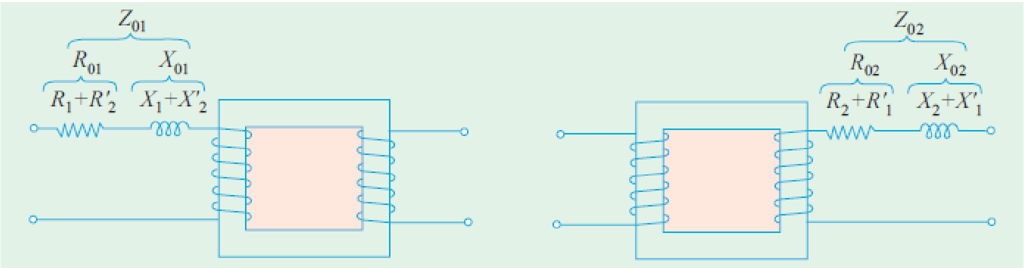
$$X_2{}' = {}^{X_2}\!/_{K^2}$$

$$X_1{}' = K^2X_1$$

$$X_{01} = X_1 + X_2{}' = X_1 + \frac{X_2}{K^2}$$

$$X_{02} = X_2 + X_1{}' = X_2 + K^2X_1$$





It is obvious that total impedance of the transformer as referred to primary is given by

$$Z_{01} = \sqrt{{R_{01}}^2 + {X_{01}}^2}$$

Similarly, total impedance of the transformer as referred to secondary is given by

$$Z_{02} = \sqrt{{R_{02}}^2 + {X_{02}}^2}$$

2.9 Equivalent circuit of a Transformer

The equivalent circuit of any device can be quite helpful in predetermination of the behavior of the device under various conditions of operation and it can be drawn if the equations describing its behavior are known. If any electrical device is to be analysed and investigated further for suitable modifications, its appropriate equivalent circuit is necessary.

Fig 5.31 shows the equivalent circuit of transformer on No-Load condition. We already known that transformer on No-Load primary current I_0 has two components

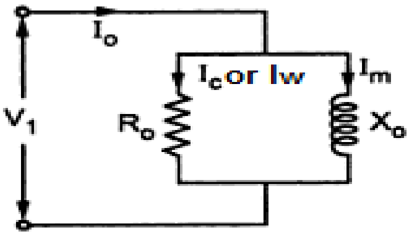


Fig 5.31 No load equivalent circuit

$$I_W = I_0 \cos\phi_0 = \textbf{active} \text{ or } \textbf{working} \text{ or } \textbf{iron} \text{ loss component}$$

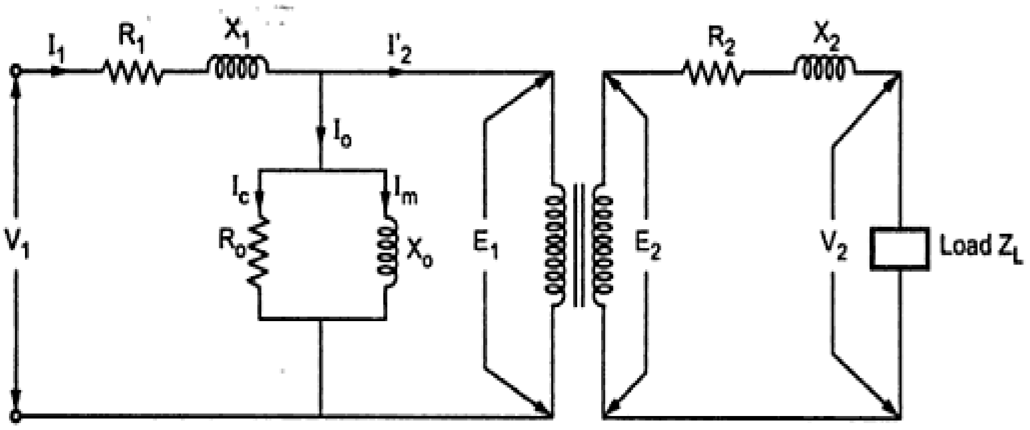
$$I_m = I_0 \sin\phi_0 = \textbf{magnetising} \text{ component}$$

From equivalent circuit we can write,

$$R_o = \frac{V_1}{I_w}$$

$$X_o = \frac{V_1}{I_m}$$

When the load is connected to the transformer then secondary current I_2 flows and operation we already discussed. So the equivalent circuit of transformer on loaded condition is given in fig 5.32.



It can be further simplified by transforming all the values to primary or secondary. Fig 5.33 shows the exact equivalent circuit of a transformer referred to primary by using transformation resistances and reactances as already discussed in previous topics.

Transforming secondary parameters to primary as follows,

$$K = \frac{N_2}{N_1}$$

$$R_2{}' = \frac{R_2}{K^2}$$

$$Z_2{}' = \frac{Z_2}{K^2}$$

$$X_2{}' = {}^{X_2}\!/_{K^2}$$

$$I_2{}' = KI_2$$

$$E_2{}' = \frac{E_2}{K}$$

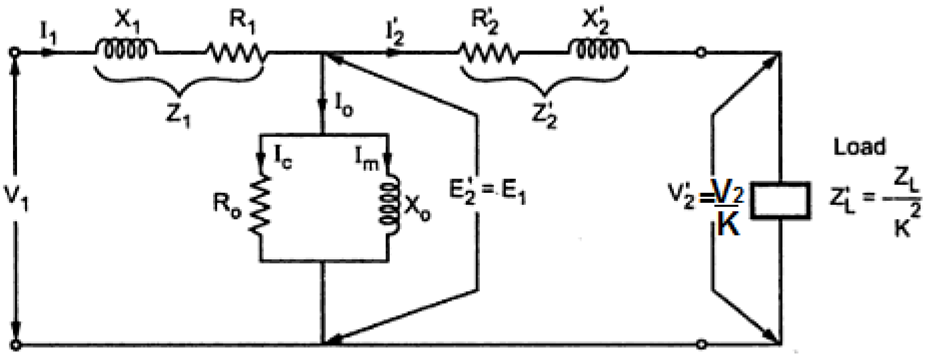


Fig 5.33 Exact equivalent circuit referred to primary

Fig 5.33 shows the exact equivalent circuit of a transformer referred to secondary

Transforming primary parameters to secondary as follows,

$$R_1{}' = K^2 R_1$$

$$X_1{}' = K^2 X_1$$

$$E_1' = KE_1$$

$$Z_1' = K^2 Z_1$$

$$I_1' = \frac{I_1}{K}$$

$$I_0' = \frac{I_0}{K}$$

$$R_0' = K^2 R_0$$

$$X_0' = K^2 X_0$$

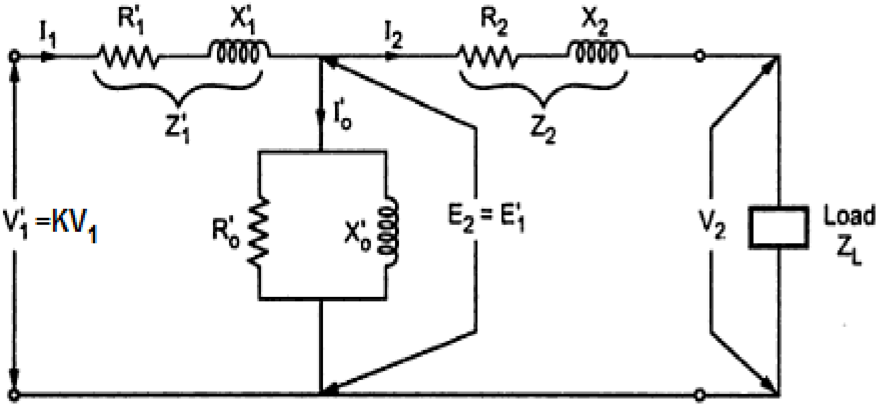


Fig 5.34 Exact equivalent circuit of transformer referred to secondary

(I) Approximate equivalent circuit

The equivalent circuit is further simplified by transferring R_0 and X_0 towards left end as shown in fig 5.35. The error introduced by doing do is very small and it is neglected. Hence such an equivalent circuit is called **approximate equivalent circuit.**

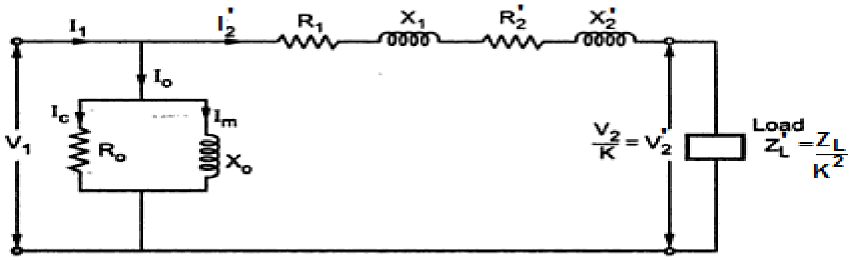
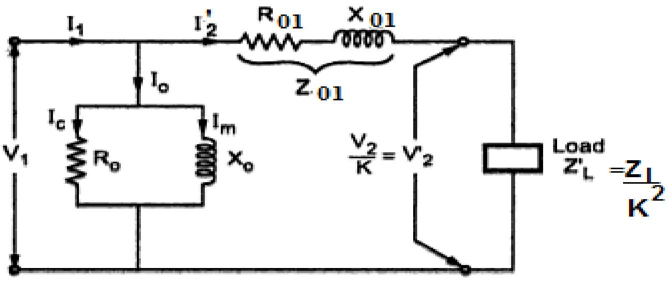


Fig 5.35 Approximate euivalent circuit of a transformer referred to primary



$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$

$$X_{01} = X_1 + X_2{}' = X_1 + \frac{X_2}{K^2}$$

$$Z_{01} = R_{01} + jX_{01}$$

(II) Total Approximate Voltage Drop in a Transformer

Consider the equivalent circuit referred to secondary as shown in fig 5.37.

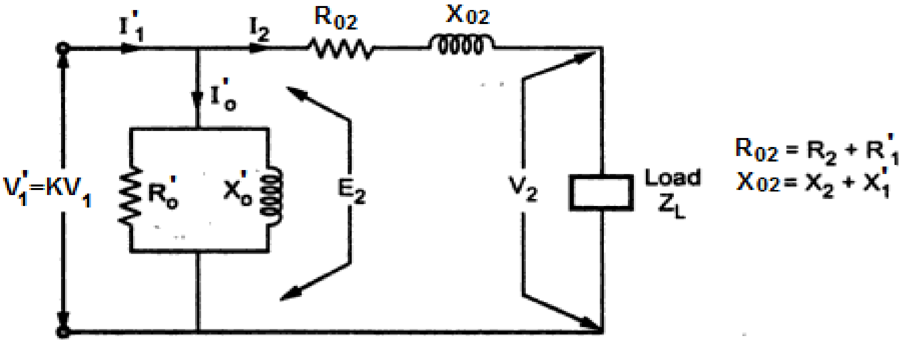
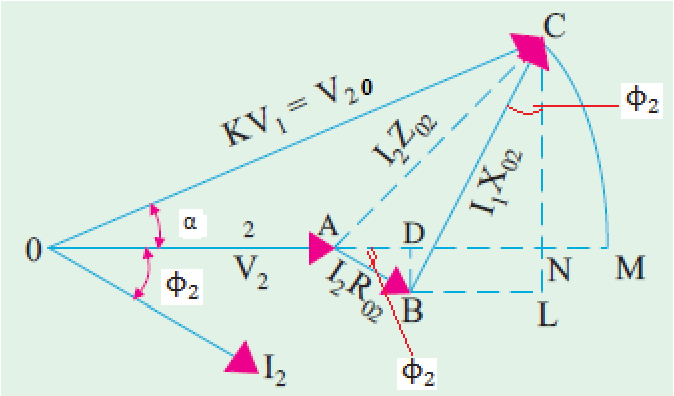


Fig 5.37 Approximate euivalent circuit of a transformer referred to secondary

When the transformer is on no-load, and then V_1 is approximately equal to E1. Hence $E_2 = KE_1 = KV_1$. Also, $E_2 = V_{20}$ where V_{20} is secondary terminal voltage on **no load**, hence no-load secondary terminal voltage is KV_1. The secondary voltage on load is V_2. The difference between the two is I_2Z_{02} as shown in Fig. 5.38. The approximate voltage drop of the transformer **as referred to secondary** is found from phasor diagram 5.38.



$$V_{20} = \text{No load terminal voltage}$$

$$V_2 = \text{Terminal voltage on load}$$

With O as the centre and radius OC draw an arc cutting OA produced at M. The total voltage drop $I_2Z_{02} = AC = AM$, which is approximately equal to AN. From B draw BD perpendicular on OA produced. Draw CN perpendicular to OM and draw BL parallel to OM.

$$AN = \text{Approximate voltage drop}$$

$$\mathrm{AM} = \text{Exact voltage drop}$$

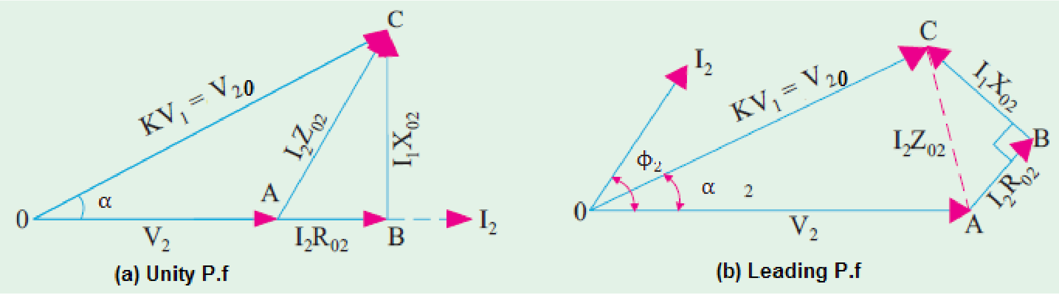
$$\begin{aligned}
\text{Approximate voltage drop} &= \mathrm{AN} \\
&= \mathrm{AD} + \mathrm{DN} \\
&= \mathrm{AB}\cos\phi_2 + \mathrm{BL} \\
&= \mathrm{AB}\ \cos\phi_2 + \mathrm{BC}\ \sin\phi_2 \\
&= \mathrm{I_2R_{02}}\cos\phi_2 + \mathrm{I_2X_{02}}\sin\phi_2
\end{aligned}$$

$$\text{Approximately } \phi_2 = \phi_1 = \phi$$

$$\text{Approximate voltage drop} = \mathrm{I_2R_{02}}\cos\phi + \mathrm{I_2X_{02}}\sin\phi$$

This is the value of approximate voltage drop for a **lagging** power factor.

The different figures for unity and leading power factors are shown in Fig. 5.39 (*a*) and (*b*) respectively.



(a) Unity P.f

(b) Leading P.f

The approximate voltage drop for **leading** power factor becomes

$$\mathrm{I_2R_{02}}\cos\phi - \mathrm{I_2X_{02}}\sin\phi$$

Approximate voltage drop=

$$\begin{aligned}
&= \mathrm{I_2R_{02}}\cos\phi + \mathrm{I_2X_{02}}\sin\phi \qquad \text{For lagging P.f} \\
&= \mathrm{I_2R_{02}}\cos\phi - \mathrm{I_2X_{02}}\sin\phi \qquad \text{For leading P.f}
\end{aligned}$$

It may be noted that approximate voltage drop as referred to primary is

$$\mathrm{I_1R_{01}}\cos\phi \pm \mathrm{I_1X_{01}}\sin\phi$$

$$\%\ \text{Voltage drop in secondary} = \frac{\mathrm{I_2R_{02}}\cos\phi \pm \mathrm{I_2X_{02}}\sin\phi}{\mathrm{V_{20}}} * 100$$

$$= \frac{\mathrm{I_2R_{02}}}{\mathrm{V_{20}}} * 100 * \cos\phi \pm \frac{\mathrm{I_2X_{02}}}{\mathrm{V_{20}}} * 100 * \sin\phi$$

$$= \mathrm{V_r} * \cos\phi \pm \mathrm{V_x} * \sin\phi$$

$$\boxed{\%\ \text{Voltage drop in secondary} = \mathrm{V_r} * \cos\phi \pm \mathrm{V_x} * \sin\phi}$$

$$V_r = \frac{I_2R_{02}}{V_{20}} * 100 = \text{Percentage resistive drop} = \frac{I_1R_{01}}{V_1} * 100$$

$$V_x = \frac{I_2X_{02}}{V_{20}} * 100 = \text{Percentage reactance drop} = \frac{I_1X_{01}}{V_1} * 100$$

(III) Exact Voltage Drop

With reference to Fig. 5.38, it is to be noted that exact voltage drop is AM and not AN. If we add the quantity NM to AN, we will get the exact value of the voltage drop.

Considering the right-angled triangle OCN, we get

$$NC^2 = OC^2 - ON^2$$

$$= (OC + ON)(OC - ON)$$

$$= (OC + OC)(OM - ON) \quad (\because OC \approx ON)$$

$$= 2\ OC * NM$$

$$NM = \frac{NC^2}{2\ OC}$$

$$NC = LC - LN = LC - BD$$

$$NC = I_2X_{02}\cos\phi - I_2R_{02}\sin\phi$$

$$NM = \frac{(I_2X_{02}\cos\phi - I_2R_{02}\sin\phi)^2}{2V_{20}}$$

For a **lagging** power factor, exact voltage drop is =AM

$$AM = AN + NM$$

$$= I_2R_{02}\cos\phi + I_2X_{02}\sin\phi + \frac{(I_2X_{02}\cos\phi - I_2R_{02}\sin\phi)^2}{2V_{20}}$$

For a **leading** power factor, exact voltage drop is

$$= I_2R_{02}\cos\phi - I_2X_{02}\sin\phi + \frac{(I_2X_{02}\cos\phi + I_2R_{02}\sin\phi)^2}{2V_{20}}$$

$$\text{Exact Voltage drop} = I_2R_{02}\cos\phi \pm I_2X_{02}\sin\phi + \frac{(I_2X_{02}\cos\phi \mp I_2R_{02}\sin\phi)^2}{2V_{20}}$$

Percentage voltage drop

$$= \frac{(I_2R_{02}\cos\phi \pm I_2X_{02}\sin\phi) * 100}{V_{20}} + \frac{(I_2X_{02}\cos\phi \mp I_2R_{02}\sin\phi)^2 * 100}{2{V_{20}}^2}$$

$$\text{Percentage voltage drop} = V_r \cos\phi \pm V_x \sin\phi + \frac{1}{200}(V_x \cos\phi \mp V_r \sin\phi)^2$$

$$\text{Percentage voltage drop} = V_r \cos\phi \pm V_x \sin\phi + \frac{1}{200}(V_x \cos\phi \mp V_r \sin\phi)^2$$

The upper signs are to be used for a **lagging** power factor and the lower ones for a **leading** power factor.

2.10 Voltage Regulation

Introduction

The voltage regulation can be defined in two ways - Regulation Down and Regulation up. These two definitions differ only in the reference voltage.

(i) Regulation down:

This is defined as “the change in terminal voltage when a load current at any power factor is applied, expressed as a fraction of the no-load terminal voltage”.

Expressed in symbolic form we have,

$$\text{Regulation } = \frac{V_{nl} - V_l}{V_{nl}}$$

V_{nl} and V_l are no-load and load terminal voltages. This is the definition normally used in the case of the transformers, the no-load voltage being the one given by the power supply provider on which the user has no say. Hence no-load voltage is taken as the reference.

(ii) Regulation up:

Here again the regulation is expressed as the ratio of the change in the terminal voltage when a load at a given power factor is thrown off, and the on load voltage. This definition if expressed in symbolic form results in

$$\text{Regulation } = \frac{V_{nl} - V_l}{V_l}$$

V_{nl} is the no-load terminal voltage. $V_l$is load voltage. Normally full load regulation is of interest as the part load regulation is going to be lower. This definition is more commonly used in the case of alternators and power systems as the user-end voltage is guaranteed by the power supply provider. He has to generate proper no-load voltage at the generating station to provide the user the voltage he has asked for. In the expressions for the regulation, only the numerical differences of the voltages are taken and not vector differences.

In the case of transformers both definitions result in more or less the same value for the regulation as the transformer impedance is very low and the power factor of operation is quite high. The power factor of the load is defined with respect to the terminal voltage on load. Hence a convenient starting point is the load voltage. Also the full load output voltage is taken from the name plate. Hence regulation up has some advantage when it comes to its application.

(iii) Voltage Regulation of a Transformer

The way in which the secondary terminal voltage varies with the load depends on the load current, the internal impedance and the load power factor. The change in secondary terminal voltage from no-load to full load is termed as inherent regulation. It is usually expressed as a percentage or a fraction of the rated no-load terminal voltage.

$$\text{percentage regulation} = \frac{\text{Terminal voltage on no load} - \text{terminal voltage on load}}{\text{Terminal voltage on no load}} * 100$$

$$= \frac{\text{Voltage drop in transformer at load}}{\text{No} - \text{load rated voltage (secondary)}} * 100$$

We already derived voltage drop in transformer at load. Here we take approximate voltage drop.

For lagging power factor

$$\%\ \text{regulation} = \frac{I_2R_{02}\cos\phi + I_2X_{02}\sin\phi}{\text{No} - \text{load rated voltage (secondary)}} * 100$$

For leading power factor

$$\%\ \text{regulation} = \frac{I_2R_{02}\cos\phi - I_2X_{02}\sin\phi}{\text{No} - \text{load rated voltage (secondary)}} * 100$$

Voltage regulation of a transformer on an average is about 4 percentage.

(iv) Condition for zero Regulation

$$\%\ \text{regulation} = \frac{I_2R_{02}\cos\phi + I_2X_{02}\sin\phi}{E_2} * 100$$

Regulation will be zero if the numerator will be equal to zero

$$I_2R_{02}\cos\phi + I_2X_{02}\sin\phi = 0$$

$$\tan\phi = \frac{-R_{02}}{X_{02}}$$

$$\boxed{\tan\phi = \frac{-R_{02}}{X_{02}}}$$

The -ve sign indicates that zero regulation occurs at a leading power factor.

(v) Condition for Maximum Regulation

Regulation will be maximum if $\frac{d}{d\phi}(\text{regulation}) = 0$

$$\frac{d}{d\phi}\left(\frac{I_2R_{02}\cos\phi + I_2X_{02}\sin\phi}{E_2}\right) = 0$$

$$-\frac{I_2R_{02}}{E_2}\sin\phi + \frac{I_2X_{02}}{E_2}\cos\phi = 0$$

$$\tan\phi = \frac{X_{02}}{R_{02}}$$

$$\boxed{\tan\phi = \frac{X_{02}}{R_{02}}}$$

Maximum regulation will occur at lagging power factor.

2.11 Losses

There are two types of power losses occur in a transformer

1. Iron loss
2. Copper loss

1. **Iron Loss:**

   This is the power loss that occurs in the iron part. This loss is due to the alternating frequency of the emf. Iron loss in further classified into two other losses.

   i. Eddy current loss
   ii. Hysteresis loss

   i. **Eddy current loss:** This power loss is due to the alternating flux linking the core, which will induced an emf in the core called the eddy emf, due to which a current called the eddy current is being circulated in the core. As there is some resistance in the core with this eddy current circulation converts into heat called the eddy current power loss. Eddy current loss is proportional to the square of the supply frequency.

$$\text{Eddy current loss} = K_eB_m{}^2f^2t^2 \text{ watts/unit volume}$$

$$\boxed{\text{Eddy current loss} = K_eB_m{}^2f^2t^2 \text{ watts/unit volume}}$$

Where, K_e = Eddy current constant

B_m = Maximum flux density

f = frequency

t = thickness of the core

ii. **Hysteresis loss:** This is the loss in the iron core, due to the magnetic reversal of the flux in the core, which results in the form of heat in the core. This loss is directly proportional to the supply frequency.

$$\text{Hysteresis loss} = K_h {B_m}^{1.67} f\, v \text{ watts}$$

$$\boxed{\text{Hysteresis loss} = K_h {B_m}^{1.67} f\, v \text{ watts}}$$

Where, K_h = Hysteresis constant

v = Volume of the core

Eddy current loss can be minimized by using the core made of thin sheets of silicon steel material, and each lamination is coated with varnish insulation to suppress the path of the eddy currents.

Hysteresis loss can be minimized by using the core material having high permeability.

2. **Copper loss:**

This is the power loss that occurs in the primary and secondary coils when the transformer is on load. This power is wasted in the form of heat due to the resistance of the coils. This loss is proportional to the sequence of the load hence it is called the Variable loss where as the Iron loss is called as the Constant loss as the supply voltage and frequency are constants.

$$\begin{aligned}\text{Total copper loss} &= {I_1}^2 R_1 + {I_2}^2 R_2 \\ &= {I_1}^2 (R_1 + R_2') \\ &= {I_2}^2 (R_1' + R_2)\end{aligned}$$

As **voltage is constant** copper losses are proportional to the square of kVA rating of transformer.

$$P_{cu} \propto I^2 \propto (\text{kVA})^2$$

Thus for transformer

$$\begin{aligned}\text{Total losses} &= \text{Iron losses} + \text{Copper losses} \\ &= P_i + P_{cu}\end{aligned}$$

Volt-Ampere rating (or) Why rating of Transformer in kVA

It is seen that iron losses depend on the supply voltage while the copper losses depend on the current. The losses are not depending on the phase angle between voltage and current. Hence the rating of transformer is expressed as a product of voltage current and called VA rating of transformer. It is not expressed in watts or kilo watts. Most of the times, rating is expressed in kVA.

2.12 Effect of variations of frequency and supply voltage on Iron losses

The iron losses of the transformer includes two types of losses

i. Eddy current loss
ii. Hysteresis loss

For given volume and thickness of laminations, these losses depend on the operating frequency, maximum flux density and voltage.

$$P_e = K_e {B_m}^2 f^2 t^2$$

$$P_h = K_h {B_m}^{1.67} f\, v$$

$$P_e \propto {B_m}^2 f^2$$

$$P_h \propto {B_m}^{1.67} f$$

We know that for transformer

$$V = 4.44\, f\, N\, \emptyset m = 4.44\, f\, N\, B_m A$$

$$B_m \propto \frac{V}{f}$$

$$\boxed{\begin{gathered} P_e \propto {B_m}^2 f^2 \\ P_h \propto {B_m}^{1.67} f \\ B_m \propto \frac{V}{f} \end{gathered}}$$

Thus voltage changes flux density changes, both eddy current and hysteresis losses will change.

If the transformer is operated with the frequency and voltage changed in the same proportion, the flux density will remain unchanged and apparently the no-load current will also remain unaffected.

The transformer can be operated safely at frequency less than rated one with correspondingly reduced voltage. In this case iron losses will reduced. But if the transformer is operated with increased voltage and frequency in the same proportion, the core losses may increase to an intolerable level. Increase in frequency with constant supply voltage will cause reduction in hysteresis loss and leave the eddy current losses unaffected. Some increase in voltage could, therefore, be tolerated at higher frequencies, but exactly how much depends on the relative magnitude of the hysteresis and eddy current losses and the grade of iron used in the transformer core.

2.13 Efficiency of a Transformer

The efficiency of any device is defined as the ratio of the power output to power input. The efficiency of a transformer at a particular load and power factor is defined as the output divided by the input. It is expressed as η

$$\eta = \frac{\text{Power output}}{\text{Power input}}$$

$$\eta = \frac{\text{Power output}}{\text{Power output} + \text{Total losses}}$$

$$\eta = \frac{\text{Power output}}{\text{Power output} + P_i + P_{cu}}$$

$$\text{Power output} = V_2 I_2 \cos\phi$$

$$\cos\phi = \text{Load power factor}$$

Transformer supplies full load current of I_2 and with terminal voltage V_2

$$P_{cu} = \text{ copper losses on full load} = {I_2}^2 R_{02}$$

$$\eta = \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + P_i + {I_2}^2 R_{02}}$$

$$V_2 I_2 = \text{VA rating of a transformer}$$

$$\%\,\eta = \frac{(\text{VA rating}) * \cos\phi}{(\text{VA rating}) * \cos\phi + P_i + {I_2}^2 R_{02}} * 100$$

$$\boxed{\%\,\eta = \frac{(\text{VA rating}) * \cos\phi}{(\text{VA rating}) * \cos\phi + P_i + {I_2}^2 R_{02}} * 100}$$

This is full load efficiency with, I_2 = full load secondary current

But if the transformer is subjected to fractional load then using the appropriate values of various quantities, the efficiency can be obtained.

$$x = \text{Fraction by which load is less than full load} = \frac{\text{Actual load}}{\text{full load}}$$

When load changes, the load current changes by same proportion.

$$\text{new } I_2 = x(I_2)F.L$$

Similarly the output power also reduces by same fraction.

Similarly as copper losses are proportional to square of current then

$$\text{new } P_{cu} = x^2(P_{cu})F.L$$

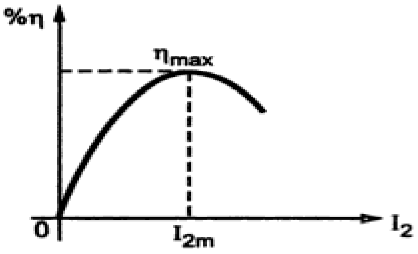
In general for fractional load the efficiency is given by,

$$\%\,\eta = \frac{x(\text{VA rating}) * \cos\phi}{x(\text{VA rating}) * \cos\phi + P_i + x^2(P_{cu})F.L} * 100$$

$$\boxed{\%\,\eta = \frac{x(\text{VA rating}) * \cos\phi}{x(\text{VA rating}) * \cos\phi + P_i + x^2(P_{cu})F.L} * 100}$$

2.14 Condition for maximum efficiency:

In general for the efficiency to be maximum for any device the losses must be minimum. Between the iron and copper losses the iron loss is the fixed loss and the copper loss is the variable loss. When these two losses are equal and also minimum the efficiency will be maximum.



Therefore the condition for maximum efficiency in a transformer is

$$\text{Iron loss} = \text{Copper loss}$$

When transformer works on a constant input voltage and frequency then efficiency varies with the load. As load increases, the efficiency increases. At a certain load current, it achieves a maximum value. If the transformer is loaded further the efficiency starts decreasing. The graph of efficiency against load current I_2 is shown in fig 5.40.

The load current at which the efficiency attains maximum value is denoted as I_{2m} and maximum efficiency is denoted as η_m.

So for maximum efficiency

$$\frac{d\eta}{dI_2} = 0$$

$$\eta = \frac{V_2 I_2 * \cos\phi_2}{V_2 I_2 * \cos\phi_2 + P_i + P_{Cu}}$$

$$\eta = \frac{V_2 I_2 * \cos\phi_2}{V_2 I_2 * \cos\phi_2 + P_i + I_2{}^2 R_{02}}$$

$$\frac{d\eta}{dI_2} = \frac{d}{dI_2}\left[\frac{V_2 I_2 * \cos\phi_2}{V_2 I_2 * \cos\phi_2 + P_i + I_2{}^2 R_{02}}\right] = 0$$

$$\left(V_2 I_2 * \cos\phi_2 + P_i + I_2{}^2 R_{02}\right)\frac{d}{dI_2}(V_2 I_2 * \cos\phi_2)$$

$$- (V_2 I_2 * \cos\phi_2)\frac{d}{dI_2}\left(V_2 I_2 * \cos\phi_2 + P_i + I_2{}^2 R_{02}\right) = 0$$

$$\left(V_2 I_2 * \cos\phi_2 + P_i + I_2{}^2 R_{02}\right)(V_2\cos\phi_2) - (V_2 I_2 * \cos\phi_2)(V_2\cos\phi_2 + 2I_2 R_{02}) = 0$$

$$(V_2\cos\phi_2)\left[V_2 I_2 * \cos\phi_2 + P_i + I_2{}^2 R_{02} - V_2 I_2 \cos\phi_2 - 2I_2{}^2 R_{02}\right] = 0$$

$$\left[V_2 I_2 * \cos\phi_2 + P_i + I_2{}^2 R_{02} - V_2 I_2 \cos\phi_2 - 2I_2{}^2 R_{02}\right] = 0$$

$$P_i - I_2{}^2 R_{02} = 0$$

$$P_i = I_2{}^2 R_{02} = P_{Cu}$$

$$\boxed{\text{Iron loss} = \text{Copper loss}}$$

(i) Load current I_{2m} at maximum efficiency

For η_{max}, $P_i = I_2{}^2 R_{02}$ but $I_2 = I_{2m}$

$$I_{2m}{}^2 R_{02} = P_i$$

$$I_{2m} = \sqrt{\frac{P_i}{R_{02}}}$$

$$I_{2\ F.L} = \text{Full load current}$$

$$I_{2m} = \frac{I_{2\ F.L}}{I_{2\ F.L}}\sqrt{\frac{P_i}{R_{02}}}$$

$$I_{2m} = I_{2\ F.L}\sqrt{\frac{P_i}{I_{2\ F.L}{}^2 R_{02}}}$$

$$I_{2m} = I_{2\ F.L}\sqrt{\frac{P_i}{P_{Cu\ (F.L)}}}$$

$$\boxed{I_{2m} = I_{2\ F.L}\sqrt{\frac{P_i}{P_{Cu\ (F.L)}}}}$$

This is the load current at η_{max} in terms of full load current.

$$\frac{I_{2m}}{I_{2\ F.L}} = \sqrt{\frac{P_i}{P_{Cu\ (F.L)}}} = X$$

$$\boxed{X = \sqrt{\frac{P_i}{P_{Cu\ (F.L)}}}}$$

X is the fraction of load maximum efficiency

(ii) kVA supplied at maximum efficiency

For constant V_2 the kVA supplied is the function of load current.

$$\text{kVA at } \eta_{max} = I_{2m} V_2 = V_2 I_{2\ F.L} \sqrt{\frac{P_i}{P_{Cu\ (F.L)}}}$$

$$\boxed{\text{kVA at } \eta_{max} = (\text{kVA rating}) * \sqrt{\frac{P_i}{P_{Cu\ (F.L)}}}}$$

Substituting condition for η_{max} in the expression of efficiency, we can write expression for η_{max} as,

$$\%\ \eta_{max} = \frac{V_2 I_{2m} \cos\phi}{V_2 I_{2m} \cos\phi + 2P_i} * 100 \qquad \text{as } P_i = P_{Cu}$$

$$\%\ \eta_{max} = \frac{\text{kVA for } \eta_{max} \cos\phi}{\text{kVA for } \eta_{max} \cos\phi + 2P_i} * 100$$

$$\boxed{\%\ \eta_{max} = \frac{\text{kVA for } \eta_{max} \cos\phi}{\text{kVA for } \eta_{max} \cos\phi + 2P_i} * 100}$$

Assignment-Cum-Tutorial Questions

A. Questions testing the remembering / understanding level of students

I) Objective Questions

1) A Transformer will work on

a) A.C only b) D.C only c) A.C as well as D.C d) None of the above

2) The Primary and Secondary of a transformer are ………… coupled

a)electrically b) magnetically c) electrically and magnetically d) None of the above.

3) A transformer is an efficient device because it ………

a) it is a static device b) uses inductive coupling c) uses capacitive coupling

d) uses electrical coupling.

4) The voltage per turn of the primary of transformer is ……. the voltage per turn of the secondary

a) more than b) less than c) the same as d) None of the above

5) The iron core is used to …….. of the transformer.

a) increase the weight b)provide tight magnetic coupling c)reduce core losses d)None of the above

6) The maximum flux produced in a core of a transformer is…..

a) directly proportional to supply frequency

b) Inversely proportional to supply frequency

c) Inversely proportional to primary voltage d) none of the above

7) When the primary of a transformer is connected to a dc supply, …….

a) primary current draws small current

b) primary leakage reactance is increased

c) core losses are increased d) primary may burn out

8) An ideal transformer is one which……

a) has no losses and leakage reactance b) does n't work c) has same no.of primary and secondary d) none of the above

9) The required thickness of lamination in a transformer decreases when

(a) The applied frequency increases (b) The applied frequency decreases

(c) The applied voltage increases (d) The applied voltage decreases

10) Laminated insulations coated with varnish are normally used in the transformer

(a) To reduce reluctance of magnetic path (b) To reduce the effect of eddy current

(c) To increase the reluctance of magnetic path (d) To reduce the hysteresis effect

11) The size and construction of bushings in a transformer depend upon the

(a) Size of winding (b) Size of tank (c) Current flowing (d)Voltage supplied

12) Transformer humming sound is reduced by the

(a) Proper bracing of transformers assemblies (b) Proper insulation

(c) Proper design (d) Proper design of winding

13) The overload capacity of a transformer depends on

(a) ratio of full load copper losses to its iron losses (b) size of the core

(c) frequency (d) none of the above

14) An air core transformer as compared to iron-core transformer has

(a) Less magnetic core loss (b) More magnetic core loss

(c) No magnetic core loss (d) Less ohmic loss

II) Descriptive Questions

1) Explain the working principle of Single Phase Transformer?
2) Explain the constructional features of different types of single phase transformers?
3) Derive an EMF Equation of a Single Phase Transformer.
4) Explain the operation of a single phase transformer with inductive load by drawing the phasor diagram?
5) What are the different losses occurred in a transformer on load? Explain how each loss varies with load current, supply voltage and frequency? How these losses are minimized?
6) Derive an expression for voltage regulation of a single phase transformer from its equivalent circuit or phasor diagram?
7) Derive the condition for maximum efficiency of a single phase transformer?

B. Question testing the ability of students in applying the concepts.

I) Objective Questions

1) The low voltage winding of a 400/230 volt, 1-phase, 50Hz transformer is to be connected to a 25Hz supply in order to keep the magnetization current at the same level as that for normal 50Hz supply at 25Hz the voltage should be……

(a) 230V (b) 460V (c) 115V (d) 65V

2) If 90 per cent of norm al voltage and 90 percent of norm al frequency are applied to a transformer, the percent charge in hysteresis losses will be

(a) 20% (b) 4.7% (c) 19% (d) 21%

3) If 110 per cent of norm al voltage and 110 per cent of normal frequency is applied to a transformer, the percentage change of eddy current losses will be

(a) 10% (b) 20% (c) 25% (d) 21%

4) A transformer has two 2,400 V primary coils and two 240 V coils. By proper connection of the windings, the transformation ratio that can be obtained is

(a) 10 (b) 5 (c) 20 (d) 9

5) A single-phase, 2,200/200 V transformer takes 1 A at the HT side or no load at a power factor of 0.385 lagging. The iron losses are

(a)167 W (b) 77 W (c) 88 W (d) 98 W

6) Neglecting resistance, at constant flux density, the power required per kilogram to magnetize the iron core of a transformer is 0.8 W at 25 Hz and 2.04 W at 60 Hz. The power required per kilogram for 100 Hz is

(a)3.8 W (b) 3.63 W (c) 3.4 W (d) 5.2 W

7) The full load copper loss of a transformer is 1600W. At half-load the copper loss will be

(a) 6400W (b)1600W (c) 800W (d) 400W

II) Descriptive Questions

1) A 2000/200v, 20 kVA transformer has 66 turns in the secondary. Calculate (i)primary turns. (ii)Primary and Secondary full load currents. Neglect losses.

2) A single Phase 50hz transformer has 20 primary turns and 273 secondary turns. The net cross sectional area of core is 400cm2. If the primary winding is connected to 230V supply, find (i) peak value of flux density in the core (ii) Voltage induced in the secondary winding.

3) A transformer takes a current of 0.6A and absorbs 64W when primary is connected to its normal supply of 200, 50Hz, the secondary being open circuited. Find the magnetizing and iron loss currents.

4) A Single phase transformer on no-load takes 4.5A at a power factor of 0.25 lagging when connected to a 230v, 50 Hz Supply. The number of turns of the primary winding is 250. Calculate (i) the magnetizing current (ii) the maximum value of flux in the core.

5) In a 50KVA transformer, the iron loss is 500W and full load copper loss is 800KW. Find the efficiency at full load and half full load at 0.8 lagging.

6) A 40KVA t/f has iron loss of 450KW and full load copper loss of 850KW. If the power factor of the load is 0.8 lagging, calculate (i) full laod efficiency (ii) the load at which the maximum efficiency occurs and (iii) the maximum efficiency.

7) A 440/110V transformer has a primary resistance of 0.032 ohms and secondary resistance of 0.02 ohms. Its iron loss at normal inputs of 150W. Determine secondary current at which maximum efficiency will occur and the value of this maximum efficiency at a unity pf load?

8) The primary and secondary windings of a 40KVA, 6600/250 v single phase transformer have a resistances of 10 ohms and 0.02 ohms respectively. The leakage reactance of transformer referred to the primary side is 35 ohms. Calculate the percentage voltage regulation of the transformer when supplying full load current at a p.f of 0.8 lagging.

C. Questions testing the analyzing / evaluating /Creative ability of students

1. A 100KVA transformer has 400 turns on the primary and 80 turns on the secondary. The primary and secondary resistances are 0.3 ohms ans 0.1 ohms respectively and the corresponding reactances are 1.1 and 0.035 ohms respectively. The supply voltage is 220V. Calculate the voltage regulation and secondary terminal voltage for full load having a p.f of (i) 0.8 lagging (ii)0.8 leading .

D.Previous GATE/IES Questions.

1. In a transformer, zero voltage regulation at full load is **GATE-2010**

(A) not possible (B) possible at unity power factor load

(C) possible at leading power factor load (D) possible at lagging power factor load

2. A single-phase, 50 kVA, 250 V/500 V two winding transformer has an efficiency of 95% at full load, unity power factor. If it is re-configured as a 500 V/750 V auto-transformer, its efficiency at its new rated load at unity power factor will be **GATE-2012**

(A) 95.752% (B) 97.851% (C) 98.276% (D) 99.241%

3. A simple phase transformer has a maximum efficiency of 90% at full load and unity power factor. Efficiency at half load at the same power factor is **GATE-2013**

(A) 86.7% (B) 88.26% (C) 88.9% (D) 87.8%

4. The core flux of a practical transformer with a resistive load **GATE-2014**

(A) is strictly constant with load changes (B) increases linearly with load

(C) increases as the square root of the load (D) decreases with increased load

5. In the protection of transformers, harmonic restraint is used to guard against **GATE-2015**

(A) magnetizing inrush current (B) unbalanced operation

(C) lightning (D) switching over-voltages

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Department of Electrical and Electronics Engineering

HANDOUT

on

ELEMENTS OF ELECTRICAL& ELECTRONICS ENGINEERING

Unit –II

DC MACHINES

Objectives:

- To familiarize the students with the constructional details of DC machine. working principles of D.C. generators and motors
- To familiarize the students with the working principles of D.C. generators and motors .

Syllabus: DC Machines: Operating principle of DC Generator – EMF equation – types of DC generators –DC motor operating principle, types of DC motors –torque equation – applications – three point starter.

Learning Outcomes:

Student will be able to

1. Demonstrate the knowledge and understanding of the fundamental principles of electromagnetism.
2. Evaluate the generated EMF of a DC generator
3. Describe the working of a dc machine for generating and motoring action

Learning Material

Unit –II
DC Machines

Construction of d.c. generator

The d.c. generators and d.c. motors have the same general construction. In fact, when the machine is being assembled, the workmen usually do not know whether it is a d.c. generator or motor. Any d.c. generator can be run as a d.c. motor and vice-versa.

All d.c. machines have five principal components viz.,

(i) field system
(ii) armature core
(iii) armature winding
(iv) commutator
(v) brushes [See Fig. 2.1].

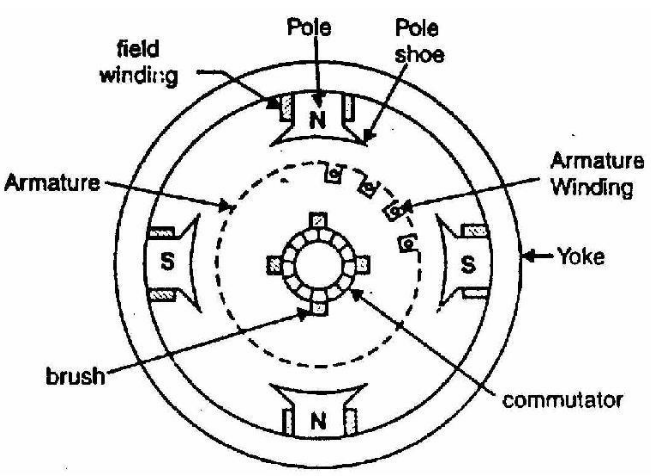


Fig. (2.1)

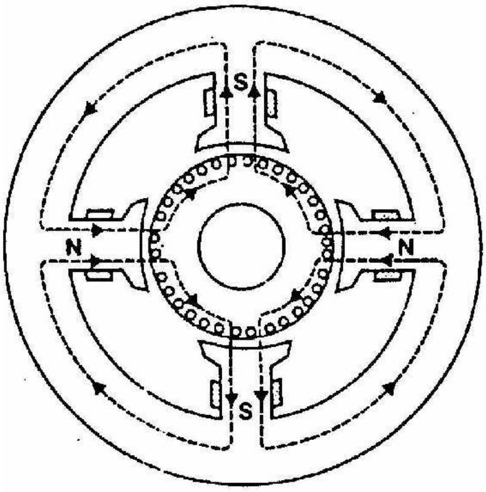


Fig. (2.2)

(i) Field system

- The function of the field system is to produce uniform magnetic field within which the armature rotates. It consists of a number of salient poles (of course, even number) bolted to the inside of circular frame (generally called yoke).
- The yoke is usually made of solid cast steel whereas the pole pieces are composed of stacked laminations. Field coils are mounted on the poles and carry the d.c. exciting current.
- The field coils are connected in such a way that adjacent poles have opposite polarity.
- The m.m.f. developed by the field coils produces a magnetic flux that passes through the pole pieces, the air gap, the armature and the frame (See Fig. 2.2).
- Practical d.c. machines have air gaps ranging from 0.5 mm to 1.5 mm. Since armature and field systems are composed of materials that have high permeability, most of the m.m.f. of field coils is required to set up

flux in the air gap. By reducing the length of air gap, we can reduce the size of field coils (i.e. number of turns).

(ii) Armature core

- The armature core is keyed to the machine shaft and rotates between the field poles. It consists of slotted soft-iron laminations (about 0.4 to 0.6 mm thick) that are stacked to form a cylindrical core as shown in Fig (2.3).
- The laminations (See Fig. 2.4) are individually coated with a thin insulating film so that they do not come in electrical contact with each other.
- The purpose of laminating the core is to reduce the eddy current loss. The laminations are slotted to accommodate and provide mechanical security to the armature winding and to give shorter air gap for the flux to cross between the pole face and the armature “teeth”.

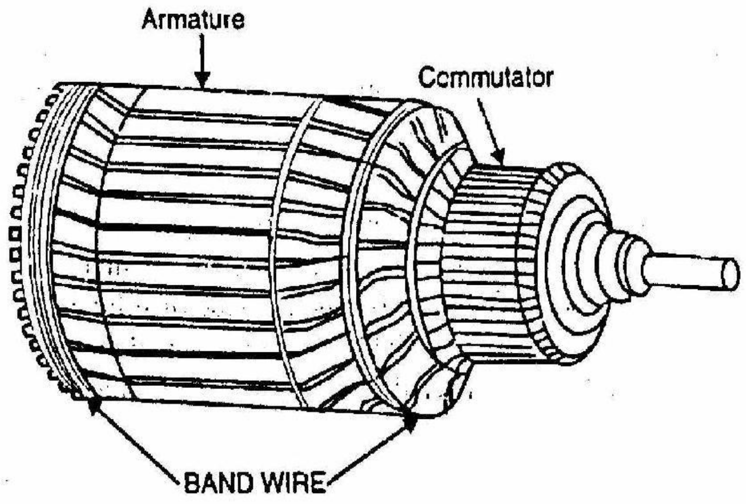


Fig. (2.3)

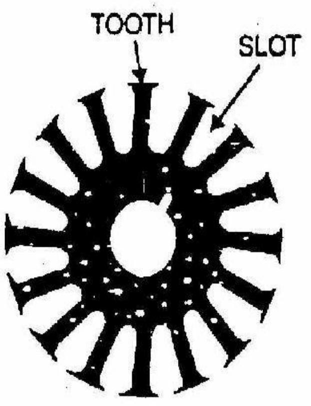


Fig. (2.4)

(iii) Armature winding

- The slots of the armature core hold insulated conductors that are connected in a suitable manner. This is known as armature winding. This is the winding in which “working” e.m.f. is induced.
- The armature conductors are connected in series-parallel; the conductors being connected in series so as to increase the voltage and in parallel paths so as to increase the current.
- The armature winding of a d.c. machine is a closed-circuit winding; the conductors being connected in a symmetrical manner forming a closed loop or series of closed loops.

(iv) Commutator

- A commutator is a mechanical rectifier which converts the alternating voltage generated in the armature winding into direct voltage across the brushes.
- The commutator is made of copper segments insulated from each other by mica sheets and mounted on the shaft of the machine (See Fig 2.5).
- The armature conductors are soldered to the commutator segments in a suitable manner to give rise to the armature winding. Depending upon the manner in which the armature conductors are connected to the commutator segments, there are two types of armature winding in a d.c. machine viz., (a) lap winding (b) wave winding.
- Great care is taken in building the commutator because any eccentricity will cause the brushes to bounce, producing unacceptable sparking.
- The sparks may bum the brushes and overheat and carbonize the commutator.

(v) Brushes

- The purpose of brushes is to ensure electrical connections between the rotating commutator and stationary external load circuit.
- The brushes are made of carbon and rest on the commutator. The brush pressure is adjusted by means of adjustable springs (See Fig. 2.6). If the brush pressure is very large, the friction produces heating of the commutator and the brushes.
- On the other hand, if it is too weak, the imperfect contact with the commutator may produce sparking.

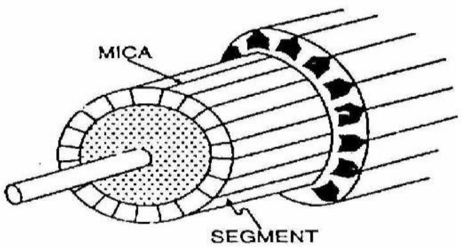


Fig. (2.5)

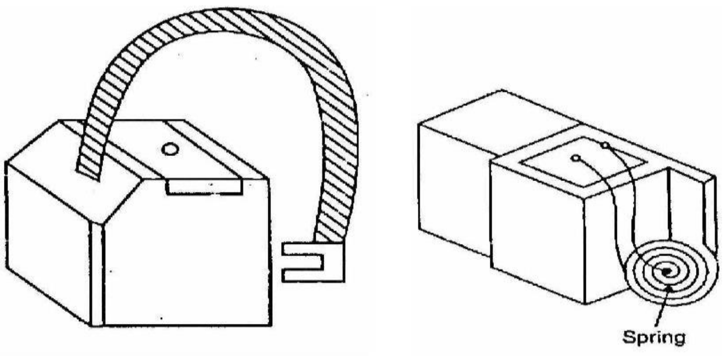


Fig. (2.6)

- Multi pole machines have as many brushes as they have poles. For example, a 4-pole machine has 4 brushes. As we go round the commutator, the successive brushes have positive and negative polarities.

- Brushes having the same polarity are connected together so that we have two terminals viz., the +ve terminal and the - ve terminal.

Generator Principle

An electric generator is a machine that converts mechanical energy into electrical energy. An electric generator is based on the principle that whenever flux is cut by a conductor, an e.m.f. is induced which will cause a current to flow if the conductor circuit is closed. The direction of induced e.m.f. (and hence current) is given by Fleming's right hand rule. Therefore, the essential components of a generator are:

(a) a magnetic field
(b) conductor or a group of conductors
(c) Motion of conductor w.r.t. magnetic field.

Simple Loop Generator

Consider a single turn loop ABCD rotating clockwise in a uniform magnetic field with a constant speed as shown in Fig.(2.7). As the loop rotates, the flux linking the coil sides AB and CD changes continuously. Hence the e.m.f. induced in these coil sides also changes but the e.m.f. induced in one coil side adds to that induced in the other.

(i) When the loop is in position no. 1 [See Fig. 2.7], the generated e.m.f. is zero because the coil sides (AB and CD) are cutting no flux but are moving parallel to it
(ii) When the loop is in position no. 2, the coil sides are moving at an angle to the flux and, therefore, a low e.m.f. is generated as indicated by point 2 in Fig. (2.8).
(iii) When the loop is in position no. 3, the coil sides (AB and CD) are at right angle to the flux and are, therefore, cutting the flux at a maximum rate. Hence at this instant, the generated e.m.f. is maximum as indicated by point 3 in Fig. (2.8).
(iv) At position 4, the generated e.m.f. is less because the coil sides are cutting the flux at an angle.
(v) At position 5, no magnetic lines are cut and hence induced e.m.f. is zero as indicated by point 5 in Fig. (2.8).
(vi) At position 6, the coil sides move under a pole of opposite polarity and hence the direction of generated e.m.f. is reversed. The maximum e.m.f. in this direction (i.e., reverse direction, See Fig. 2.8) will be when the loop is at position 7 and zero when at position 1. This cycle repeats with each revolution of the coil.

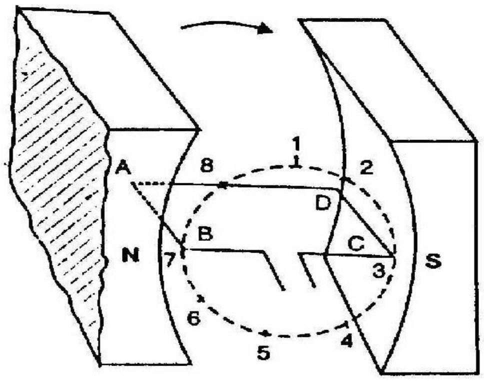


Fig. (2.7)

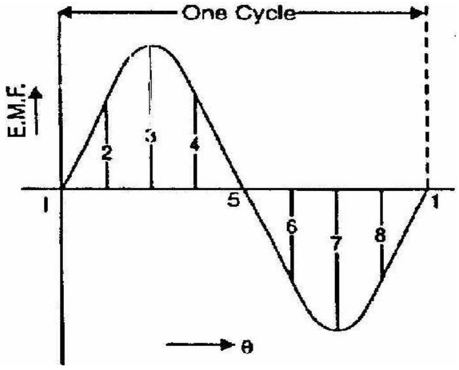


Fig. (2.8)

- Note that e.m.f. generated in the loop is alternating one. It is because any coil side; say AB has e.m.f. in one direction when under the influence of N-pole and in the other direction when under the influence of S-pole.
- If a load is connected across the ends of the loop, then alternating current will flow through the load. The alternating voltage generated in the loop can be converted into direct voltage by a device called commutator. In fact, a commutator is a mechanical Rectifier.

E.M.F. Equation of a D.C. Generator

Let Ø= flux/pole in Wb
Z = total number of armature conductors P =
number of poles
A = number of parallel paths
= 2 ... for wave winding
= P ... for lap winding
N = speed of armature in r.p.m.
E_g = e.m.f. of the generator = e.m.f./parallel path
Flux cut by one conductor in one revolution of the armature,
dØ = PØ webers
Time taken to complete one revolution, dt = 60/N second
e.m.f/conductor = dØ / dt = $\frac{\emptyset PN}{60}$
e.m.f. of generator,
E_g = e.m.f. per parallel path
= (e.m.f/conductor) * No. of conductors in series per parallel path

$$E_g = \frac{\emptyset PNZ}{60A}$$

Types of D.C. Generators

The magnetic field in a d.c. generator is normally produced by electromagnets rather than permanent magnets. Generators are generally classified according to their methods of field excitation. On this basis, d.c. generators are divided into the following two classes:

(i) Separately excited d.c. generators
(ii) Self-excited d.c. generators

The behaviour of a d.c. generator on load depends upon the method of field excitation adopted.

Separately Excited D.C. Generators

- A d.c. generator whose field magnet winding is supplied from an independent external d.c. source (e.g., a battery etc.) is called a separately excited generator. Fig. (2.9) shows the connections of a separately excited generator.
- The voltage output depends upon the speed of rotation of armature and the field current ($E_g = P\Phi ZN/60\ A$). The greater the speed and field current, greater is the generated e.m.f.
- It may be noted that separately excited d.c. generators are rarely used in practice. The d.c. generators are normally of self-excited type.

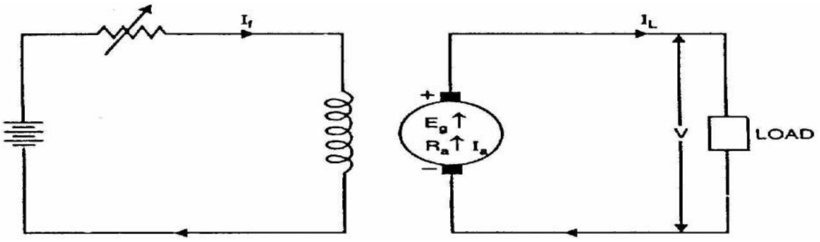


Fig. (2.9)

Armature current, $I_a = I_L$

Terminal voltage, $V = E_g - I_a R_a$

Electric power developed = $E_g I_a$

Power delivered to load = $E_g I_a - I_a^2 R_a$

Self-Excited D.C. Generators

- A d.c. generator whose field magnet winding is supplied current from the output of the generator itself is called a self-excited generator.

- There are three types of self-excited generators depending upon the manner in which the field winding is connected to the armature, namely
  (i) Series generator
  (ii) Shunt generator
  (iii) Compound generator

(i) Series generator

- In a series wound generator, the field winding is connected in series with armature winding so that whole armature current flows through the field winding as well as the load.
- Fig. (2.10) shows the connections of a series wound generator. Since the field winding carries the whole of load current, it has a few turns of thick wire having low resistance. Series generators are rarely used except for special purposes e.g., as boosters.

Armature current, $I_a = I_{se} = I_L = I$(say)

Terminal voltage, $V = E_G - I(R_a + R_{se})$

Power developed in armature = E_gI_a

Power delivered to load= $E_gI_a - I^2{}_a(R_a + R_{se}) = I_a[E_g - I_a(R_a + R_{se})$

$$= VI_a or VI_L$$

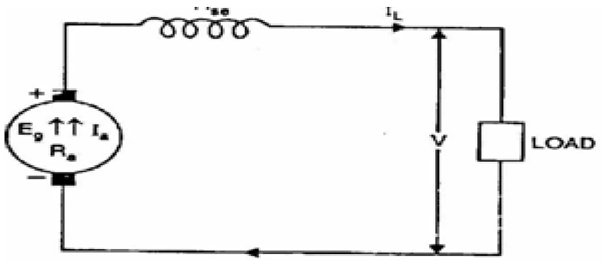


Fig. (2.10)

(ii) Shunt generator

- In a shunt generator, the field winding is connected in parallel with the armature winding so that terminal voltage of the generator is applied across it.
- The shunt field winding has many turns of fine wire having high resistance. Therefore, only a part of armature current flows through shunt field winding and the rest flows through the load.
- Fig. (2.11) shows the connections of a shunt-wound generator.

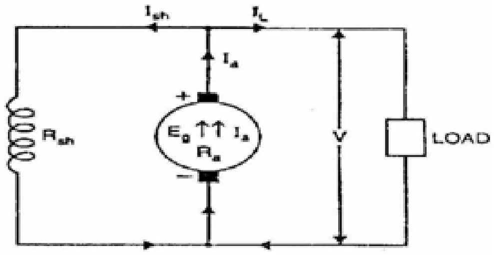


Fig. (2.11)

Shunt field current, $I_{sh} = V/R_{sh}$
Armature current, $I_a = I_L + I_{sh}$
Terminal voltage, $V = E_g - I_aR_a$
Power developed in armature = E_gI_a
Power delivered to load = VI

(iii) Compound generator

- In a compound-wound generator, there are two sets of field windings on each pole one is in series and the other in parallel with the armature. A compound wound generator may be:

(a) Short Shunt in which only shunt field winding is in parallel with the armature winding [See Fig. 2.12(i)].

(b) Long Shunt in which shunt field winding is in parallel with both series field and armature winding [See Fig. 2.12 (ii)].

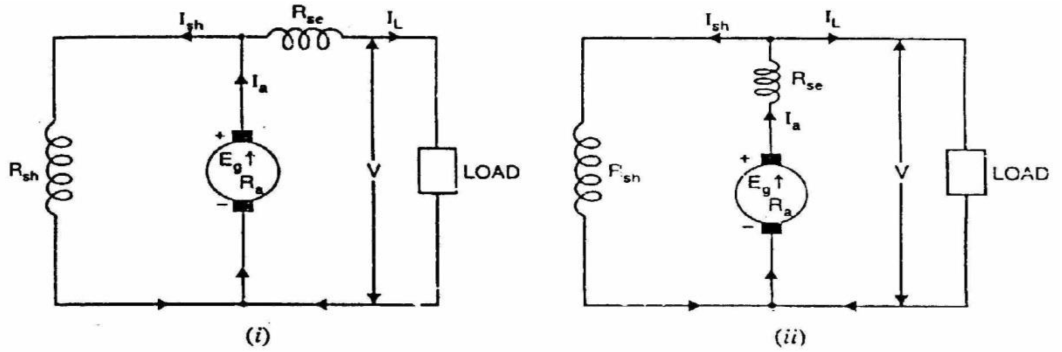


Fig. (2.12)

Short shunt Generator

Series field current, $I_{se} = I_L$
Shunt field current, Ish = $(E_g - I_aR_a)/R_{sh}$
Terminal voltage, $V = E_g - I_aR_a - I_{se}R_{se}$
Power developed in armature = E_gI_a
Power delivered to load = VI_L

Long shunt Generator

Series field current, $I_{se} = I_a = I_L + I_{sh}$

Shunt field current, $I_{sh} = V/R_{sh}$

Terminal voltage, $V = E_g - I_a (R_a + R_{se})$

Power developed in armature = $E_g I_a$

Power delivered to load = VI_L

Power delivered to load = VI_L

Depends on the flux directions of series and shunt windings, compound machines are classified as

(i) Cumulative compound machine:

Here both series and shunt field fluxes are adding each other

$$\therefore \Phi_{net} = \Phi_{sh} + \Phi_{se}$$

(ii) Differential compound:

Here series field and shunt field fluxes are opposing with each other

$$\therefore \Phi_{net} = \Phi_{sh} \sim \Phi_{se}$$

Principle of Operation of dc motor:

- DC motor operates on the principle that when a current carrying is placed in a magnetic field, it experiences a mechanical force give n by F = BIL newton. Where 'B' = flux density in wb/m^2', 'I' is the current and 'L' is the length of the conductor.
- The direction of force can be found by Fleming's left hand rule. Constructionally, there is no difference between a DC generator and DC motor.

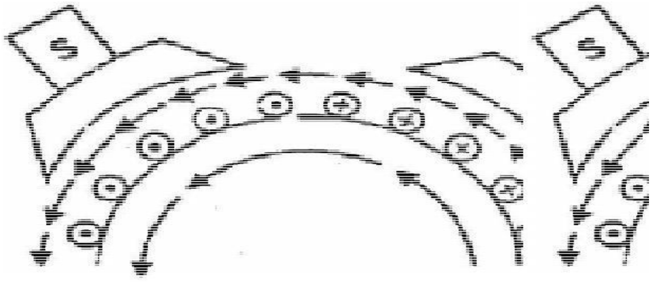


Figure 2.13

- Figure 2.13 shows a multipolar DC motor. Armature conductors are carrying current downwards under North Pole and upwards under South Pole.
- When the field coils are excited, with current carrying armature conductors, a force is experienced by each armature conductor whose direction can be found by Fleming's left hand rule.

- This is shown by arrows on top of the conductors. The collective force produces a driving torque which sets the armature into rotation.
- The function of a commutator in DC motor is to provide a continuous and unidirectional torque.
- In DC generator the work done in overcoming the magnetic drag is converted into electrical energy.
- Conversion of energy from electrical form to mechanical form by a DC motor takes place by the work done in overcoming the opposition which is called the 'back emf'.

Back Emf:

- Back emf is the dynamically induced emf in the armature conductors of a dc motor when the armature is rotated. The direction of the induced emf as found by Flemings right hand rule is in opposition to the applied voltage. Its value is same as that of the induced emf in a DC generator i.e. is

$$E_b = \frac{\Phi ZNP}{60A}\ Volts$$

- This emf is called as back emf E_b'The work done in overcoming this opposition is converted into mechanical energy.

Significance of back Emf:

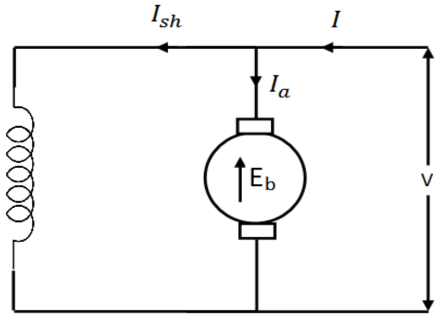


Fig.2.14

- Figure 2.14 shows a DC shunt motor. The rotating armature generating the back emfE_b' is like a battery of emf E_b'connected across a supply voltage of 'V' volts.

$$I_a = \frac{V - E_b}{r_a} \text{ where } r_a = armature\ resistance.$$

$$E_b = \frac{\phi ZNP}{60A} \text{Volts. } E_b \propto N.$$

- If E_b is large, armature current will be less and vice versa. Hence E_b acts like a governor i.e., it makes the motor self- regulating so that it draws as much current as required by the motor.

voltage equation of a motor:

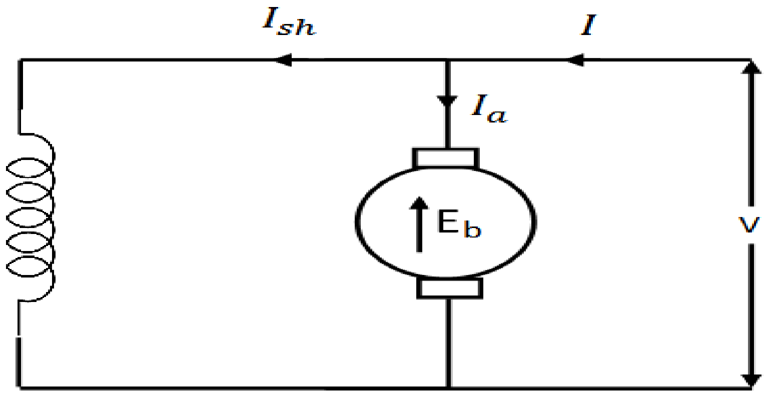


Figure 2.15

The voltage 'V' applied across the motor armature has to

i) Overcome the back emf E_b and
ii) Supply the armature ohmic drop $I_a r_a$

Hence $V = E_b + I_a r_a$.....1 This is known as voltage equation of DC motor.

Multiplying both sides of voltage equation by I_a

$VI_a = E_b I_a + I_a^2 r_a$.......2

$VI_a = electrical\ input\ to\ the\ armature.$

$E_b I_a = P_m = electrical\ equivalent\ of\ mechanical\ power\ developed\ in\ the\ armature.$

$I_a^2 r_a = armature\ copper\ loss$

Torque Equation

- The turning or twisting moment of a force about an axis is called torque. It is measured by the product of the force and the radius at which this force acts.

- Consider a pulley of radius meter acted upon by a circumferential force of newton which causes it to rotate at rpm.

Then torque $T = F \times r$ newton-metre(N-m

Work done by this force in one revolution

=Force $\times$ distance

$= F \times 2\pi r$ joule

Power developed $= F \times 2\pi r \times N$ joule/second or watt $= (F \times r) \times 2\pi N$ watt

Now, $2\pi N =$ angular velocity ω in radian per second and $F \times r =$torque T

Hence, power developed $= T \times \omega$ watt or $P = T\omega$ watt

Moreover, if N is in rpm, then

$\omega = 2\pi N / 60$ rad/s

Hence, $P = \dfrac{2\pi N}{60} \times T$ or $P = \dfrac{2\pi}{60} NT = \dfrac{NT}{9.55}$

Armature torque of a motor

Let Ta be the torque developed by the motor running at Nr.p.s

Power developed=Ta*2πN Watt

We also know that electrical power converted into mechanical power in the armature $= E_b I_a$ watt.

Comparing above equations, we get $T_a \times 2\pi N = E_b I_a$

After simplification, if N in rps, $T_a = \dfrac{E_b I_a}{2\pi N}$

If N is in rpm, then $T_a = 9.55 \dfrac{E_b I_a}{N}$ N-m

Also, $T_a = 0.159 \phi Z I_a \times (P/A)$ N-m

Shaft torque

The whole of the armature torque, as calculated above, is not available for doing useful work, because of iron and friction losses in the motor. The torque which is available for doing useful work is known as shaft torque T_{sh}. The motor output is given by

Output $= T_{sh} \times 2\pi N$ watt provided T_{sh} is in N-m and N in rps.

Let T_a be the torque developed by the armature of a motor running at N rps. If T_a is in N-m, then

Hence, $T_{sh} = \dfrac{Output\_in\_watts}{2\pi N}$, if N is in rps

And, if N is in rpm, then $T_{sh} = \dfrac{Output\_in\_watts}{2\pi N/60} = 9.55 \dfrac{Output}{N}$

Types of Motors

1.Separately excited machine

- The armature and field winding are electrically separate from each other.
- The field winding is excited by a separate DC source.

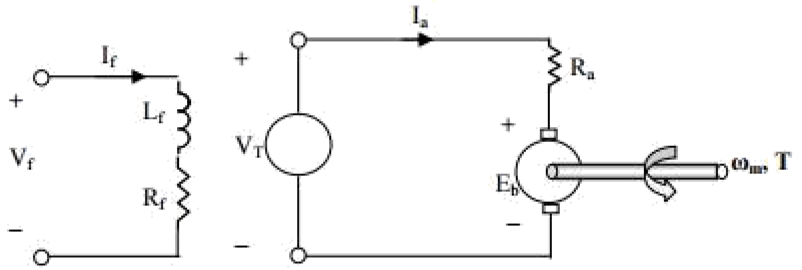


Fig2.16 Separately excited dc motor

The voltage and power equations for this machine are same as those derived in the previous section. Note that the total input power = Vf If + VT Ia

2. Self excited machines

In these machines, instead of a separate voltage source, the field winding is connected across the main voltage terminals.

1.Shunt motor

- The armature and field winding are connected in parallel.
- The armature voltage and field voltage are the same.

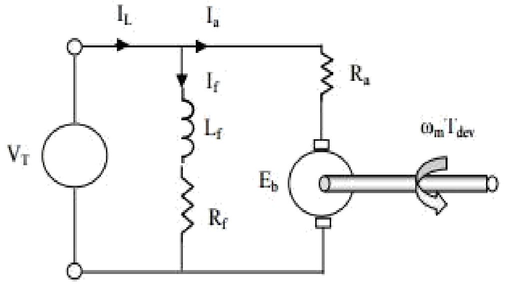


Fig 2.17 shunt motor

Total current drawn from the supply, IL = If + Ia

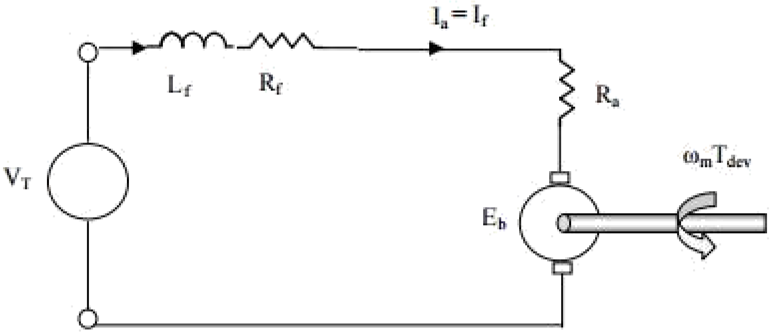
Total input power = VT IL

Series DC machine

The field winding and armature winding are connected in series.

- The field winding carries the same current as the armature winding.

A series wound motor is also called a universal motor. It is universal in the sense that it will run equally well using either an ac or a dc voltage source.



2.18 Series motor

Compound DC machine

If both series and shunt field windings are used, the motor is said to be compounded. In a compound machine, the series field winding is connected in series with the armature, and the shunt field winding is connected in parallel. Two types of arrangements are possible in compound motors:

- Cumulative compounding - If the magnetic fluxes produced by both series and shunt field windings are in the same direction (i.e., additive), the machine is called cumulative compound.
- Differential compounding - If the two fluxes are in opposition, the machine is differential compound.

In both these types, the connection can be either short shunt or long shunt.

Three Point Starter:

- It consists of resistances arranged in steps,R1 to R5 connected in series with the armature of the shunt motor. Field winding is connected across the supply through a protective device called 'NO – Volt Coil'. Another protection given to the motor in this starter is 'over load release coil'. The arrangement is shown in Figure 2.19

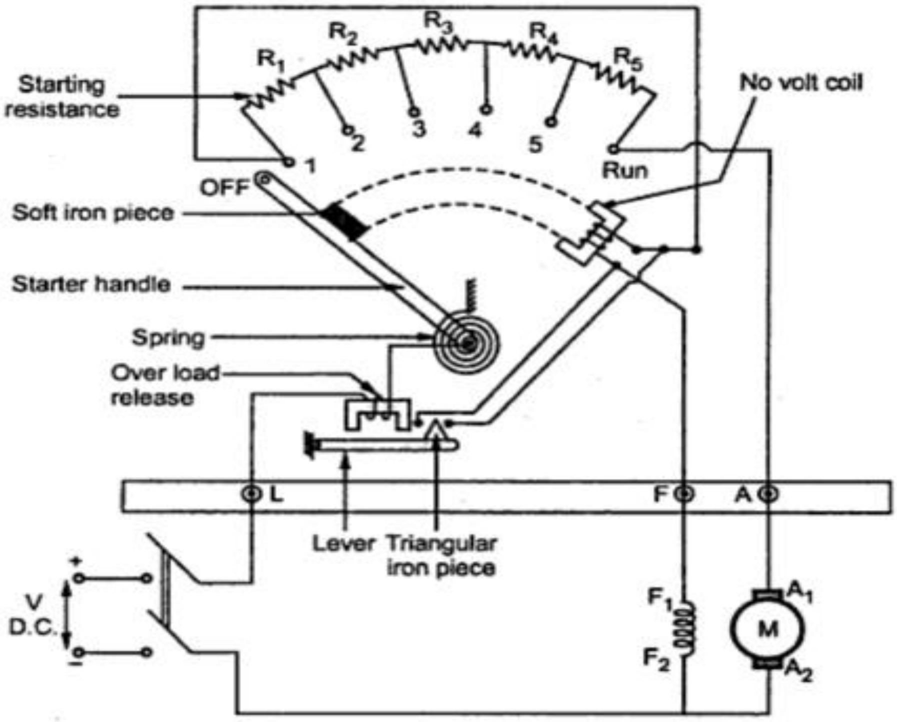


Figure 2.19 Three Point Starter

- To start the motor the starter handle is moved from OFF position to run position gradually against the tension of a hinged spring. An iron piece is attached to the starter handle which is kept hold by the No-volt coil at Run position.
- The function of No volt coil is to get de-energized and release the handle when there is failure or disconnection or a break in the field circuit so that on restoration of supply, armature of the motor will not be connected across the lines without starter resistance.
- If the motor is over loaded beyond a certain predetermined value, then the electromagnet of overload release will exert a force enough to attract the lever which short circuits the electromagnet of No volt coil. Short circuiting of No volt coil results in deenergization of it and hence the starter handle will be released and return to its off position due to the tension of the spring.
- In this type of starter, the shunt field current has to flow back through the starter resistance thus decreasing the shunt field current. This can be avoided by placing a brass arc on which the handle moves as shown in Figure

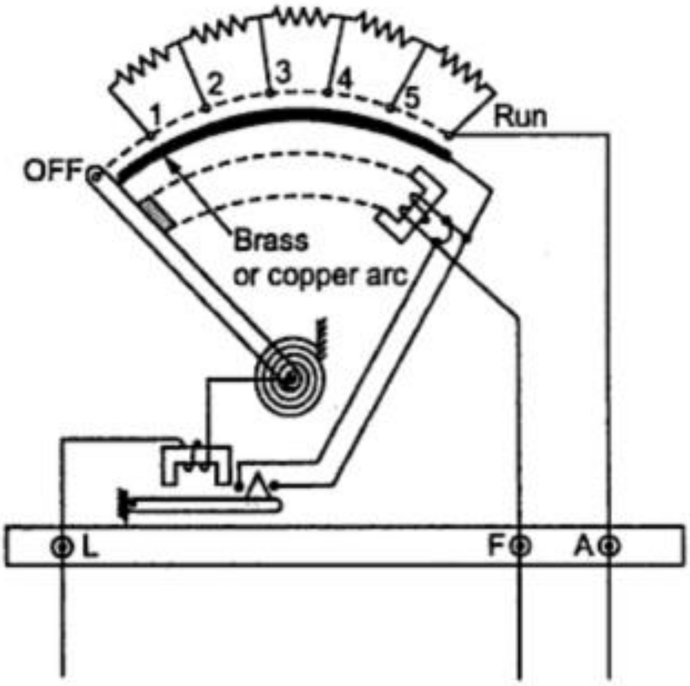


Figure 2.20

Construction of 3 Point Starter

Construction wise a starter is a variable resistance, integrated into number of sections as shown in the figure beside. The contact points of these sections are called studs and are shown separately as OFF, 1, 2,3,4,5, RUN. Other than that there are 3 main points, referred to as

1. 'L' Line terminal. (Connected to positive of supply.)
2. 'A' Armature terminal. (Connected to the armature winding.)
3. 'F' Field terminal. (Connected to the field winding.)

And from there it gets the name 3 point starter.

- Now studying the construction of 3 point starter in further details reveals that, the point 'L' is connected to an electromagnet called overload release (OLR) as shown in the figure. The other end of 'OLR' is connected to the lower end of conducting lever of starter handle where a spring is also attached with it and the starter handle contains also a soft iron piece housed on it.
- This handle is free to move to the other side RUN against the force of the spring. This spring brings back the handle to its original OFF position under the influence of its own force.
- Another parallel path is derived from the stud '1', given to the another electromagnet called No Volt Coil (NVC) which is further connected to terminal 'F'.

- The starting resistance at starting is entirely in series with the armature. The OLR and NVC acts as the two protecting devices of the starter.

Working of Three Point Starter

- Having studied its construction, let us now go into the working of the 3 point starter. To start with the handle is in the OFF position when the supply to the DC motor is switched on.
- Then handle is slowly moved against the spring force to make a contact with stud No. 1. At this point, field winding of the shunt or the compound motor gets supply through the parallel path provided to starting resistance, through No Voltage Coil.
- While entire starting resistance comes in series with the armature. The high starting armature current thus gets limited as the current equation at this stage becomes $Ia = E/(Ra+Rst)$.
- As the handle is moved further, it goes on making contact with studs 2, 3, 4 etc., thus gradually cutting off the series resistance from the armature circuit as the motor gathers speed.
- Finally when the starter handle is in 'RUN' position, the entire starting resistance is eliminated and the motor runs with normal speed.This is because back emf is developed consequently with speed to counter the supply voltage and reduce the armature current.
- So the external electrical resistance is not required anymore, and is removed for optimum operation. The handle is moved manually from OFF to the RUN position with development of speed.

Working of No Voltage Coil of 3 Point Starter

- The supply to the field winding is derived through no voltage coil. So when field current flows, the NVC is magnetized.
- Now when the handle is in the 'RUN' position, soft iron piece connected to the handle and gets attracted by the magnetic force produced by NVC, because of flow of current through it.
- The NVC is designed in such a way that it holds the handle in 'RUN' position against the force of the spring as long as supply is given to the motor. Thus NVC holds the handle in the 'RUN' position and hence also called hold on coil.Now when there is any kind of supply failure, the current flow through NVC is affected and it immediately looses its magnetic property and is unable to keep the soft iron piece on the handle, attracted.
- At this point under the action of the spring force, the handle comes back to OFF position, opening the circuit and thus switching off the motor.

- So due to the combination of NVC and the spring, the starter handle always comes back to OFF position whenever there is any supply problems. Thus it also acts as a protective device safeguarding the motor from any kind of abnormality.

Working of over load coil of 3 Point Starter

- If any fault occurs on motor or overload, it will draw extreme current from the source. This current raise the ampere turns of OLR coil (over load relay) and pull the armature Coil, in consequence short circuiting the NVR coil (No volt relay coil).
- The NVR coil gets demagnetized and handle comes to the rest position under the influence of spring. Therefore the motor disconnected from the supply automatically.

Drawback of three point starter:

- The use of a three point starter presents a problem. The speed of the motor is controlled by means of the field rheostat. To increase the speed of motor necessitates the setting of the field rheostat to higher resistance value.
- The current through the shunt field is reduced, and so is the current through the coil of the holding electromagnet. The reduced current through the coil weakens the strength of magnet and makes susceptible to line voltage variations.
- In the weakened condition a slight reduction in line voltage would further weaken the holding magnet, releasing the arm of the starter and thus disconnecting the motor from the line. Unscheduled stoppages of the motor make the three point starter quite unpopular.

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Department of Electricaland Electronics Engineering

HANDOUT

on

ELEMENTS OF ELECTRICAL ENGINEERING

UNIT –III

Three Phase Induction Motors

Objectives:

1. To familiarize the students with the constructional details and working principle of Three Phase Induction Motors
2. To familiarize the students with Torque –slip Characteristics
3. To familiarize the students with different types of starters like auto transformer starter,DOL starter etc.,

Syllabus:

Principle of operation of three phase induction motors-Slip ring and squirrel cage motors, Slip-Torque characteristics- Efficiency calculation.

Learning Outcomes:

After the completion of this unit, students will be to

5. Explain the various types of Induction motors.
6. Describe the working of a Three Phase Induction motor.
7. Draw the Torque –slip characteristics.
8. Explain different starting methods of Three phase induction motor.

Learning Material

Three Phase Induction Motors

INTRODUCTION

An **induction motor** (IM) is a type of asynchronous AC motor where power is supplied to the rotating device by means of electromagnetic induction.

The induction motor with a wrapped rotor was invented by Nikola Tesla Nikola Teslain1882inFrancebuttheinitialpatentwasissuedin1888afterTeslahadmovedto theUnitedStates.Inhisscientificwork,Teslalaidthefoundationsforunderstandingthe way the motor operates. The induction motor with a cage was invented by Mikhail Dolivo-DobrovolskyaboutayearlaterinEurope.Technologicaldevelopmentinthefield hasimprovedtowherea100hp(74.6kW)motorfrom1976takesthesamevolumeashp (5.5 kW) motor did in 1897. Currently, the most common induction motor is the cage rotormotor.

An electric motor converts electrical power to mechanical power in its rotor (rotating part). There are several ways to supply power to the rotor. In a DC motor this powerissuppliedtothearmaturedirectlyfromaDCsource,whileinaninductionmotor this power is induced in the rotating device. An induction motor is sometimes called a *rotatingtransformer*becausethestator(stationarypart)isessentiallytheprimarysideof the transformer and the rotor (rotating part) is the secondary side. Induction motors are widely used, especially polyphase induction motors, which are frequently used in industrialdrives.

Induction motors are now the preferred choice for industrial motors due to their rugged construction, absence of brushes(which are required inmost DC motors) and the ability to control the speed of the motor.

CONSTRUCTION

A typical motor consists of two parts namely stator and rotor like other type ofmotors.

1. An outside stationary stator having coils supplied with AC current to produce a rotating magnetic field,

2. An inside rotor attached to the output shaft that is given a torque by the rotating field.

Stator:

Stator of an Induction Machine

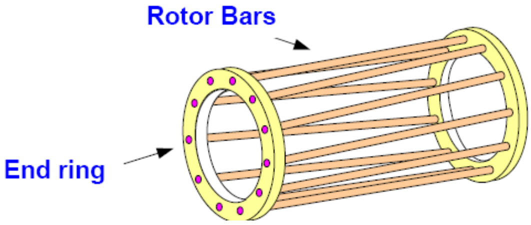
The stator of an induction motor is ,in principle ,the same as that of synchronous motor or generator.It is made up of a number of stampings,which are slotted to receive the windings.The stator carrie s 3-phase winding and is fed from a 3 phase supply.It is wound for a definite number of poles,the exact numbers of poles being determined by the requirements of speed.Greater the number of poles ,lesser the speed and vice-versa.The stator windings ,when supplied with 3-phase currents ,produce a magnetic flux,which is of constant magnitude but which at a speed of synchronous speed. This revolving magnetic flux induces an e.m.f in the rotor by mutual induction.

Type of rotors

Rotor is of two different types.

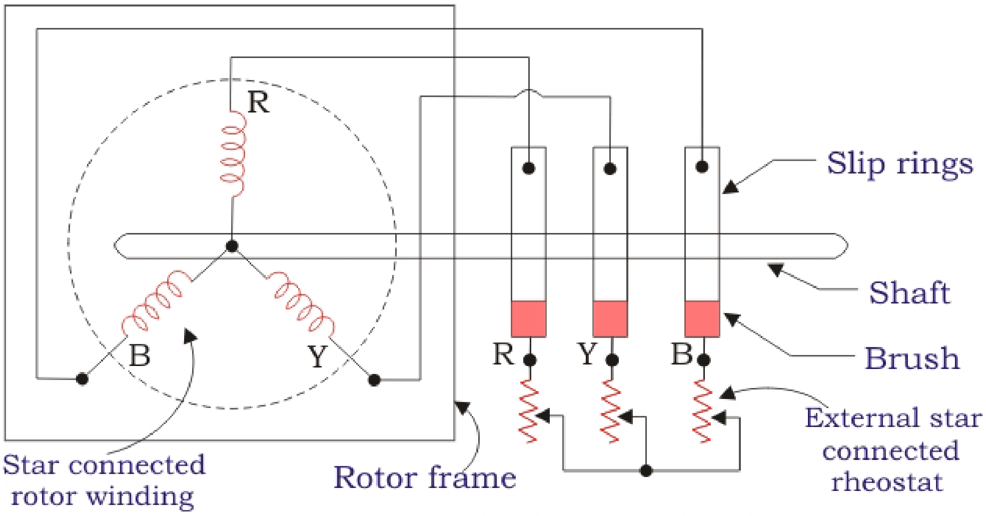
1. Squirrel cagerotor
2. Woundrotor

1.Squirrel-Cage Rotor



The rotor of the squirrel cage three phase induction motor is cylindrical in shape and have slots on its periphery. The slots are not made parallel to each other but are bit skewed (skewing is not shown in the figure of squirrel cadge rotor beside) as the skewing prevents magnetic locking of stator and rotor teeth and makes the working of motor more smooth and quieter. The squirrel cage rotor consists of aluminum, brass or copper bars (copper bras rotor is shown in the figure beside). These aluminum, brass or copper bars are called rotor conductors and are placed in the slots on the periphery of the rotor. The rotor conductors are permanently shorted by the copper or aluminum rings called the end rings. In order to provide mechanical strength these rotor conductor are braced to the end ring and hence form a complete closed circuit resembling like a cage and hence got its name as "squirrel cage induction motor". The squirrel cage rotor winding is made symmetrical. As the bars are permanently shorted by end rings, the rotor resistance is very small and it is not possible to add external resistance as the bars are permanently shorted.The absence of slip ring and brushes make the construction of Squirrel cage three phase induction motor very simple and robust and hence widely used three phase induction motor. These motors have the advantage of adapting any number of pole pairs

2.Wound Rotor



Slip Ring Three Phase Induction Motor

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In this type of three phase induction motor the rotor is wound for the same number of poles as that of stator but it has less number of slots and has less turns per phase of a heavier conductor.The rotor also carries star or delta winding similar to that of stator winding. The rotor consists of numbers of slots and rotor winding are placed inside these slots. The three end terminals are connected together to form star connection. As its name indicates three phase slip ring induction motor consists of slip rings connected on same shaft as that of rotor. The three ends of three phase windings are permanently connected to these slip rings. The external resistance can be easily connected through the brushes and slip rings and hence used for speed control and improving the starting torque of three phase induction motor. The brushes are used to carry current to and from the rotor winding. These brushes are further connected to three phase star connected resistances. At starting, the resistance are connected in rotor circuit and is gradually cut out as the rotor pick up its speed. When the motor is running the slip ring are shorted by connecting a metal collar, which connect all slip ring together and the brushes are also removed. This reduces wear and tear of the brushes. Due to presence of slip rings and brushes the rotor construction becomes somewhat complicated therefore it is less used as compare to squirrel cage induction motor.

PRINCIPLE OF OPERATION

- An AC current is applied in the stator armature which generates a flux in the statormagneticcircuit.
- This flux induces an emf in the conducting bars of rotor as they are “cut” by the fluxwhilethemagnetisbeingmoved($E=BV$L(Faraday’sLaw))
- A current flows in the rotor circuit due to the induced emf, which in term producesaforce,($F=BI$L)canbechangedtothetorqueastheoutput.

In a 3-phase induction motor, the three-phase currents ia, ib and ic, each of equal magnitude, but differing in phase by 120°. Each phase current produces a magnetic flux and there is physical 120 °shift between each flux. The total flux in the machine is the sum of the three fluxes. The summation of the three ac fluxes results in a rotating flux, which turns with constant speed and has constant amplitude. Such a magnetic flux produced by balanced three phase currents flowing in thee-phase windings is called a rotating magnetic flux or rotating magnetic field (RMF).RMF rotates with a constant speed (Synchronous Speed).

Existence of a RFM is an essential condition for theoperation of an induction motor.

Ifstatorisenergizedbyanaccurrent,RMFisgeneratedduetotheappliedcurrent to the stator winding. This flux produces magnetic field and the field revolves in the air gap between stator and rotor. So, the magnetic field induces a voltage in the short-circuitedbarsoftherotor.Thisvoltagedrivescurrentthroughthebars.Theinteractionof therotatingfluxandtherotorcurrentgeneratesaforcethatdrivesthemotorandatorque isdevelopedconsequently.Thetorqueisproportionalwiththefluxdensityandtherotor bar current (F=BLI). The motor speed is less than the synchronous speed. The direction of the rotation of the rotor is the same as the direction of the rotation of the revolving magnetic field in the air gap.

However,forthesecurrentstobeinduced,thespeedofthephysicalrotorandthe speed of the rotating magnetic field in the stator must be different, or else the magnetic field will not be moving relative to the rotor conductors and nocurrents will be induced. If by some chance this happens, the rotor typically slows slightly until a current is re- induced and then the rotor continues as before. This difference between the speed of the rotorandspeedoftherotatingmagneticfieldinthestatoriscalled*slip*.Itisunitlessand is the ratio between the relative speed of the magnetic field as seen by the rotor the (*slip speed*) to the speed of the rotating stator field. Due to this an induction motor is sometimesreferredtoasanasynchronousmachine.

SLIP:

The relationship between the supply frequency, *f*, the number of poles, *p*, and the synchronous speed (speed of rotating field), n_s is given by

$$n_s = \frac{120f}{p}$$

The stator magnetic field (rotating magnetic field) rotates at a speed, n_S, the synchronousspeed.If,n=speedoftherotor,theslip,sforaninductionmotorisdefined asS=Ns-N/Ns

At standstill, rotor does not rotate ,n=0,

So s=1. Atsynchronousspeed,$n=n_S$,s=0

The mechanical speed of the rotor, in terms of slip and synchronous speed is given by,

$n=(1-s)\,n_s$

Frequency of Rotor Current and Voltage

With the rotor at stand-still, the frequency of the induced voltages and currents is the same as that of the stator (supply) frequency, f_e.

If the rotor rotates at speed of n, then the relative speed is the slip speed:

$n_{slip}=n_s-n$

n_{slip} is responsible for induction.

Hence, the frequency of the induced voltages and currents in the rotor is, $f_r=sf_e$.

Torque Equation of Three Phase Induction Motor

The torque produced by three phase induction motor depends upon the following three factors: Firstly the magnitude of rotor current, secondly the flux which interact with the rotor of three phase induction motor and is responsible for producing emf in the rotor part of induction motor, lastly the power factor of rotor of the three phase induction motor. Combining all these factors together we get the equation of torque as-

$$T \propto \phi I_2 \cos\theta_2$$

Where, T is the torque produced by induction motor,

φ is flux responsible for producing induced emf,

I_2 is rotor current, $\cos\theta_2$ is the power factor of rotor circuit.

The flux φ produced by the stator is proportional to stator emf E_1. i.e

$$\varphi \propto E_1$$

We know that transformation ratio K is defined as the ratio of secondary voltage (rotor voltage) to that of primary voltage (stator voltage)

$$K = \frac{E_2}{E_1}$$

$$or,\ K = \frac{E_2}{\phi}$$

$$or,\ E_2 = \phi$$

Rotor current I_2 is defined as the ratio of rotor induced emf under running condition , sE_2 to total impedance, Z_2 of rotor side,

$$i.e\ I_2 = \frac{sE_2}{Z_2}$$

and total impedance Z_2 on rotor side is given by ,

$$Z_2 = \sqrt{R_2^2 + (sX_2)^2}$$

Putting this value in above equation we get,

$$I_2 = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

s= slip of Induction motor

We know that power factor is defined as ratio of resistance to that of impedance. The power factor of the rotor circuit is

$$\cos\theta_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Putting the value of flux φ, rotor current I_2, power factor $\cos\theta_2$ in the equation of torque we get,

$$T \propto E_2 \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Combining similar term we get,

$$T \propto sE_2^2 \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Removing proportionality constant we get,

$$T = KsE_2^2 \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$This\ comstant\ K = \frac{3}{2\pi n_s}$$

Where n_s is synchronous speed in r. p. s, $n_s = N_s / 60$. So, finally the equation of torque becomes,

Derivation of K in torque equation.

In case of three phase induction motor, there occur copper losses in rotor. These rotor copper losses are expressed as $P_c = 3I_2^2R^2$We know that rotor current, Substitute this value of I_2 in the equation of rotor copper losses, P_c. So, we get

$$P_m = \frac{1}{s} \times \frac{(1-s)3R_2s^2E_2^2}{R_2^2 + (sX_2)^2}$$

On simplifying we get,

$$P_m = \frac{(1-s)3R_2sE_2^2}{R_2^2 + (sX_2)^2}$$

The mechanical power developed $P_m = T\omega$,

$$\omega = \frac{2\pi N}{60}$$

$$or\ P_m = T\frac{2\pi N}{60}$$

Substituting the value of P_m

We know that the rotor speed $N = N_s(1 - s)$

Substituting this value of rotor speed in above equation we get,

$$T = \frac{1}{s} \times \frac{(1-s)3R_2s^2E_2^2}{R_2^2 + (sX_2)^2} \times \frac{60}{2\pi N_s(1-s)}$$

N_s is speed in revolution per minute (rpm) and n_s is speed in revolution per sec (rps) and the relation between the two is

$$\frac{N_s}{60} = n_s$$

Substitute this value of N_s in above equation and simplifying it we get

$$Toeque,\ T = \frac{s\ E_2^2R_2}{R_2^2 + (sX_2)^2} \times \frac{3}{2\pi N_s}$$

$$or,\ T = KsE_2^2\frac{R_2}{R_2^2 + (sX_2)^2}$$

Comparing both the equations, we get, constant $K = 3 / 2\pi n_s$

Equation of Starting Torque of Three Phase Induction Motor

Starting torque is the torque produced by induction motor when it is started. We know that at start the rotor speed, N is zero

$$So,\ slip\ s = \frac{N_s - N}{N_s}\ becomes\ 1$$

.

So, the equation of starting torque is easily obtained by simply putting the value of s = 1 in the equation of torque of the three phase induction motor,

$$T = \frac{E_2^2 R_2}{R_2^2 + X_2^2} \times \frac{3}{2\pi n_s} \ N - m$$

The starting torque is also known as standstill torque.

Maximum Torque Condition for Three Phase Induction Motor

In the equation of torque, $T = \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \times \frac{3}{2\pi n_s}$

he rotor resistance, rotor inductive reactance and synchronous speed of induction motor remains constant . The supply voltage to the three phase induction motor is usually rated and remains constant so the stator emf also remains the constant. The transformation ratio is defined as the ratio of rotor emf to that of stator emf. So if stator emf remains constant then rotor emf also remains constant.

If we want to find the maximum value of some quantity then we have to differentiate that quantity with respect to some variable parameter and then put it equal to zero. In this case we have to find the condition for maximum torque so we have to differentiate torque with respect to some variable quantity which is slip, s in this case as all other parameters in the equation of torque remains constant. So, for torque to be maximum

$$\frac{dT}{ds} = 0$$

$$T = KsE_2^2 \frac{R_2}{R_2^2 + (sX_2)^2}$$

Now differentiate the above equation by using division rule of differentiation. On differentiating and after putting the terms equal to zero we get, Neglecting the

negative value of slip we get

$$s^2 = \frac{R_2^2}{X_2^2}$$

So, when slip s = R_2 / X_2, the torque will be maximum and this slip is called maximum slip Sm and it is defined as the ratio of rotor resistance to that of rotor reactance. NOTE : At starting S = 1, so the maximum starting torque occur when rotor resistance is equal to rotor reactance.

Equation of Maximum Torque

The equation of torque is $T = \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$

The torque will be maximum when slip s = R_2 / X_2

Substituting the value of this slip in above equation we get the maximum value of torque as,

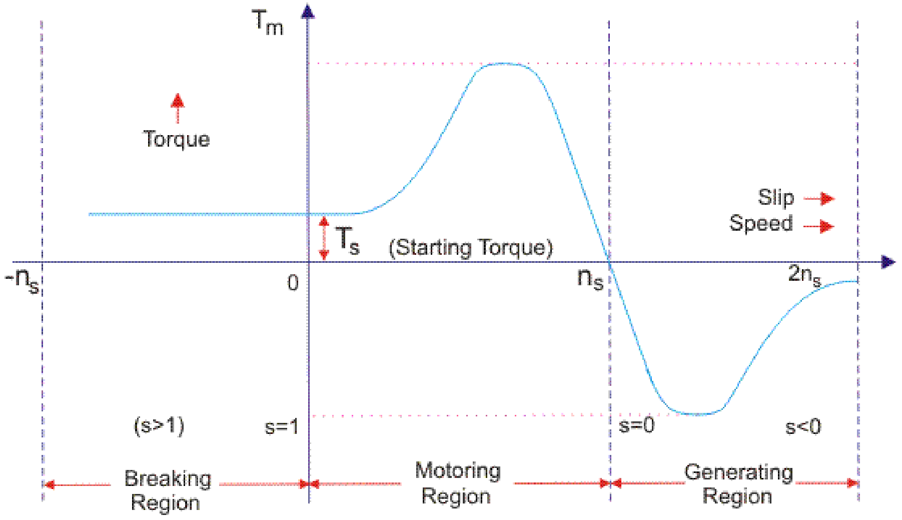
$$T_{max} = K\frac{E_2^2}{2X_2}N - m$$

In order to increase the starting torque, extra resistance should be added to the rotor circuit at start and cut out gradually as motor speeds up.

Torque Slip Characteristics of Three Phase Induction Motor

The torque slip curve for an induction motor gives us the information about the variation of torque with the slip. The slip is defined as the ratio of difference of synchronous speed and actual rotor speed to the synchronous speed of the machine. The variation of slip can be obtained with the variation of speed that is when speed varies the slip will also vary and the torque corresponding to that speed will also vary.

The curve can be described in three modes of operation-



Torque Slip Curve for Three Phase Induction Motor

Motoring Mode: In this mode of operation, supply is given to the stator sides and the motor always rotates below the synchronous speed. The induction motor torque varies from zero to full load torque as the slip varies. The slip varies from zero to one. It is zero at no load and one at standstill. From the curve it is seen that the torque is directly proportional to the slip. That is, more is the slip, more will be the torque produced and vice-versa. The linear relationship simplifies the calculation of motor parameter to great extent.

Generating Mode: In this mode of operation induction motor runs above the synchronous speed and it should be driven by a prime mover. The stator winding is connected to a three phase supply in which it supplies electrical energy. Actually, in this case, the torque and slip both are negative so the motor receives mechanical energy and delivers electrical energy. Induction motor is not much used as generator because it requires reactive power for its operation. That is, reactive power should be supplied from outside and if it runs below the synchronous speed by any means, it consumes electrical energy rather than giving it at the output. So, as far as possible, induction generators are generally avoided.

Braking Mode: In the Braking mode, the two leads or the polarity of the supply voltage is changed so that the motor starts to rotate in the reverse direction and as a result the motor stops. This method of braking is known as plugging. This method is used when it is required to stop the motor within a very short period of time. The kinetic energy stored in the revolving load is dissipated as heat. Also, motor is still receiving power from the stator which is also dissipated as heat. So as a result of which motor develops enormous heat energy. For this stator is disconnected from the supply before motor enters the braking mode.

If load which the motor drives accelerates the motor in the same direction as the motor is rotating, the speed of the motor may increase more than synchronous speed. In this case, it acts as an induction generator which supplies electrical energy to the mains which tends to slow down the motor to its synchronous speed, in this case the motor stops. This type of breaking principle is called dynamic or regenerative breaking.

Losses and Efficiency of Induction Motor

There are two types of losses occur in three phase induction motor. These losses are,

1. Constant or fixed losses,
2. Variable losses

1.Constant or Fixed Losses

Constant losses are those losses which are considered to remain constant over normal working range of induction motor. The fixed losses can be easily obtained by performing no-load test on the three phase induction motor. These losses are further classified as-

1. Iron or core losses,
2. Mechanical losses,
3. Brush friction losses.

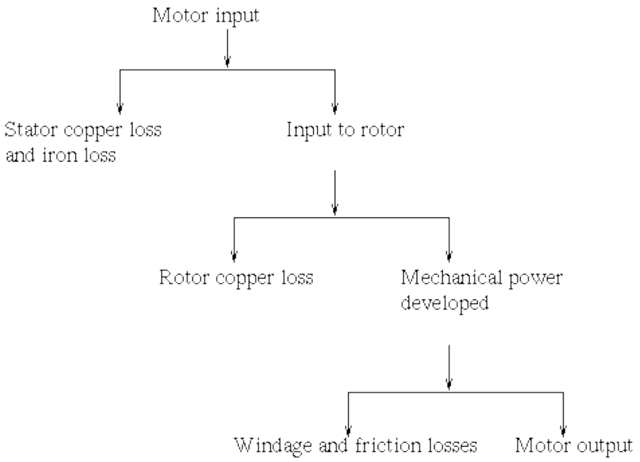
iron or Core Losses

Iron or core losses are further divided into hysteresis and eddy current losses. Eddy current losses are minimized by using lamination on core. Since by laminating the core, area decreases and hence resistance increases, which results in decrease in eddy currents. Hysteresis losses are minimized by using high grade silicon steel. The core losses depend upon frequency of the supply voltage. The frequency of stator is always supply frequency, f and the frequency of rotor is slip times the supply frequency, (sf) which is always less than the stator frequency. For stator frequency of 50 Hz, rotor frequency is about 1.5 Hz because under normal running condition slip is of the order of 3 %. Hence the rotor core loss is very small as compared to stator core loss and is usually neglected in running conditions.

Mechanical and Brush Friction Losses

Mechanical losses occur at the bearing and brush friction loss occurs in wound rotor induction motor. These losses are zero at start and with increase in speed these losses increases. In three phase induction motor the speed usually remains constant. Hence these losses almost remains constant.

Variable Losses



These losses are also called copper losses. These losses occur due to current flowing in stator and rotor windings. As the load changes, the current flowing in rotor and stator winding also changes and hence these losses also changes. Therefore these losses are called variable losses. The copper losses are obtained by performing blocked rotor test on three phase induction motor.

The main function of induction motor is to convert an electrical power into mechanical power. During this conversion of electrical energy into mechanical energy the power flows through different stages. This power flowing through different stages is shown by power flow diagram. As we all know the input to the three phase induction motor is three phase supply. So, the three phase supply is given to the stator of three phase induction motor.
Let, P_{in} = electrical power supplied to the stator of three phase induction motor,
V_L = line voltage supplied to the stator of three phase induction motor,
I_L = line current, Cosφ = power factor of the three phase induction motor.
Electrical power input to the stator,

$$P_{in} = \sqrt{3} V_L I_L \cos\varphi$$

A part of this power input is used to supply stator losses which are stator iron loss and stator copper loss. The remaining power i.e(inputelectrical power – stator losses) are supplied to rotor as rotor input.
So, rotor input $P_2 = P_{in}$ – stator losses (stator copper loss and stator iron loss).
Now, the rotor has to convert this rotor input into mechanical energy but this complete input cannot be converted into mechanical output as it has to supply rotor losses. As explained earlier the rotor losses are of two types rotor iron loss and rotor copper loss. Since the iron loss depends upon the rotor frequency, which is very small when the rotor rotates, so it is usually neglected. So, the rotor has only rotor copper loss. Therefore the rotor input has to supply these rotor copper losses. After supplying the rotor copper losses, the remaining part of Rotor input, P_2 is converted into mechanical power, P_m.
Let P_c be the rotor copper loss,
I_2 be the rotor current under running condition,
R_2 is the rotor resistance,
P_m is the gross mechanical power developed.

$$P_c = 3I_2^2R_2$$
$$P_m = P_2 - P_c$$

Now this mechanical power developed is given to the load by the shaft but there occur some mechanical losses like friction and windage losses. So, the gross mechanical power developed has to be supplied to these losses. Therefore the net output power developed at the shaft, which is finally given to the load is P_{out}.

$P_{out} = P_m$ – Mechanical losses (friction and windage losses).

P_{out} is called the shaft power or useful power.

Efficiency of Three Phase Induction Motor

Efficiency is defined as the ratio of the output to that of input,

$$Efficiency,\ \eta = \frac{output}{input}$$

Rotor efficiency of the three phase induction motor ,

$$= \frac{rotor\ output}{rotor\ input}$$

= Gross mechanical power developed / rotor input

$$= \frac{P_m}{P_2}$$

Three phase induction motor efficiency,

$$= \frac{power\ developed\ at\ shaft}{electrical\ input\ to\ the\ motor}$$

Three phase induction motor efficiency

$$\eta = \frac{P_{out}}{P_{in}}$$

Assignment-Cum-Tutorial Questions

A. Questions testing the remembering / understanding level of students

I) Objective Questions

1) The principle of operation of a 3-phase induction motor is most similar to that of a [ ]

a) synchronous motor b)repulsion –start induction motor

c) transformer with shorted secondary d) capacitor start induction motor

2) Regarding skewing of motor bars in squirrel induction motor which statement is false.[ ]

a)it prevents cogging b)it increases starting torque

c)it produces more uniform torque d) it reduces motor hum during its operation

3) The effect of increasing the length of air-gap in an induction motor will be to increase the

[ ]

a)Power factor b)speed c) magnetising current d) air-gap flux

4) In a 3-Phase induction motor ,the relative speed of stator flux with respect to …is zero[ ]

a)stator winding b)rotor c) rotor flux d) space

5) In a 3 –Phase induction motor ,the rotor field rotates at synchronous speed with respect to [ ]

a) stator b) rotor c) stator flux d) none of the above

6) Irrespective of the supply frequency,the torque developed by SCIM is the same when ever is the same [ ]

a) supply voltage b) external load c) rotor resistance d) slip speed

7) Which of the following rotor quantity in a SCIM does not depend on its slip? [ ]

a) reactance b) speed c) induced emf d) frequency

8) The efficiency of 3-Phase induction motor is approximately proportional to [ ]

a) (1-s) b) s c) N d) Ns

9)In a 3-Phase slip ring induction motor ,3-Phase balanced supply is given to the rotor and stator winding is short-circuited .The rotor would [ ]

a) not run b) run in the direction of rotating field

c) run against the direction of rotating field d)run at half the synchronous speed

10)squirrel cage induction motor is [ ]

a) low at light loads b) low at heavy loads

c) low at light and heavy loads both d) low at rated load only.

II) Descriptive Questions

1) Explain the principle of operation of 3-phase induction motor
2) Explain how the rotating magnetic field is developed in a 3-phase induction Motor
3) Explain the constructional details of a 3-phase Induction motor
4) Derive the torque equation of a three phase induction motor.
5) Derive the condition for maximum torque and starting torque for 3-phase Induction motor
6) Draw and explain the torque –slip characteristics of three phase induction motor
7) List the factors governing the performance of induction motors.?
8) Derive the expression of rotor frequency in terms of main supply frequency and slip.
9) Explain the constructional difference between squirrel cage and slip ring induction motor

B. Question testing the ability of students in applying the concepts.

I) Objective Questions

1) A induction motor with an open circuited to rotor [ ]

a)runs on no load b) runs at reduced speed c) does not run d) gets damaged.

2) A 6 pole ,50 Hz induction motor can not run at a speed of [ ]

a) 500r.p.m. b) 750 r.p.m c) 950r.p.m d) 1000 r.p.m

3) The speed of rotating field due to rotor current relative to rotor surface is [ ]

a) Ns b) SNs c) N d) none.

4)The synchronous speed of an I-M can be increased by [ ]

a) reducing the friction at the bearings b) increasing the number of poles

c) decreasing the frequency d) increasing the frequency

5)The reactance under running condition is less than its stand still value.this is due to

the reduction in [ ]

a)rotor inductance b) rotor frequency c)mutual flux d)stator flux

6) The air gap between the stator and rotor of a 3 –phase induction motor ranges from [ ]

a)2cm to 4cm b) 0.4mm to 4 mm c) 1cm to 2 cm d) 4cm to 6cm

7) when the rotor of a 3-phase induction motor is blocked ,the slip is ……… [ ]

a)zero b) 0.5 c) 0.1 d) 1

8) A wound rotor is mainly used in applications where …… [ ]

a) high starting torque is required b) speed control is requored

c) less costly motor is not required d) high rotor resistance is required during running.

9) The rotor winding of a 3-phase wound rotor induction motor is generally ….. [ ]

a) star b)delta c) partly star and partly delta d)none.

10) If Ns is the speed of rotating flux and N the speed of the rotor ,then the rate at which the flux cuts the rotor conductors is directly proportional to …… [ ]

a) Ns b)N c)Ns-N d)N-Ns

II) Descriptive Questions

1. A 3-phase , 6 pole, 50Hz cage motor is running with a slip of 4%. Find

(a) Speed of rotating field relative to stator winding

(b) Motor speed

(c) slip speed

(d) Frequency of the emf induced in the rotor

(e) Speed of rotation of rotor mmf relative to rotor winding

(f) Speed of rotor of rotor mmf relative to stator winding

2. The power input to the rotor of a 440V, 50Hz, 3-phase, 6-pole induction motor. is 50kW.It is observed that the rotor emf makes 120 complete cycles per minute. Calculate

(i)Slip

(ii) Rotor speed

(iii) Rotor copper losses per phase

(iv)Mechanical power developed

(v)Rotor resistance per phase if the rotor current is 50A

3. A 600Hp three phase,440volts,50Hz induction motor with 6 poles as rotor current frequency of 2Hz. Compute the operating slip and actual speed of the machine

4.A 3-phase, 6 pole, 50Hz induction motor has a slip of 1% at no load and 3% at full load. Find the synchronous speed, no load speed, frequency of rotor current at standstill and frequency of rotor current at full load?

5.A 4 pole ,3 phase induction motor operates from a supply whose frequency is 50Hz ,calculate

(i) the speed at which the magnetic field of the stator is rotating

(ii) the speed of the rotor when the slip is 0.04

(iii) the frequency of the rotor currents when the slip is 0.03

(iv) the frequency of the rotor currents at stand still.

6. A 24 pole ,50 Hz ,star connected induction motor has rotor resistance of 0.016 ohms per phase and rotor reactance of 0.265 ohm per phase at stand still. It is achieving its full load torque at a speed of 247r.p.m.calculte the ratio of i) full load torque to maximum torque ,ii) starting torque to maximum torque.

7. A 3 phase, 4 pole, 50 Hz star connected induction motor running on full load develops a use full torque of 300N-m,The rotor emf is completing 120 cycles per minute. If torque lost in friction is 50Nm Calculate i) slip, ii) Net output power, iii)Rotor copper losses per phase iv)Rotor efficiency v)Rotor resistance per phase if the rotor current is 60 A in running condition.

C. Questions testing the analyzing / evaluating /Creative ability of students

1. A 400 v ,4 pole, 3 phase ,50 Hz star connected induction motor has a rotor resistance and reactance per phase equal to 0.01ohm and 0.1ohmrespectively.Determine i)starting torque, ii)slip at which maximum torque will occur, iii)speed at which maximum torque will occur iv) Maximum torque,v) full load torque if full load slip is 4%. Assume ratio of stator to rotor turns ratio 4.
2. A 3 phase ,50 Hz, 400V, induction motor has 4 pole star connected stator winding rptor resistance and stands still reactance per pahse are 0.1Ω and 1Ω respectively.The full load slip is 4%.Calculate (a) the total torque developed (b)the horse power developed
   (c) maximum torque developed (d)the speed at maximum torque . Assume that the stator to rotor turns ratio is 2:1

D.Previous GATE/IES Questions.

1)The slip of an induction motor normally does not depend on **GATE-2011**

(A) rotor speed (B) synchronous speed (C) shaft torque (D) core-loss component

2) A three-phase 440 V, 6 pole, 50 Hz, squirrel cage induction motor is running at a slip of 5%. The speed of stator magnetic field to rotor magnetic field and speed of rotor with respect of stator magnetic field are **GATE-2012**

(A) zero, −5 rpm (B) zero, 955 rpm (C) 1000 rpm, −5 rpm (D) 1000 rpm, 955 rpm

3) A 400 V, 50 Hz 30 hp, three-phase induction motor is drawing 50 A current at 0.8 power factor lagging. The stator and rotor copper losses are 1.5 kW and 900 W respectively. The friction and windage losses are 1050 W and the core losses are 1200 W. The air-gap power of the motor will be **GATE-2012**

(A) 23.06 kW (B) 24.11 Kw (C) 25.01 kW (D) 26.21 kW

4) The speed of rotation of stator magnetic field with respect to rotor structure will be **GATE-2013**

(A) 90 rpm in the direction of rotation

(B) 90 rpm in the opposite direction of rotation

(C) 1500 rpm in the direction of rotation

(D) 1500 rpm in the opposite direction of rotation

5) A three-phase squirrel cage induction motor has a starting torque of 150% and a maximum torque of 300% with respect to rated torque at rated voltage and rated frequency. Neglect the stator resistance and rotational losses. The value of slip for maximum torque is **GATE-2014**

(A) 13.48% (B) 16.42% (C) 18.92% (D) 26.79%

6) In a single phase induction motor driving a fan load, the reason for having a high resistance rotor is to achieve **GATE-2015**

(A) low starting torque (B) quick acceleration (C) high efficiency (D) reduced size

7) A 3-phase induction motor is driving a constant torque load at rated voltage and frequency. If both voltage and frequency are halved, following statements relate to the new condition if stator resistance, leakage reactance and core loss are ignored 1. The difference between synchronous speed and actual speed remains same 2. The airgap flux remains same 3. The stator current remains same 4. The p.u. slip remains same Among the above, current statements are **GATE-2015**

(A) All (B) 1, 2 and 3 (C) 2, 3 and 4 (D) 1 and 4

8) If a 400 V, 50 Hz, star connected, 3-phase squirrel cage induction motor is operated from a 400 V, 75 Hz supply, the torque that the motor can now provide while drawing rated current from the supply

GATE-2016

(A) reduces (B) increases (C) remains the same (D) increases or reduces depending upon the rotor resistance

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Department of Electrical and Electronics Engineering

HANDOUT

on

ELEMENTS OF ELECTRICAL& ELECTRONICS ENGINEERING

Unit –III

Transformers

Objectives:

- To familiarize the students with the constructional details, principle of operation single phase transformers, EMF equation.
- To familiarize the students to understand the losses, efficiency and regulation of single phase transformer.

Syllabus: Transformers: Principle of operation of Single phase transformers-EMF equation-losses in transformer -efficiency and regulation..

Learning Outcomes:

Student will be able to

3. Demonstrate the knowledge and understanding of the fundamental principles of electromagnetism.

2.Evaluate the generated EMF of a Single phase transformers

3.Calculate the efficiency and regulation of a Single phase transformers

Transformers

Introduction

- The transformer is probably one of the most useful electrical devices ever invented. It can change the magnitude of alternating voltage or current from one value to another.
- This useful property of transformer is mainly responsible for the widespread use of alternating currents rather than direct currents i.e., electric power is generated, transmitted and distributed in the form of alternating current.
- Transformers have no moving parts, rugged and durable in construction, thus requiring very little attention. They also have a very high efficiency—as high as 99%.
- A transformer is a static piece of equipment used either for raising or lowering the voltage of an a.c. supply with a corresponding decrease or increase in current. It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in Fig. (1).
- The winding connected to the a.c. source is called primary winding (or primary) and the one connected to load is called secondary winding (or secondary). The alternating voltage V_1 whose magnitude is to be changed is applied to the primary.
- Depending upon the number of turns of the primary (N_1) and secondary (N_2), an alternating e.m.f. E_2 is induced in the secondary. This induced e.m.f. E_2 in the secondary causes a secondary current I_2. Consequently, terminal voltage V_2 will appear across the load. If $V_2 > V_1$, it is called a step up-transformer.
- On the other hand, if $V_2 < V_1$, it is called a step-down transformer.

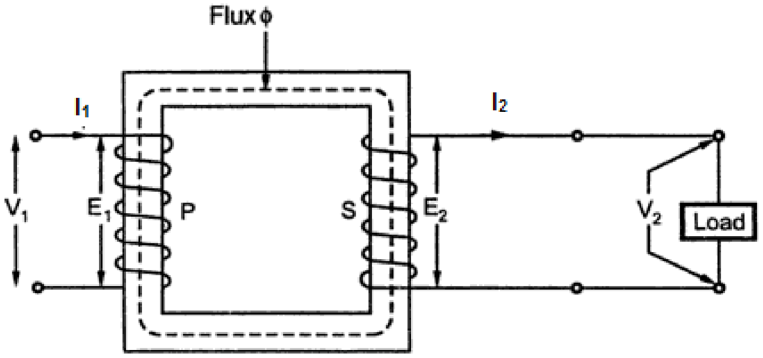


Figure 14

➢ Working Principle of a Transformer

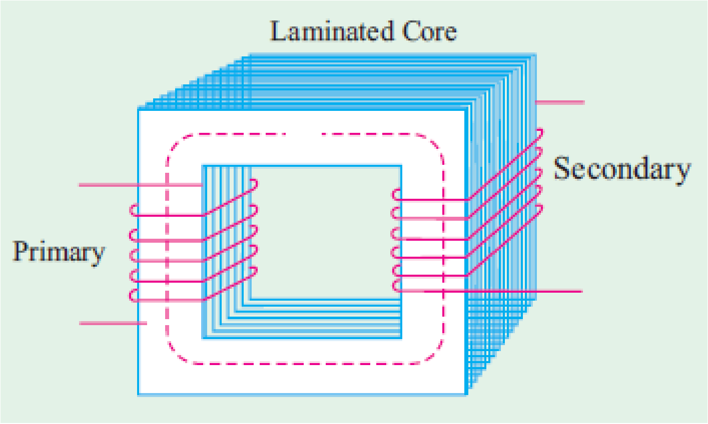


Figure 15

- A transformer is a static (or stationary) piece of apparatus by means of which electric power in one circuit is transformed into electric power of the same frequency in another circuit.
- It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. The physical basis of a transformer is **mutual induction** between two circuits linked by a common magnetic flux. In its simplest form, it consists of two inductive coils which are electrically separated but magnetically linked through a path of low reluctance as shown in Fig. 2.
- The two coils possess high mutual inductance. If one coil is connected to a source of alternating voltage, an alternating flux is set up in the laminated core, most of which is linked with the other coil in which it

produces mutually-induced e.m.f. (according to Faraday's Laws of Electromagnetic Induction **$e = MdI/dt$**).

- If the second coil circuit is closed, a current flows in it and so electric energy is transferred (entirely magnetically) from the first coil to the second coil. The first coil, in which electric energy is fed from the a.c. supply mains, is called **primary** winding and the other from which energy is drawn out, is called **secondary** winding.

  In brief, a transformer is a device that

5. transfers electric power from one circuit to another
6. it does so without a change of frequency
7. it accomplishes this by electromagnetic induction and
8. where the two electric circuits are in mutual inductive influence of each other.

When an alternating voltage V_1 is applied to the primary, an alternating flux Ø is set up in the core. This alternating flux links both the windings and induces e.m.f.s E_1 and E_2 in them according to Faraday's laws of electromagnetic induction. The e.m.f. E_1 is termed as primary e.m.f. and e.m.f. E_2 is termed as secondary e.m.f.

- **The losses that occur in a transformer are:**

c) Core losses—eddy current and hysteresis losses

d) Copper losses—in the resistance of the windings

In practice, these losses are very small so that output power is nearly equal to the input primary power. In other words, a transformer has very high efficiency.

- **Transformer Construction**

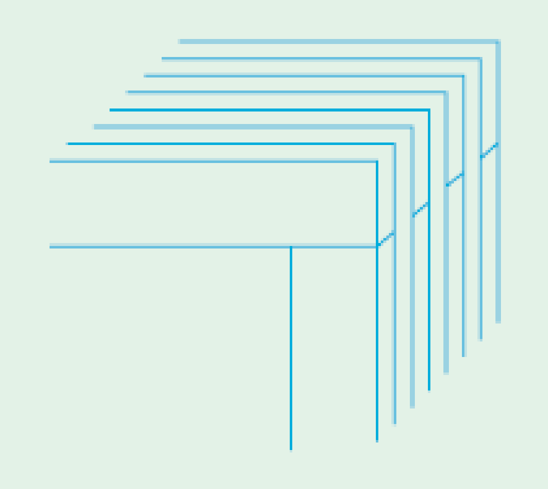


Figure 16

- The simple elements of a transformer consist of two coils having mutual inductance and a laminated steel core. The two coils are insulated from each other and the steel core.
- Other necessary parts are: some suitable container for assembled core and windings; a suitable medium for insulating the core and its windings from its container; suitable bushings (either of porcelain, oil-filled or capacitor-type) for insulating and bringing out the terminals of windings from the tank.
- In all types of transformers, the core is constructed of transformer sheet steel laminations assembled to provide a continuous magnetic path with a minimum of air-gap included.
- The steel used is of high silicon content, sometimes heat treated to produce a high permeability and a low hysteresis loss at the usual operating flux densities. The eddy current loss is minimised by laminating the core, the laminations being insulated from each other by a light coat of core-plate varnish or by an oxide layer on the surface.

➢ Types of Transformers

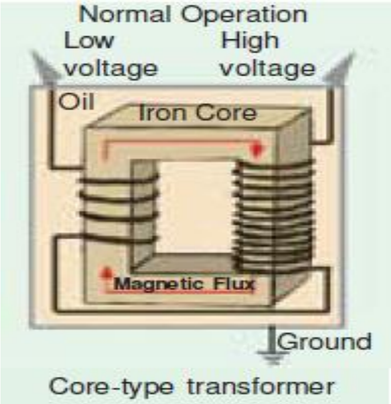


Figure 17

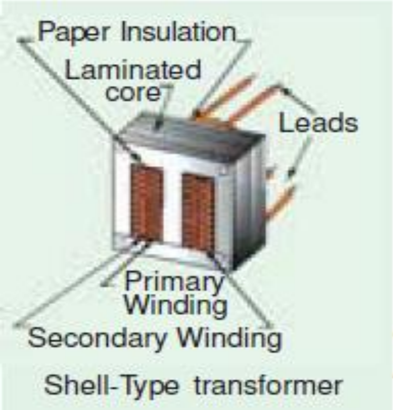


Figure 18

- Constructionally, the transformers are of two general types, distinguished from each other merely by the manner in which the primary and secondary coils are placed around the laminated core.

The two types are known as

(iv) core-type and

(v) shelltype.

(vi) Another recent development is spiral-core or wound-core type, the trade name being spirakore transformer.

- In the so-called core type transformers, the windings surround a considerable part of the core whereas in shell-type transformers, the core surrounds a considerable portion of the windings as shown schematically in Fig. 6.(a) and (b) respectively.

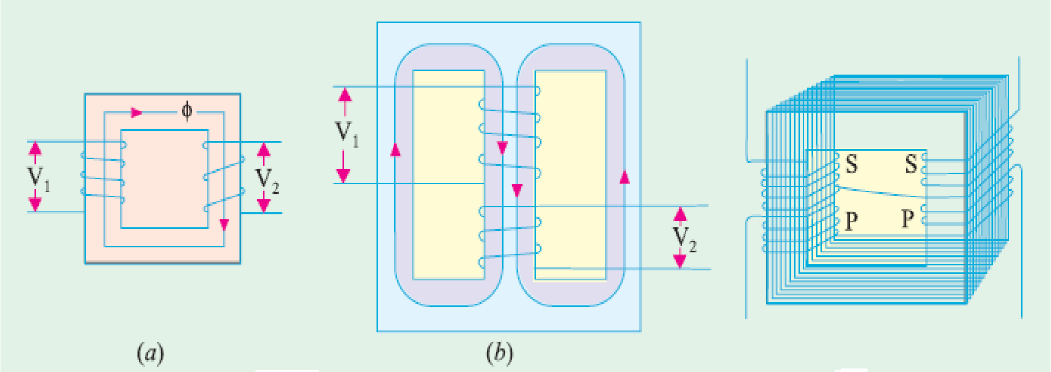


Figure 19

- In the simplified diagram for the core type transformers [Fig.4], the primary and secondary winding are shown located on the opposite legs (or limbs) of the core, but in actual construction, these are always interleaved to reduce leakage flux.
- Half of the primary and half of the secondary winding have been placed side by side or concentrically on each limb, not primary on one limb (or leg) and the secondary on the other.

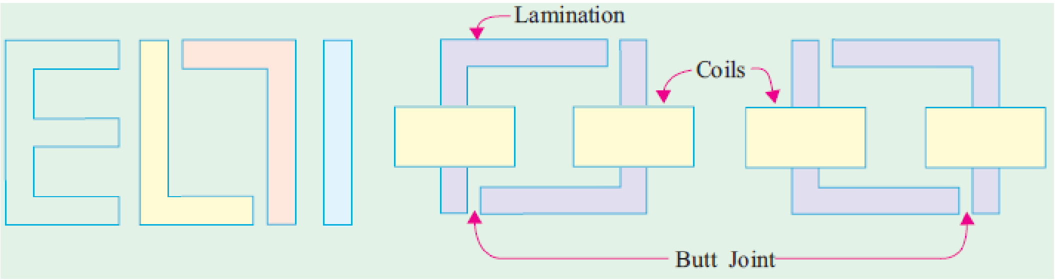


Figure 20

- In both core and shell-type transformers, the individual laminations are cut in the form of long strips of L's, E's and I's as shown in Fig. 7.

The assembly of the complete core for the two types of transformers is shown in Fig. 8.

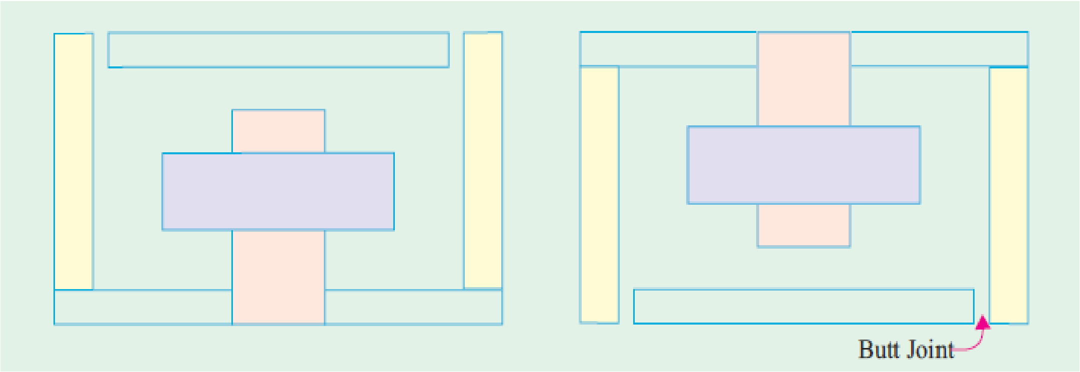


Figure 21

- As said above, in order to avoid high reluctance at the joints where the laminations are butted against each other.

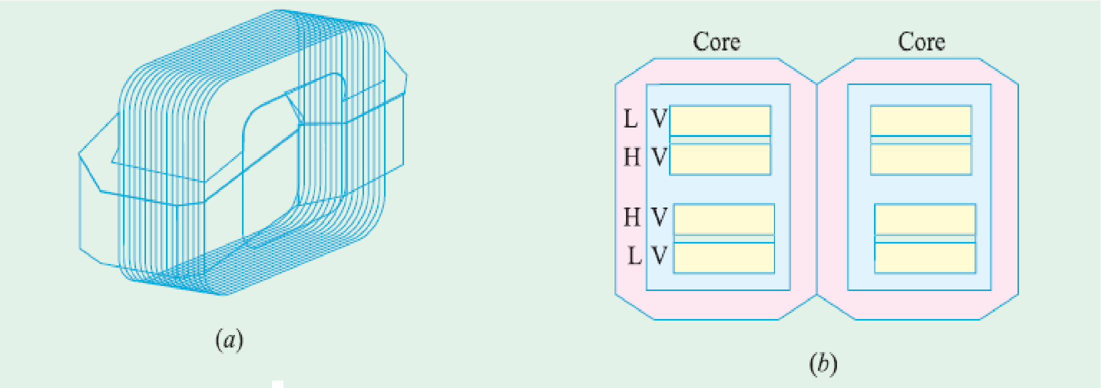
(iii) Core-type Transformers

Figure 22

- The coils used are form-wound and are of the cylindrical type. The general form of these coils may be circular or oval or rectangular. In small size core-type transformers, a simple rectangular core is used with cylindrical coils which are either circular or rectangular in form. But for large-size core-type transformers, round or circular cylindrical coils are used which are so wound as to fit over a cruciform core section as shown in Fig. 9.
- The circular cylindrical coils are used in most of the core-type transformers because of their mechanical strength. Such cylindrical coils are wound in helical layers with the different layers insulated from each other by paper, cloth, micarta board or cooling ducts.

(iv) Shell-type Transformers

- In these case also, the coils are form-would but are multi-layer disc type usually wound in the form of pancakes. The different layers of such multi-layer discs are insulated from each other by paper.
- The complete winding consists of stacked discs with insulation space between the coils–the spaces forming horizontal cooling and insulating ducts. A shell-type transformer may have a simple rectangular form as shown in Fig.12(a) or it may have distributed form as shown in Fig.12(b).



(a) (b)

Figure 23

➢ Concept of ideal transformer

An ideal transformer is one which has

4. Its windings have no ohmic resistance and hence which has no I^2R losses.
5. There is no magnetic leakage and hence which has no core losses. In other words, an ideal transformer consists of two purely inductive coils wound on a loss-free core (or Leakage flux is zero i.e. 100% flux produced by primary links with the secondary).
6. Permeability of core is so high that negligible current is required to establish the flux in it.

- Although ideal transformer cannot be physically realized, yet its study provides a very powerful tool in the analysis of a practical transformer.

In fact, practical transformers have properties that approach very close to an ideal transformer.

- **E.M.F. Equation of a Transformer**

Let $N_1 = \text{No. of turns in primary}$

$N_2 = \text{No. of turns in secondary}$

$\emptyset m = \text{Maximum flux in core in webers}$

$= B_m \times A$

$f = \text{Frequency of a.c. input in Hz}$

As shown in Fig. 5.14, flux increases from its zero value to maximum value Øm in one quarter of the cycle i.e. in 1/4 f second.

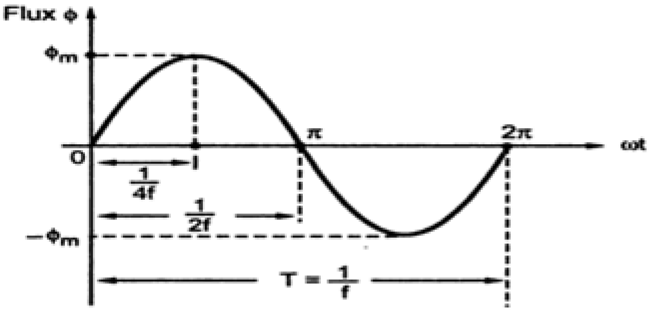


Fig.13

$$\therefore \text{ Average rate of change of flux } = \frac{\emptyset m}{1/4f}$$

$$= 4\,f\,\emptyset m \text{ Wb/s or volt}$$

Now, rate of change of flux per turn means induced e.m.f. in volts.

$$\therefore \text{ Average e.m.f./turn } = 4\,f\,\emptyset m \text{ volt}$$

If flux Φ varies sinusoidally, then r.m.s. value of induced e.m.f. is obtained by multiplying the average value with form factor.

$$\text{Form factor } = \frac{\text{r.m.s value}}{\text{average value}} = 1.11$$

$$\therefore \text{ r.m.s. value of e.m.f./ turn} = 1.11 \times 4\,f\,\emptyset m = 4.44\,f\,\emptyset m \text{ volt}$$

Now,r.m.s. value of the induced e.m.f. in the whole of primary winding

= (induced e.m.f/turn) ×No. of primary turns

$$E1 = 4.44\,f\,N_1\,\emptyset m = 4.44\,f\,N_1\,B_m A \qquad (1)$$

Similarly, r.m.s. value of the e.m.f. induced in secondary is,

$$E2 = 4.44\,f\,N_2\,\emptyset m = 4.44\,f\,N_2\,B_m A \qquad (2)$$

It is seen from (1) and (2) that

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44\,f\,\emptyset m$$

It means that e.m.f./turn is the same in both the primary and secondary windings.

In an ideal transformer on no-load, $V_1 = E_1$ and $V_2 = E_2$, where V_2 is the terminal voltage.

Voltage Transformation Ratio (K)

From equations (1) and (2), we get

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = K$$

This constant K is known as voltage transformation ratio.

iii. If $N_2 > N_1$ i.e. $K > 1$, then transformer is called **step-up** transformer.

iv. If $N_2 < N_1$ i.e. $K < 1$, then transformer is known as **step-down** transformer.

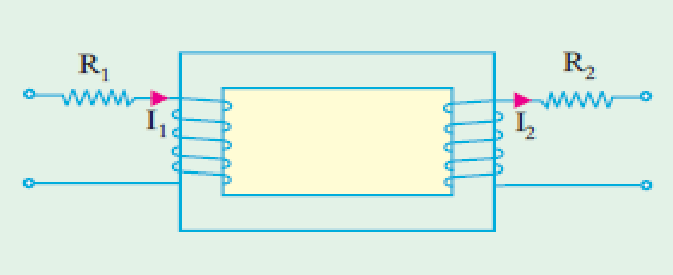
Again, for an ideal transformer, input VA = output VA.

$$V_1 I_1 = V_2 I_2$$

$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence, currents are in the inverse ratio of the (voltage) transformation ratio.

Equivalent Resistance



In Fig. 5.19 a transformer is shown whose primary and secondary windings have resistances of R_1 and R_2 respectively. The resistances have been shown external to

Fig.14

the windings. It would now be shown that the resistances of the two windings can be transferred to any one of the two windings.

- The advantage of concentrating both the resistances in one winding is that it makes calculations very simple and easy because one has then to work in one winding only. It will be proved that a resistance of R_2 in secondary is equivalent to $\frac{R_2}{K^2}$ in primary. The value $\frac{R_2}{K^2}$ will be denoted by $R_2^{'}$ – **the equivalent secondary resistance as referred to primary**. The copper loss in secondary is $I_2{}^2R_2$.
- This loss is supplied by primary which takes a current of I_1. Hence if $R_2^{'}$ is the **equivalent resistance in primary which would have caused the same loss** as R_2 in secondary, then

$$I_1{}^2R_2^{'} = I_2{}^2R_2$$

$$R_2^{'} = \frac{I_2{}^2}{I_1{}^2}R_2 = \frac{R_2}{K^2}$$

$$R_2^{'} = \frac{R_2}{K^2}$$

Similarly, equivalent primary resistance as referred to secondary is

$$R_1^{'} = K^2R_1$$

In Fig. 15, secondary resistance has been transferred to primary side leaving secondary circuit resistance less. The resistance $R_1 + R_2^{'} = R_1 + \frac{R_2}{K^2}$ is known as the **equivalent or effective resistance of the transformer as referred to primary** and may be designated as R_{01}.

$$\therefore R_{01} = R_1 + R_2^{'} = R_1 + \frac{R_2}{K^2}$$

Similarly, the **equivalent resistance of the transformer as referred to secondary is**

$$\therefore R_{02} = R_2 + R_1^{'} = R_2 + K^2R_1$$

This fact is shown in Fig. 15 where all the resistances of the transformer has been concentrated in the secondary winding.

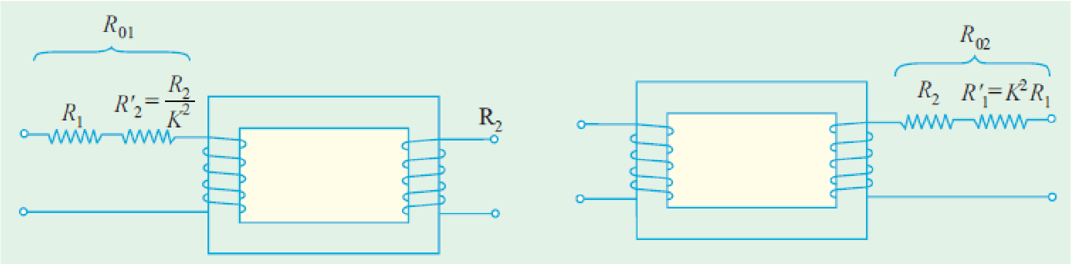


Fig.15

It is to be noted that

5. A resistance of R_1 in primary is equivalent to K^2R_1 in secondary. Hence, it is called **equivalent resistance as referred to secondary** i.e. $R_1{'}$.
6. A resistance of R_2 in secondary is equivalent to $\frac{R_2}{K^2}$ in primary. Hence, it is called the **equivalent secondary resistance as referred to primary** i.e. $R_2{'}$.
7. Total or effective resistance of the transformer as referred to primary is

R_{01} = primary resistance + equivalent secondary resistance as referred to primary

$$R_{01} = R_1 + R_2{'} = R_1 + \frac{R_2}{K^2}$$

8. Similarly, total transformer resistance as referred to secondary is,

R_{02} = secondary resistance + equivalent primary resistance as referred to secondary

$$R_{02} = R_2 + R_1{'} = R_2 + K^2R_1$$

Note: It is important to remember that

d. When shifting any primary resistance to the secondary, **multiply** it by K^2 i.e. (transformation ratio)^2.
e. When shifting secondary resistance to the primary, **divide** it by K^2.
f. However, when shifting any voltage from one winding to another only K is used.

(ii) Magnetic Leakage

- In the preceding discussion, it has been assumed that all the flux linked with primary winding also links the secondary winding. But, in practice, it is impossible to realize this condition.

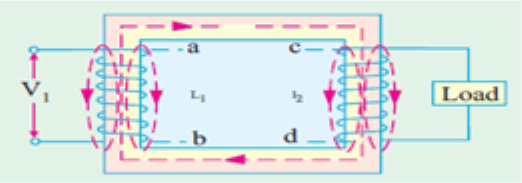


Fig.16

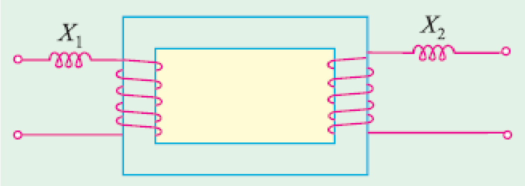


Fig.17

- It is found, however, that all the flux linked with primary does not link the secondary but part of it i.e. $\emptyset_{L_1}$ completes its 1magnetic circuit by passing through air rather than around the core, as shown in Fig.16. This leakage flux is produced when the m.m.f. due to primary ampere-turns existing between points a and b, acts along the leakage paths. Hence, this flux is known as **primary leakage flux** and is proportional to the primary ampere-turns alone because the secondary turns do not link the magnetic circuit of $\emptyset_{L_1}$ The flux $\emptyset_{L_1}$ is in time phase with I_1. It induces an e.m.f. E_{L_1} in primary but not in secondary.
- Similarly, secondary ampere-turns (or m.m.f.) acting across points c and d set up leakage flux $\emptyset_{L_2}$, which is linked with secondary winding alone (and not with primary turns).
- This flux $\emptyset_{L_2}$ is in time phase with I_2 and produces a self-induced e.m.f. E_{L_2} in secondary (but not in primary).
- At no load and light loads, the primary and secondary ampere-turns are small, hence leakage fluxes are negligible. But when load is increased, both primary and secondary windings carry huge currents. Hence, large m.m.f.'s are set up which, while acting on leakage paths, increase the leakage flux.
- As said earlier, the leakage flux linking with each winding produces a self-induced e.m.f. in that winding. Hence, in effect, it is equivalent to a small choker or inductive coil in series with each winding such that voltage drops in each series coil is equal to that produced by leakage flux.
- In other words, **a transformer with magnetic leakage is equivalent to an ideal transformer with inductive coils connected in both primary and secondary circuits** as shown in Fig. 17 such that the

internal e.m.f. in each inductive coil is equal to that due to the corresponding leakage flux in the actual transformer.

$$X_1 = \frac{E_{L1}}{I_1} = \frac{2\pi f L_1 I_1}{I_1} = 2\pi f L_1$$

$$X_2 = \frac{E_{L2}}{I_2} = \frac{2\pi f L_2 I_2}{I_2} = 2\pi f L_2$$

The terms X_1 and X_2 are known as primary and secondary leakage reactance's respectively.

Following few points should be kept in mind:

4. The leakage flux links one or the other winding but **not both**, hence it in no way contributes to the transfer of energy from the primary to the secondary winding.

5. The primary voltage V_1 will have to supply reactive drop I_1X_1 in addition to I_1R_1. Similarly E_2 will have to supply I_2R_2 and I_2X_2.

6. In an actual transformer, the primary and secondary windings are not placed on separate legs or limbs as shown in Fig. 5.23 because due to their being widely separated, large primary and secondary leakage fluxes would result. These leakage fluxes are minimized by sectionalizing and interleaving the primary and secondary windings as in Fig. 5.22 or Fig. 5.24.

- **Transformer with Resistance and Leakage Reactance**

In Fig.18 the primary and secondary windings of a transformer with reactances taken out of the windings are shown. The primary impedance is given by

$$Z_1 = \sqrt{R_1^{\,2} + X_1^{\,2}}$$

Similarly, secondary impedance is given by

$$Z_2 = \sqrt{R_2^{\,2} + X_2^{\,2}}$$

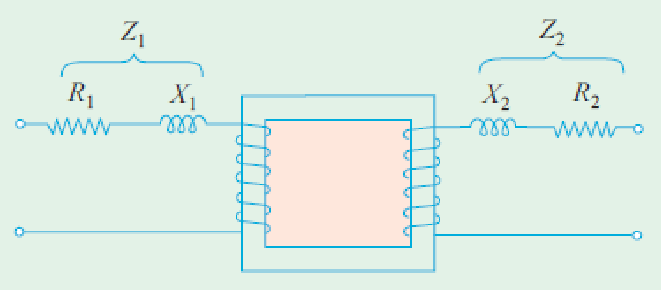


Fig.18

- The resistance and leakage reactance of each winding is responsible for some voltage drop in each winding. In primary, the leakage reactance drop is I_1X_1 (usually 1 or 2% of V_1).Hence

$$V_1 = -E_1 + I_1(R_1 + jX_1)$$

Similarly, there are I_2R_2 and I_2X_2 drops in secondary which combine with V_2 to give E_2.

$$E_2 = V_2 + I_2(R_2 + jX_2)$$

- The vector diagram for such a transformer for different kinds of loads is shown in Fig. 19. In these diagrams, vectors for resistive drops are drawn parallel to current vectors whereas reactive drops are perpendicular to the current vectors. The angle ϕ_1 between V_1 and I_1 gives the power factor angle of the transformer.

It may be noted that leakage reactances can also be transferred from one winding to the other in the same way as resistance.

$$X_2{}' = {}^{X_2}/_{K^2}$$

$$X_1{}' = K^2X_1$$

$$X_{01} = X_1 + X_2{}' = X_1 + \frac{X_2}{K^2}$$

$$X_{02} = X_2 + X_1{}' = X_2 + K^2X_1$$

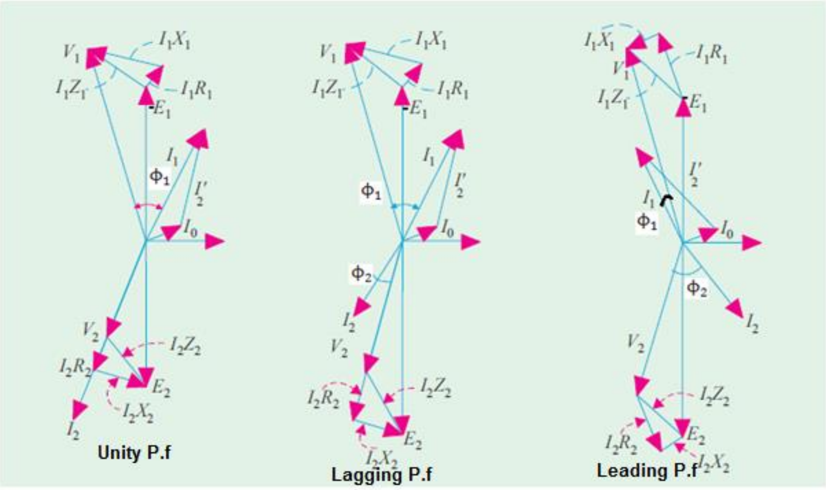


Fig.19

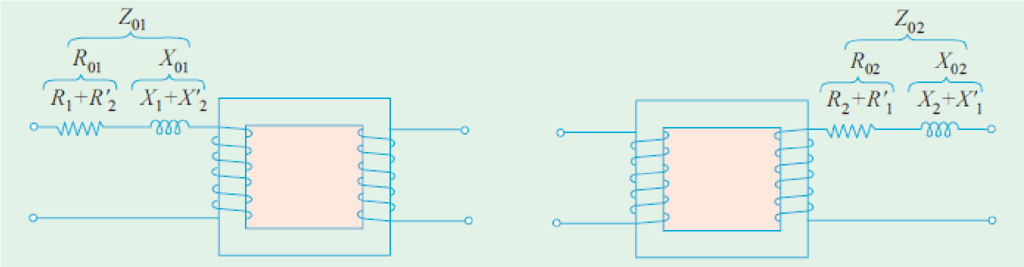


Fig.20

It is obvious that total impedance of the transformer as referred to primary is given by

$$Z_{01} = \sqrt{R_{01}^{\ 2} + X_{01}^{\ 2}}$$

Similarly, total impedance of the transformer as referred to secondary is given by

$$Z_{02} = \sqrt{R_{02}^{\ 2} + X_{02}^{\ 2}}$$

➢ Equivalent circuit of a Transformer

The equivalent circuit of any device can be quite helpful in predetermination of the behavior of the device under various conditions of operation and it can be drawn if the equations describing its behavior are known. If any electrical device is to be analysed and investigated further for suitable modifications its appropriate equivalent circuit is necessary.

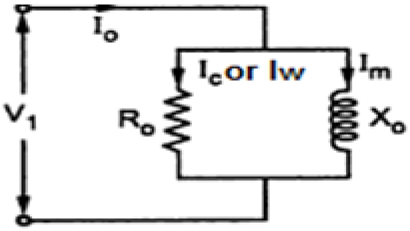


Fig.21

Fig .21 shows the equivalent circuit of transformer on No-Load condition. We already known that transformer on No-Load primary current I_0 has two components

$I_W = I_0 \cos\phi_0 =$ **active** or **working** or **iron** loss component

$I_m = I_0 \sin\phi_0 =$ **magnetising** component

From equivalent circuit we can write,

$$R_o = \frac{V_1}{I_w}$$

$$X_o = \frac{V_1}{I_m}$$

When the load is connected to the transformer then secondary current I_2 flows and operation we already discussed. So the equivalent circuit of transformer on loaded condition is given in fig. 22.

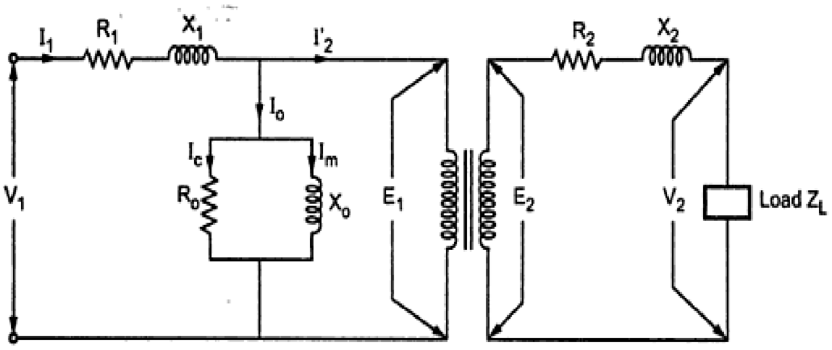


Fig.22

It can be further simplified by transforming all the values to primary or secondary. Fig 23 shows the exact equivalent circuit of a transformer referred to primary by using transformation resistances and reactances as already discussed in previous topics.

Transforming secondary parameters to primary as follows,

$$K = \frac{N_2}{N_1}$$

$$R_2' = \frac{R_2}{K^2} \qquad Z_2' = \frac{Z_2}{K^2}$$

$$X_2' = {}^{X_2}/_{K^2}$$

$$I_2' = KI_2$$

$$E_2' = \frac{E_2}{K}$$

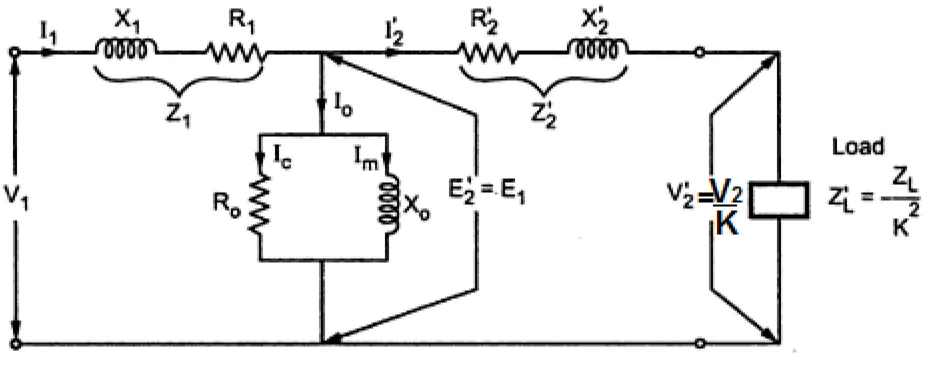


Fig.23 exact equivalent circuit of a transformer referred to primary

Fig. 24 shows the exact equivalent circuit of a transformer referred to secondary

Transforming primary parameters to secondary as follows,

$$R_1' = K^2R_1$$

$$X_1' = K^2X_1$$

$$E_1' = KE_1$$

$$Z_1' = K^2Z_1$$

$$I_1' = \frac{I_1}{K}$$

$$I_0' = \frac{I_0}{K}$$

$$R_0' = K^2R_0$$

$$X_0' = K^2X_0$$

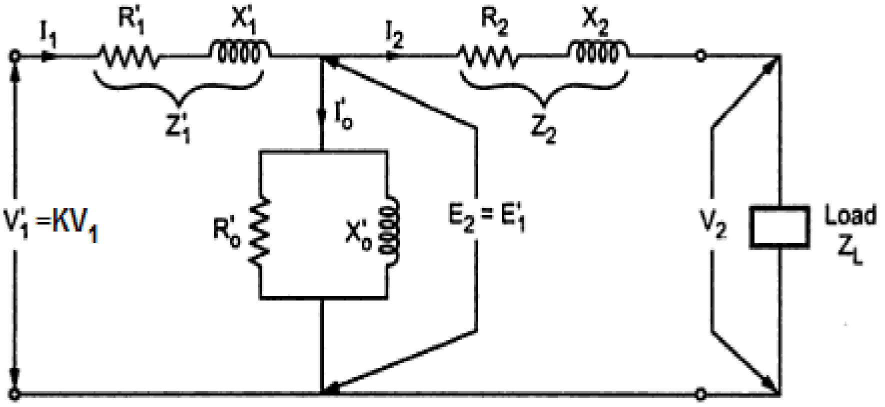


Fig.24. exact equivalent circuit of a transformer referred to secondary

(IV) Approximate equivalent circuit

The equivalent circuit is further simplified by transferring R_0 and X_0 towards left end as shown in fig. 25. The error introduced by doing do is very small and it is neglected. Hence such an equivalent circuit is called **approximate equivalent circuit.**

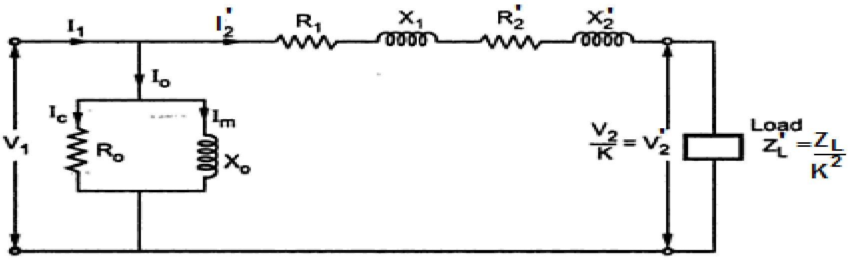


Fig.25

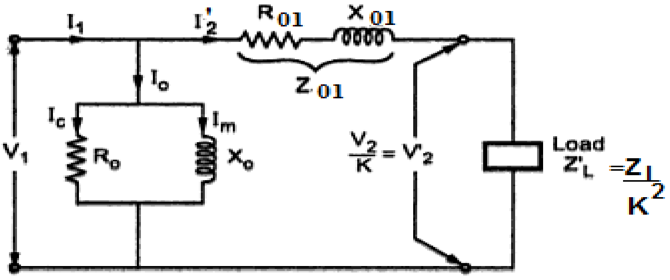


Fig.26

$$R_{01} = R_1 + R_2{}' = R_1 + \frac{R_2}{K^2}$$

$$X_{01} = X_1 + X_2{}' = X_1 + \frac{X_2}{K^2}$$

$$Z_{01} = R_{01} + jX_{01}$$

(V) Total Approximate Voltage Drop in a Transformer

Consider the equivalent circuit referred to secondary as shown in fig 27.

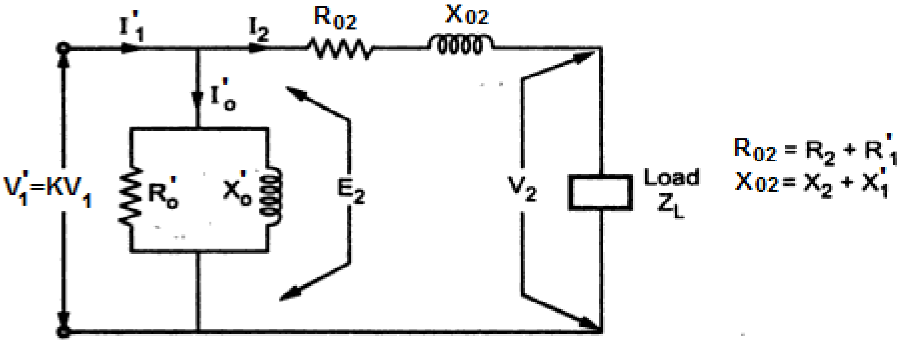


Fig.27

When the transformer is on no-load, and then V_1 is approximately equal to E1. Hence $E_2 = KE_1 = KV_1$. Also, $E_2 = V_{20}$ where V_{20} is secondary terminal voltage on **no load**, hence no-load

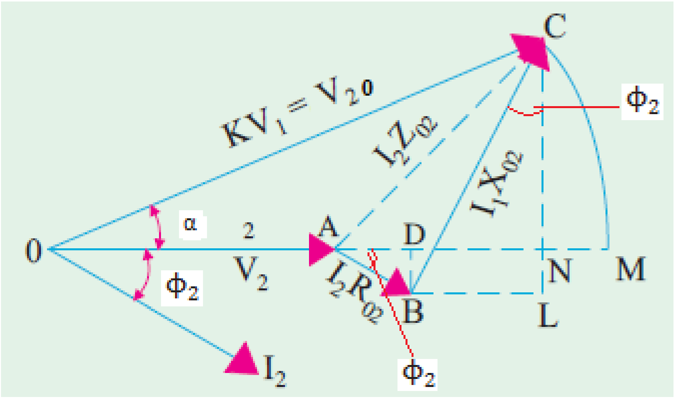


Fig.28

secondary terminal voltage is KV_1. The secondary voltage on load is V_2. The difference between the two is I_2Z_{02} as shown in Fig. 28. The approximate voltage drop of the transformer **as referred to secondary** is found from phasor diagram 28.

$$V_{20} = \text{No load terminal voltage}$$

$$V_2 = \text{Terminal voltage on load}$$

With O as the centre and radius OC draw an arc cutting OA produced at M. The total voltage drop $I_2Z_{02} = AC = AM$, which is approximately equal to AN. From B draw BD perpendicular on OA produced. Draw CN perpendicular to OM and draw BL parallel to OM.

$$AN = \text{Approximate voltage drop}$$

$$AM = \text{Exact voltage drop}$$

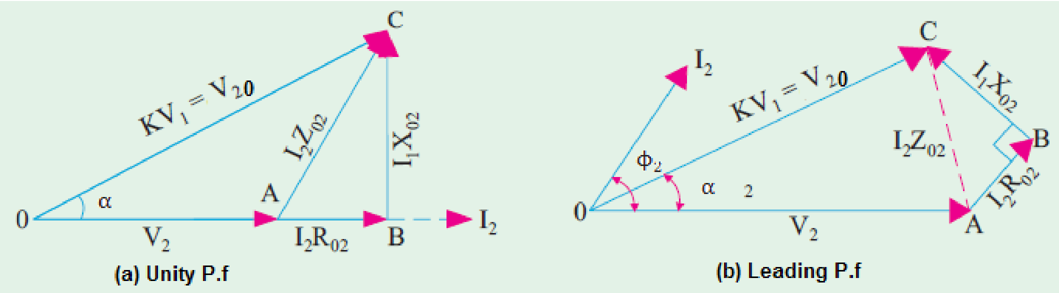
$$\begin{aligned}\text{Approximate voltage drop} &= AN\\ &= AD + DN\\ &= AB\cos\phi_2 + BL\\ &= AB\ \cos\phi_2 + BC\ \sin\phi_2\\ &= I_2R_{02}\cos\phi_2 + I_2X_{02}\sin\phi_2\end{aligned}$$

$$\text{Approximately } \phi_2 = \phi_1 = \phi$$

$$\text{Approximate voltage drop} = I_2R_{02}\cos\phi + I_2X_{02}\sin\phi$$

This is the value of approximate voltage drop for a **lagging** power factor.

The different figures for unity and leading power factors are shown in Fig.29 (*a*) and (*b*) respectively.



(a) Unity P.f

(b) Leading P.f

Fig.29

The approximate voltage drop for **leading** power factor becomes

$$I_2R_{02}\cos\phi - I_2X_{02}\sin\phi$$

Approximate voltage drop=

$$= I_2R_{02}\cos\phi + I_2X_{02}\sin\phi \qquad \text{For lagging P.f}$$

$$= I_2R_{02}\cos\phi - I_2X_{02}\sin\phi \qquad \text{For leading P.f}$$

It may be noted that approximate voltage drop as referred to primary is

$$I_1R_{01}\cos\phi \pm I_1X_{01}\sin\phi$$

$$\%\ \text{Voltage drop in secondary} = \frac{I_2R_{02}\cos\phi \pm I_2X_{02}\sin\phi}{V_{20}} * 100$$

$$= \frac{I_2R_{02}}{V_{20}} * 100 * \cos\phi \pm \frac{I_2X_{02}}{V_{20}} * 100 * \sin\phi$$

$$= V_r * \cos\phi \pm V_x * \sin\phi$$

$$\boxed{\%\ \text{Voltage drop in secondary} = V_r * \cos\phi \pm V_x * \sin\phi}$$

$$V_r = \frac{I_2R_{02}}{V_{20}} * 100 = \text{Percentage resistive drop} = \frac{I_1R_{01}}{V_1} * 100$$

$$V_x = \frac{I_2X_{02}}{V_{20}} * 100 = \text{Percentage reactance drop} = \frac{I_1X_{01}}{V_1} * 100$$

(VI) Exact Voltage Drop

With reference to Fig. 28, it is to be noted that exact voltage drop is AM and not AN. If we add the quantity NM to AN, we will get the exact value of the voltage drop.

Considering the right-angled triangle OCN, we get

$$NC^2 = OC^2 - ON^2$$

$$= (OC + ON)(OC - ON)$$

$$= (OC + OC)(OM - ON) \quad (\because OC \approx ON)$$

$$= 2\ OC * NM$$

$$NM = \frac{NC^2}{2\ OC}$$

$$NC = LC - LN = LC - BD$$

$$\mathrm{NC} = \mathrm{I_2X_{02}cos\phi - I_2R_{02}sin\phi}$$

$$\mathrm{NM} = \frac{(\mathrm{I_2X_{02}cos\phi - I_2R_{02}sin\phi})^2}{2\mathrm{V_{20}}}$$

For a **lagging** power factor, exact voltage drop is =AM

$$\mathrm{AM} = \mathrm{AN} + \mathrm{NM}$$

$$= \mathrm{I_2R_{02}cos\phi + I_2X_{02}sin\phi} + \frac{(\mathrm{I_2X_{02}cos\phi - I_2R_{02}sin\phi})^2}{2\mathrm{V_{20}}}$$

For a **leading** power factor, exact voltage drop is

$$= \mathrm{I_2R_{02}cos\phi - I_2X_{02}sin\phi} + \frac{(\mathrm{I_2X_{02}cos\phi + I_2R_{02}sin\phi})^2}{2\mathrm{V_{20}}}$$

$$\text{Exact Voltage drop} = \mathrm{I_2R_{02}cos\phi \pm I_2X_{02}sin\phi} + \frac{(\mathrm{I_2X_{02}cos\phi \mp I_2R_{02}sin\phi})^2}{2\mathrm{V_{20}}}$$

Percentage voltage drop

$$= \frac{(\mathrm{I_2R_{02}cos\phi \pm I_2X_{02}sin\phi}) * 100}{\mathrm{V_{20}}} + \frac{(\mathrm{I_2X_{02}cos\phi \mp I_2R_{02}sin\phi})^2 * 100}{2\mathrm{V_{20}}^2}$$

$$\text{Percentage voltage drop} = \mathrm{V_r}\ \mathrm{cos\phi} \pm \mathrm{V_x}\ \mathrm{sin\phi} + \frac{1}{200}(\mathrm{V_x}\ \mathrm{cos\phi} \mp \mathrm{V_r}\ \mathrm{sin\phi})^2$$

$$\boxed{\text{Percentage voltage drop} = \mathrm{V_r}\ \mathrm{cos\phi} \pm \mathrm{V_x}\ \mathrm{sin\phi} + \frac{1}{200}(\mathrm{V_x}\ \mathrm{cos\phi} \mp \mathrm{V_r}\ \mathrm{sin\phi})^2}$$

The upper signs are to be used for a **lagging** power factor and the lower ones for a **leading** power factor.

- **Voltage Regulation**

Introduction

The voltage regulation can be defined in two ways - Regulation Down and Regulation up. These two definitions differ only in the reference voltage.

(vi) Regulation down:

- This is defined as "the change in terminal voltage when a load current at any power factor is applied, expressed as a fraction of the no-load terminal voltage".

Expressed in symbolic form we have,

$$\text{Regulation } = \frac{V_{nl} - V_{l}}{V_{nl}}$$

V_{nl} and V_{l} are no-load and load terminal voltages. This is the definition normally used in the case of the transformers, the no-load voltage being the one given by the power supply provider on which the user has no say. Hence no-load voltage is taken as the reference.

(vii) Regulation up:

- Here again the regulation is expressed as the ratio of the change in the terminal voltage when a load at a given power factor is thrown off, and the on load voltage. This definition if expressed in symbolic form results in

$$\text{Regulation } = \frac{V_{nl} - V_{l}}{V_{l}}$$

V_{nl} is the no-load terminal voltage. $V_{l}$is load voltage. Normally full load regulation is of interest as the part load regulation is going to be lower. This definition is more commonly used in the case of alternators and power systems as the user-end voltage is guaranteed by the power supply provider. He has to generate proper no-load voltage at the generating station to provide the user the voltage he has asked for. In the expressions for the regulation, only the numerical differences of the voltages are taken and not vector differences.

In the case of transformers both definitions result in more or less the same value for the regulation as the transformer impedance is very low and the power factor of operation is quite high. The power factor of the load is defined with respect to the terminal voltage on load. Hence a convenient starting point is the load voltage. Also the full load output voltage is taken from the name plate. Hence regulation up has some advantage when it comes to its application.

(viii) Voltage Regulation of a Transformer

The way in which the secondary terminal voltage varies with the load depends on the load current, the internal impedance and the load power factor. The change in secondary terminal voltage from no-load to full load is termed as inherent regulation. It is usually expressed as a percentage or a fraction of the rated no-load terminal voltage.

$$\text{percentage regulation} = \frac{\text{Terminal voltage on no load} - \text{terminal voltage on load}}{\text{Terminal voltage on no load}} * 100$$

$$= \frac{\text{Voltage drop in transformer at load}}{\text{No} - \text{load rated voltage (secondary)}} * 100$$

We already derived voltage drop in transformer at load. Here we take approximate voltage drop.

For lagging power factor

$$\%\ \text{regulation} = \frac{I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi}{\text{No} - \text{load rated voltage (secondary)}} * 100$$

For leading power factor

$$\%\ \text{regulation} = \frac{I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi}{\text{No} - \text{load rated voltage (secondary)}} * 100$$

Voltage regulation of a transformer on an average is about 4 percentage.

(ix) Condition for zero Regulation

$$\%\ \text{regulation} = \frac{I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi}{E_2} * 100$$

Regulation will be zero if the numerator will be equal to zero

$$I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi = 0$$

$$\tan\phi = \frac{-R_{02}}{X_{02}}$$

$$\boxed{\tan\phi = \frac{-R_{02}}{X_{02}}}$$

The -ve sign indicates that zero regulation occurs at a leading power factor.

(x) Condition for Maximum Regulation

Regulation will be maximum if $\frac{d}{d\phi}(\text{regulation}) = 0$

$$\frac{d}{d\phi}\left(\frac{I_2R_{02}\cos\phi + I_2X_{02}\sin\phi}{E_2}\right) = 0$$

$$-\frac{I_2R_{02}}{E_2}\sin\phi + \frac{I_2X_{02}}{E_2}\cos\phi = 0$$

$$\tan\phi = \frac{X_{02}}{R_{02}}$$

$$\boxed{\tan\phi = \frac{X_{02}}{R_{02}}}$$

Maximum regulation will occur at lagging power factor.

- **Losses**

There are two types of power losses occur in a transformer

3. Iron loss
4. Copper loss

3. **Iron Loss:**

This is the power loss that occurs in the iron part. This loss is due to the alternating frequency of the emf. Iron loss in further classified into two other losses.

iii. Eddy current loss
iv. Hysteresis loss

iii. **Eddy current loss:** This power loss is due to the alternating flux linking the core, which will induced an emf in the core called the eddy emf, due to which a current called the eddy current is being circulated in the core. As there is some resistance in the core with this eddy current circulation converts into heat called the eddy current power loss. Eddy current loss is proportional to the square of the supply frequency.

$$\text{Eddy current loss} = K_e {B_m}^2 f^2 t^2 \text{ watts/unit volume}$$

$$\boxed{\text{Eddy current loss} = K_e {B_m}^2 f^2 t^2 \text{ watts/unit volume}}$$

Where, K_e = Eddy current constant

B_m = Maximum flux density

f = frequency

t = thickness of the core

iv. **Hysteresis loss:** This is the loss in the iron core, due to the magnetic reversal of the flux in the core, which results in the form of heat in the core. This loss is directly proportional to the supply frequency.

$$\text{Hysteresis loss} = K_h {B_m}^{1.67} f\, v \text{ watts}$$

$$\boxed{\text{Hysteresis loss} = K_h {B_m}^{1.67} f\, v \text{ watts}}$$

Where, K_h = Hysteresis constant

v = Volume of the core

Eddy current loss can be minimized by using the core made of thin sheets of silicon steel material, and each lamination is coated with varnish insulation to suppress the path of the eddy currents.

Hysteresis loss can be minimized by using the core material having high permeability.

4. **Copper loss:**

This is the power loss that occurs in the primary and secondary coils when the transformer is on load. This power is wasted in the form of heat due to the resistance of the coils. This loss is proportional to the sequence of the load hence it is called the Variable loss where as the Iron loss is called as the Constant loss as the supply voltage and frequency are constants.

$$\begin{aligned}\text{Total copper loss} &= {I_1}^2 R_1 + {I_2}^2 R_2 \\ &= {I_1}^2 (R_1 + R_2{}') \\ &= {I_2}^2 (R_1{}' + R_2)\end{aligned}$$

As **voltage is constant** copper losses are proportional to the square of kVA rating of transformer.

$$P_{cu} \propto I^2 \propto (kVA)^2$$

Thus for transformer

$$\text{Total losses} = \text{Iron losses} + \text{Copper losses}$$

$$= P_i + P_{cu}$$

Volt-Ampere rating (or) Why rating of Transformer in kVA

It is seen that iron losses depend on the supply voltage while the copper losses depend on the current. The losses are not depending on the phase angle between voltage and current. Hence the rating of transformer is expressed as a product of voltage current and called VA rating of transformer. It is not expressed in watts or kilo watts. Most of the times, rating is expressed in kVA.

- **Effect of variations of frequency and supply voltage on Iron losses**

The iron losses of the transformer includes two types of losses

iii. Eddy current loss

iv. Hysteresis loss

For given volume and thickness of laminations, these losses depend on the operating frequency, maximum flux density and voltage.

$$P_e = K_e {B_m}^2 f^2 t^2$$

$$P_h = K_h {B_m}^{1.67} f\, v$$

$$P_e \propto {B_m}^2 f^2$$

$$P_h \propto {B_m}^{1.67} f$$

We know that for transformer

$$V = 4.44\, f\, N\, \emptyset m = 4.44\, f\, N\, B_m A$$

$$B_m \propto \frac{V}{f}$$

$$\boxed{\begin{array}{c} P_e \propto {B_m}^2 f^2 \\ P_h \propto {B_m}^{1.67} f \\ B_m \propto \frac{V}{f} \end{array}}$$

Thus voltage changes flux density changes, both eddy current and hysteresis losses will change.

- If the transformer is operated with the frequency and voltage changed in the same proportion, the flux density will remain unchanged and apparently the no-load current will also remain unaffected.
- The transformer can be operated safely at frequency less than rated one with correspondingly reduced voltage. In this case iron losses will reduced. But if the transformer is operated with increased voltage and frequency in the same proportion, the core losses may increase to an intolerable level.
- Increase in frequency with constant supply voltage will cause reduction in hysteresis loss and leave the eddy current losses unaffected. Some increase in voltage could, therefore, be tolerated at higher frequencies, but exactly how much depends on the relative magnitude of the hysteresis and eddy current losses and the grade of iron used in the transformer core.
- **Efficiency of a Transformer**

The efficiency of any device is defined as the ratio of the power output to power input. The efficiency of a transformer at a particular load and power factor is defined as the output divided by the input. It is expressed as η

$$\eta = \frac{\text{Power output}}{\text{Power input}}$$

$$\eta = \frac{\text{Power output}}{\text{Power output} + \text{Total losses}}$$

$$\eta = \frac{\text{Power output}}{\text{Power output} + P_i + P_{cu}}$$

$$\text{Power output} = V_2 I_2 \cos\phi$$

$$\cos\phi = \text{Load power factor}$$

Transformer supplies full load current of I_2 and with terminal voltage V_2

$$P_{cu} = \text{ copper losses on full load} = {I_2}^2 R_{02}$$

$$\eta = \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + P_i + {I_2}^2 R_{02}}$$

$$V_2 I_2 = \text{VA rating of a transformer}$$

$$\%\, \eta = \frac{(\text{VA rating}) * \cos\phi}{(\text{VA rating}) * \cos\phi + P_i + {I_2}^2 R_{02}} * 100$$

$$\boxed{\%\, \eta = \frac{(\text{VA rating}) * \cos\phi}{(\text{VA rating}) * \cos\phi + P_i + {I_2}^2 R_{02}} * 100}$$

This is full load efficiency with, $I_2 = \text{full load secondary current}$

But if the transformer is subjected to fractional load then using the appropriate values of various quantities, the efficiency can be obtained.

$$x = \text{Fraction by which load is less than full load} = \frac{\text{Actual load}}{\text{full load}}$$

When load changes, the load current changes by same proportion.

$$\text{new } I_2 = x(I_2)F.L$$

Similarly the output power also reduces by same fraction.

Similarly as copper losses are proportional to square of current then

$$\text{new } P_{cu} = x^2 (P_{cu})F.L$$

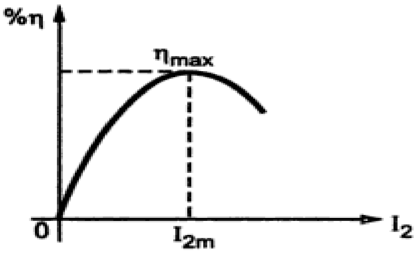
In general for fractional load the efficiency is given by,

$$\%\, \eta = \frac{x(\text{VA rating}) * \cos\phi}{x(\text{VA rating}) * \cos\phi + P_i + x^2 (P_{cu})F.L} * 100$$

$$\boxed{\%\, \eta = \frac{x(\text{VA rating}) * \cos\phi}{x(\text{VA rating}) * \cos\phi + P_i + x^2 (P_{cu})F.L} * 100}$$

- **Condition for maximum efficiency:**

In general for the efficiency to be maximum for any device the losses must be minimum. Between the iron and copper losses the iron loss is the fixed loss and the copper loss is the variable loss. When these two losses are equal and also minimum the efficiency will be maximum.



Therefore the condition for maximum efficiency in a transformer is

$$\text{Iron loss} = \text{Copper loss}$$

When transformer works on a constant input voltage and frequency then efficiency varies with the load. As load increases, the efficiency increases. At a certain load current, it achieves a maximum value. If the transformer is loaded further the efficiency starts decreasing. The graph of efficiency against load current I_2 is shown in fig 5.40.

The load current at which the efficiency attains maximum value is denoted as I_{2m} and maximum efficiency is denoted as η_m.

So for maximum efficiency

$$\frac{d\eta}{dI_2} = 0$$

$$\eta = \frac{V_2 I_2 * \cos\phi_2}{V_2 I_2 * \cos\phi_2 + P_i + P_{Cu}}$$

$$\eta = \frac{V_2 I_2 * \cos\phi_2}{V_2 I_2 * \cos\phi_2 + P_i + I_2{}^2 R_{02}}$$

$$\frac{d\eta}{dI_2} = \frac{d}{dI_2}\left[\frac{V_2 I_2 * \cos\phi_2}{V_2 I_2 * \cos\phi_2 + P_i + I_2{}^2 R_{02}}\right] = 0$$

$$\left(V_2 I_2 * \cos\phi_2 + P_i + I_2{}^2 R_{02}\right)\frac{d}{dI_2}(V_2 I_2 * \cos\phi_2)$$

$$- (V_2 I_2 * \cos\phi_2)\frac{d}{dI_2}\left(V_2 I_2 * \cos\phi_2 + P_i + I_2{}^2 R_{02}\right) = 0$$

$$\left(V_2 I_2 * \cos\phi_2 + P_i + I_2{}^2 R_{02}\right)(V_2 \cos\phi_2) - (V_2 I_2 * \cos\phi_2)(V_2 \cos\phi_2 + 2 I_2 R_{02}) = 0$$

$$(V_2 \cos\phi_2)\left[V_2 I_2 * \cos\phi_2 + P_i + I_2{}^2 R_{02} - V_2 I_2 \cos\phi_2 - 2 I_2{}^2 R_{02}\right] = 0$$

$$\left[V_2 I_2 * \cos\phi_2 + P_i + I_2{}^2 R_{02} - V_2 I_2 \cos\phi_2 - 2 I_2{}^2 R_{02}\right] = 0$$

$$P_i - I_2{}^2 R_{02} = 0$$

$$P_i = I_2{}^2 R_{02} = P_{Cu}$$

$$\boxed{\text{Iron loss} = \text{Copper loss}}$$

(i) Load current I_{2m} at maximum efficiency

For η_{max}, $P_i = I_2{}^2 R_{02}$ but $I_2 = I_{2m}$

$$I_{2m}{}^2 R_{02} = P_i$$

$$I_{2m} = \sqrt{\frac{P_i}{R_{02}}}$$

$$I_{2\,F.L} = \text{Full load current}$$

$$I_{2m} = \frac{I_{2\,F.L}}{I_{2\,F.L}}\sqrt{\frac{P_i}{R_{02}}}$$

$$I_{2m} = I_{2\,F.L}\sqrt{\frac{P_i}{{I_{2\,F.L}}^2 R_{02}}}$$

$$I_{2m} = I_{2\,F.L}\sqrt{\frac{P_i}{P_{Cu\,(F.L)}}}$$

$$\boxed{I_{2m} = I_{2\,F.L}\sqrt{\frac{P_i}{P_{Cu\,(F.L)}}}}$$

This is the load current at η_{max} in terms of full load current.

$$\frac{I_{2m}}{I_{2\,F.L}} = \sqrt{\frac{P_i}{P_{Cu\,(F.L)}}} = X$$

$$\boxed{X = \sqrt{\frac{P_i}{P_{Cu\,(F.L)}}}}$$

X is the fraction of load maximum efficiency

(ii) kVA supplied at maximum efficiency

For constant V_2 the kVA supplied is the function of load current.

$$\text{kVA at } \eta_{max} = I_{2m} V_2 = V_2 I_{2\,F.L}\sqrt{\frac{P_i}{P_{Cu\,(F.L)}}}$$

$$\boxed{\text{kVA at } \eta_{max} = (\text{kVA rating}) * \sqrt{\frac{P_i}{P_{Cu\,(F.L)}}}}$$

Substituting condition for η_{max} in the expression of efficiency, we can write expression for η_{max} as,

$$\% \, \eta_{max} = \frac{V_2 I_{2m} \cos\phi}{V_2 I_{2m} \cos\phi + 2P_i} * 100 \qquad \text{as } P_i = P_{Cu}$$

$$\% \, \eta_{max} = \frac{\text{kVA for } \eta_{max} \cos\phi}{\text{kVA for } \eta_{max} \cos\phi + 2P_i} * 100$$

$$\boxed{\% \, \eta_{max} = \frac{\text{kVA for } \eta_{max} \cos\phi}{\text{kVA for } \eta_{max} \cos\phi + 2P_i} * 100}$$

Unit- IV

AC MACHINES

Objectives

- To familiarize the students with the constructional details, working principles of alternator and induction motors
- To familiarize the students about the slip-torque characteristics of induction motors
- To familiarize the students about the real time applications of induction motors

Syllabus : AC MACHINES

Principle of operation of alternators - principle of operation of induction motor – slip-torque characteristics-applications-single phase induction motor- constructional features-principle of operation

Learning outcomes :

Student will be able to

- Describe the constructional features of alternator and induction motor
- Understand the working principle of induction motor
- Describe the characteristics of induction motor
- Select an appropriate machine to meet specified performance requirements for a particular application

Unit- IV

AC MACHINES

An alternator is an electrical machine which converts mechanical energy into alternating electric energy. They are also known as synchronous generators.

How Does An AC Generator Work?

The working principle of an alternator or AC generator is similar to the basic working principle of a DC generator.

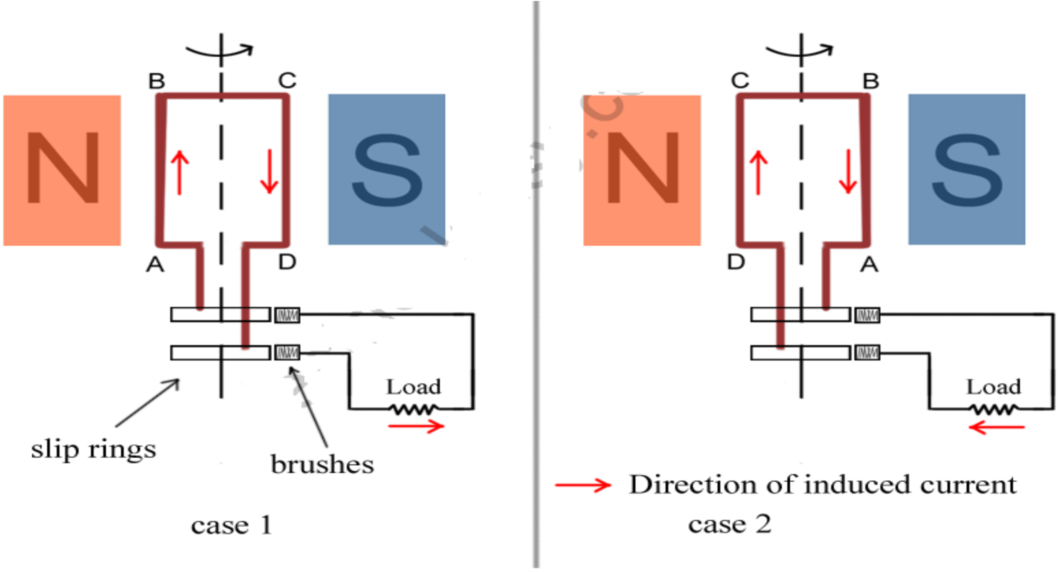


Fig.4.1 Working principle of alternator

Above figure helps you understanding how an alternator or AC generator works. According to theFaraday's law of electromagnetic induction, whenever a conductor moves in a magnetic field EMF gets induced across the conductor. If the close path is provided to the conductor, induced emf causes current to flow in the circuit.

Now, see the above figure. Let the conductor coil ABCD is placed in a magnetic field. The direction of magnetic flux will be form N pole to S pole. The coil is connected to slip rings, and the load is connected through brushes resting on the slip rings.

Now, consider the case 1 from above figure. The coil is rotating clockwise, in this case the direction of induced current can be given by Fleming's right hand rule, and it will be along A-B-C-D. As the coil is rotating clockwise, after half of the time period, the position of the coil will be as in second case of above figure. In this case, the direction of the induced current according to Fleming's right hand rule will be along D-C-B-A. It shows that, the direction of the current changes after half of the time period, that means we get an alternating current.

Construction Of AC Generator (Alternator)

Main parts of the alternator, obviously, consists of stator and rotor. But, the unlike other machines, in most of the alternators, field exciters are rotating and the armature coil is stationary.

Stator: Unlike in DC machine stator of an alternator is not meant to serve path for magnetic flux. Instead, the stator is used for holding armature winding. The stator core is made up of lamination of steel alloys or magnetic iron, to minimize the eddy current losses.

Why Armature Winding Is Stationary In An Alternator?

- At high voltages, it easier to insulate stationary armature winding, which may be as high as 30 kV or more.
- The high voltage output can be directly taken out from the stationary armature. Whereas, for a rotary armature, there will be large brush contact drop at higher voltages, also the sparking at the brush surface will occur.
- Field exciter winding is placed in rotor, and the low dc voltage can be transferred safely.
- The armature winding can be braced well, so as to prevent deformation caused by the high centrifugal force.

Rotor: There are two types of rotor used in an AC generator / alternator:
(i) Salient and (ii) Cylindrical type

1. Salient pole type: Salient pole type rotor is used in low and medium speed alternators. Construction of AC generator of salient pole type rotor is shown in the figure above. This type of rotor consists of large number of projected poles (called salient poles), bolted on a magnetic wheel. These poles are also laminated to minimize the eddy current losses. Alternators featuring this type of rotor are large in diameters and short in axial length.
2. Cylindrical type: Cylindrical type rotors are used in high speed alternators, especially in turbo alternators. This type of rotor consists of a smooth and solid steel cylinder havingg slots along its outer periphery. Field windings are placed in these slots.

The DC suppy is given to the rotor winding through the slip rings and and brushes arrangement.

Working Principle and Types of Induction Motors

Induction Motors are the most commonly used motors in many applications. These are also called asAsynchronous Motors, because an induction motor always runs at a speed lower than synchronous speed. Synchronous speed means the speed of the rotating magnetic field in the stator.

There are basically two types of induction motor depending upon the type of input supply –
(i) Single phase induction motor and (ii) Three phase induction motor.

Or they can be divided according to type of rotor - (i) Squirrel cage motor and (ii) Slip ring motor or wound type

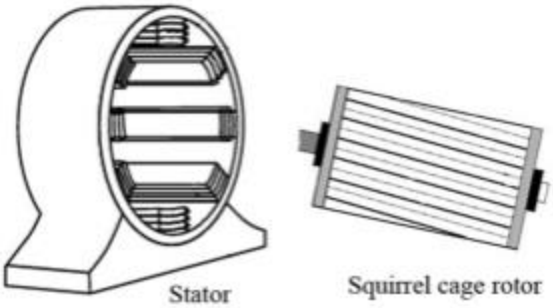


Fig.4.2 Types of rotors

Construction of a 3 Phase Induction Motor

Just like any other motor, a 3 phase induction motor also consists of a stator and a rotor. Basically there are two types of 3 phase IM - 1. Squirrel cage induction motor and 2. Phase Wound induction motor (slip-ring induction motor). Both types have similar constructed rotor, but they differ in construction of rotor. This is explained further

Stator

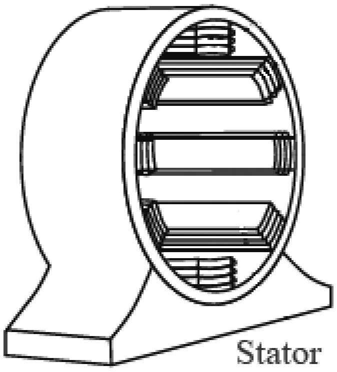


Fig.4.3 stator

The stator of a 3 phase IM (Induction Motor) is made up with number of stampings, and these stampings are slotted to receive the stator winding. The stator is wound with a 3 phase winding which is fed from a 3 phase supply. It is wound for a defined number of poles, and the number of poles is determined from the required speed. For greater speed, lesser number of poles is used and vice versa. When stator windings are supplied with 3 phase ac supply, they produce alternating flux which revolves with synchronous speed. The synchronous speed is inversely proportional to number of poles ($N_s = 120f / P$). This revolving or rotating magnetic flux induces current in rotor windings according to Faraday's law of mutual induction.

Rotor

As described earlier, rotor of a 3 phase induction motor can be of either two types, squirrel cage rotorand phase wound rotor (or simply - wound rotor).

Squirrel Cage Rotor

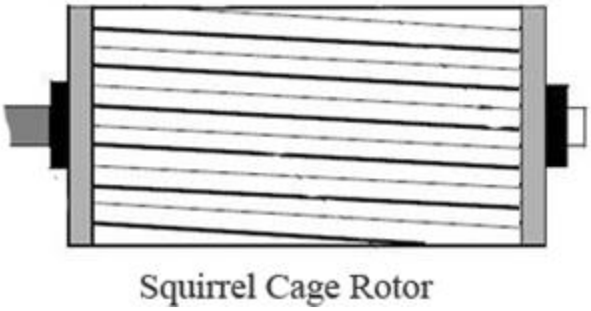


Fig.4.4 squirrel cage rotor

Most of the induction motors (upto 90%) are of squirrel cage type. Squirrel cage type rotor has very simple and almost indestructible construction. This type of rotor consist of a cylindrical laminated core, having parallel slots on it. These parallel slots carry rotor conductors. In this type of rotor, heavy bars of copper, aluminum or alloys are used as rotor conductors instead of wires. Rotor slots are slightly skewed to achieve following advantages -

1. it reduces locking tendency of the rotor, i.e. the tendency of rotor teeth to remain under stator teeth due to magnetic attraction.

2. increases the effective transformation ratio between stator and rotor

3. increases rotor resistance due to increased length of the rotor conductor

The rotor bars are brazed or electrically welded to short circuiting end rings at both ends. Thus this rotor construction looks like a squirrel cage and hence we call it. The rotor bars are permanently short circuited, hence it is not possible to add any external resistance to armature circuit.

Phase Wound Rotor

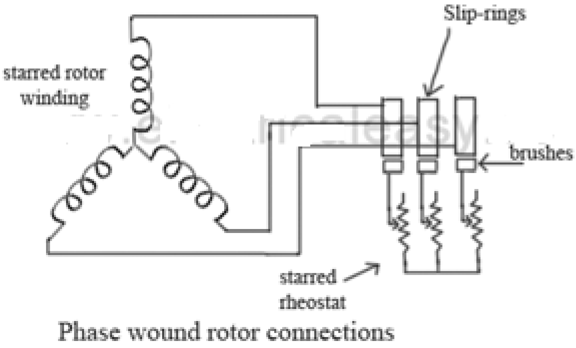


Fig.4.5 phase wound rotor

Phase wound rotor is wound with 3 phase, double layer, distributed winding. The number of poles of rotor are kept same to the number of poles of the stator. The rotor is always wound 3 phase even if the stator is wound two phase.The three phase rotor winding is internally star connected. The other three terminals of the winding are taken out via three insulated sleep rings mounted on the shaft and the brushes resting on them. These three brushes are connected to an external star connected rheostat. This arrangement is done to introduce an external resistance in rotor circuit for starting purposes and for changing the speed/torque characteristics.
When motor is running at its rated speed, slip rings are automatically short circuited by means of a metal collar and brushes are lifted above the slip rings to minimize the frictional losses.

Single Phase Induction Motor

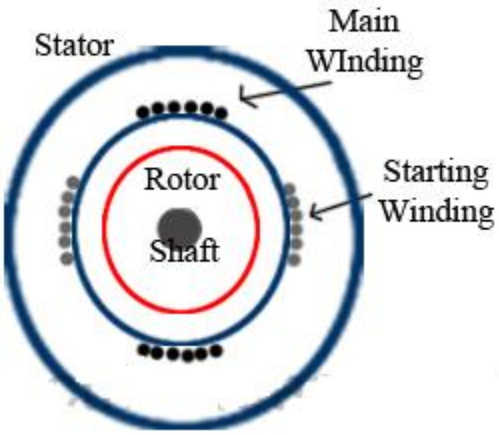


Fig.4.6 Schematic of single phase induction motor

Construction of a single phase induction motor is similar to the construction of three phase induction motor having squirrel cage rotor, except that the stator is wound for single phase supply. Stator is also provided with a 'starting winding' which is used only for starting purpose. This can be understood from the schematic of single phase induction motor above.

Basic Working Principle Of An Induction Motor

In a DC motor, supply is needed to be given for the stator winding as well as the rotor winding. But in aninduction motor only the stator winding is fed with an AC supply.

- Alternating flux is produced around the stator winding due to AC supply. This alternating flux revolves with synchronous speed. The revolving flux is called as "Rotating Magnetic Field" (RMF).
- The relative speed between stator RMF and rotor conductors causes an induced emf in the rotor conductors, according to the Faraday's law of electromagnetic induction. The rotor conductors are short circuited, and hence rotor current is produced due to induced emf. That is why such motors are called as induction motors.
  (This action is same as that occurs in transformers, hence induction motors can be called as rotating transformers.)

- Now, induced current in rotor will also produce alternating flux around it. This rotor flux lags behind the stator flux. The direction of induced rotor current, according to Lenz's law, is such that it will tend to oppose the cause of its production.
- As the cause of production of rotor current is the relative velocity between rotating stator flux and the rotor, the rotor will try to catch up with the stator RMF. Thus the rotor rotates in the same direction as that of stator flux to minimize the relative velocity. However, the rotor never succeeds in catching up the synchronous speed. This is the basic working principle of induction motor of either type, single phase of 3 phase.

Synchronous Speed:

The rotational speed of the rotating magnetic field is called as synchronous speed.

$$Ns = \frac{120 \text{ x } f}{P} \quad \text{(RPM)}$$

where, f = frequency of the spply

P = number of poles

Slip:

Rotor tries to catch up the synchronous speed of the stator field, and hence it rotates. But in practice, rotor never succeeds in catching up. If rotor catches up the stator speed, there wont be any relative speed between the stator flux and the rotor, hence no induced rotor current and no torque production to maintain the rotation. However, this won't stop the motor, the rotor will slow down due to lost of torque, the torque will again be exerted due to relative speed. That is why the rotor rotates at speed which is always less the synchronous speed.

The difference between the synchronous speed (N_s) and actual speed (N) of the rotor is called as slip.

$$\% \text{ slip} \quad s = \frac{Ns - N}{Ns} \text{ x } 100$$

How rotating magnetic field produced in induction motor?

the induction motor does not have any direct supply onto the rotor; instead, a secondary current is induced in the rotor. To achieve this, stator windings are arranged around the rotor so that when energized with a polyphase supply they create a rotating magnetic field pattern which sweeps past the rotor. This changing magnetic field pattern induces current in the rotor conductors. These currents interact with the rotating magnetic field created by the stator and in effect causes a rotational motion on the rotor.

Why Single Phase Induction Motor Is Not Self Starting?

The stator of a single phase induction motor is wound with single phase winding. When the stator is fed with a single phase supply, it produces alternating flux (which alternates along one space axis only). Alternating flux acting on a squirrel cage rotor can not produce rotation, only revolving flux can. That is why a single phase induction motor is not self starting.

How to make Single Phase Induction Motor Self Starting?

- As explained above, single phase induction motor is not self-starting. To make it self-starting, it can be temporarily converted into a two-phase motor while starting. This can be achieved by introducing an additional 'starting winding' also called as auxillary winding.
- Hence, stator of a single phase motor has two windings: (i) Main winding and (ii) Starting winding (auxillary winding). These two windings are connected in parallel across a single phase supply and are spaced 90 electrical degrees apart. Phase difference of 90 degree can be achieved by connecting a capacitor in series with the starting winding.
- Hence the motor behaves like a two-phase motor and the stator produces revolving magnetic field which causes rotor to run. Once motor gathers speed, say upto 80 or 90% of its normal speed, the starting winding gets disconnected form the circuit by means of a centrifugal switch, and the motor runs only on main winding.

Depending upon the methods for making asynchronous motor as Self Starting Motor, there are mainly four types of single phase induction motor namely,

1. Split phase induction motor,
2. Capacitor start inductor motor,
3. Capacitor start capacitor run induction motor,
4. Shaded pole induction motor.
5. Permanent split capacitor motor or single value capacitor motor.

Three Phase Induction Motor

A three phase induction motor runs on a three phase AC supply. This machine is called as asynchronous machine because it never runs at synchronous speed. Stator winding is made up of copper. Rotor winding depends on the type of rotor used.

Stator winding is connected to three phase supply that setup rotating magnetic field in the airgap. This magnetic field revolves at a speed called as synchronous speed.

Like any electric motor, a 3-phase induction motor has a stator and a rotor. The stator carries a 3-phase winding (called stator winding) while the rotor carries a short-circuited winding (called rotor winding). Only the stator winding is fed from 3-phase supply. The rotor winding derives its voltage and power from the externally energized stator winding through electromagnetic induction and hence the name. The induction motor may be considered to be a transformer with a rotating secondary and it can, therefore, be described as a transformer type a.c. machine in which electrical energy is converted into mechanical energy.

Advantages:

It has simple and rugged construction.
It is relatively cheap.
It requires little maintenance.

It has high efficiency and reasonably good power factor.
It has self starting torque

Disadvantages:

It is essentially a constant speed motor and its speed cannot be changed easily.
Its starting torque is inferior to d.c. shunt motor

Synchronous speed:

The speed at which magnetic field produced by primary current rotates is called as synchronous speed. This magnetic field cuts the rotor conductors, and then emf will be induced in the rotor conductors. Since the rotor circuit is closed, current will flow in rotor conductors which set ups another magnetic field. Due to interaction of air gap field and rotor magnetic field torque will develop. When rotor is at rest rotor speed is zero. Relative speed between stator field rotor is Ns

According to Lenz's law affect must oppose the cause.

Rotor starts rotating to reduce the relative speed, i.e. rotor rotates in the direction of air gap field.

Frequency of stator field (f) = $\frac{P}{2} \times \frac{Ns}{60}$

$$Ns = \frac{120f}{P}$$

Frequency of rotor current (f_2) = $\frac{P}{2} \times \frac{Ns-Nr}{60}$

$$= \frac{P}{2} \times \frac{SNs}{60}$$

$$f_2 = S.f$$

when rotor speed(Nr) reaches synchronous speed, relative speed becomes zero. Emf induced in rotor conductor is zero then torque becomes zero. Machine should stop working abruptly. Due to inertia does not stop suddenly. Therefore rotor slows down. Due to relative speed, again magnetic field cuts the rotor conductors. Therefore the torque will zero. Let us consider Nr is rotor speed, Ns is the synchronous speed. The difference in speed between Ns and Nr is called slip speed.

$$Ns - Nr = S\ Ns$$

$$Slip(S) = \frac{Ns-Nr}{Ns}$$

Rotor current:

Rotor current (I_2) = SE2$\frac{SE_2}{\sqrt{{r_2}^2+(Sx_2)^2}}$

Where r_2, x_2 are rotor resistance and leakage reactance.

Rotor power factor $(\cos\Theta_2) = \frac{R_2}{\sqrt{r_2{}^2+(Sx_2)^2}}$

Air gap power $(p_g) = 3E_2 \times I_2 \times \cos\Theta$

$$= 3\ E_2 \times \frac{SE_2}{\sqrt{r_2{}^2+(Sx_2)^2}} \times \frac{R_2}{\sqrt{r_2{}^2+(Sx_2)^2}}$$

$$= \frac{3 \times S \times R_2 E_2^2}{r_2^2+(Sx_2^2)}$$

$$=3\left(\frac{SE_2}{\sqrt{r_2{}^2+(Sx_2)^2}}\right)^2 \times \frac{r_2}{S}$$

$$P_g = 3\ I_2{}^2 \times \frac{r_2}{S}$$

Air gap power = rotor copper loss + mechanical power developed in rotor.

$$3\ I_2{}^2 \times \frac{r_2}{S} = 3I_2{}^2\, r_2 + 3I_2{}^2\, r_2\left(\frac{1-s}{s}\right)$$

Air gap power: rotor copper loss: mechanical power developed in rotor =

$$3\ I_2{}^2 \times \frac{r_2}{S} : 3I_2{}^2\, r_2 : 3I_2{}^2\, r_2\left(\frac{1-s}{s}\right)$$

$$Pg:P_{cu}:P_{mech} = 1:S:(1-S)$$

Mechanical power developed in rotor $(P_{mech}) = Te \times \omega_r = (1-S)Pg$

$$Te = \frac{(1-S)Pg}{\omega r} = \frac{(1-S)Pg}{\omega s(1-S)} = \frac{Pg}{\omega s}$$

Shaft power (Psh) = mechanical power developed in rotor – windage and friction losses

Shaft torque $(Tsh) = \frac{Psh}{\omega r}$

Power flow diagram of induction motor:

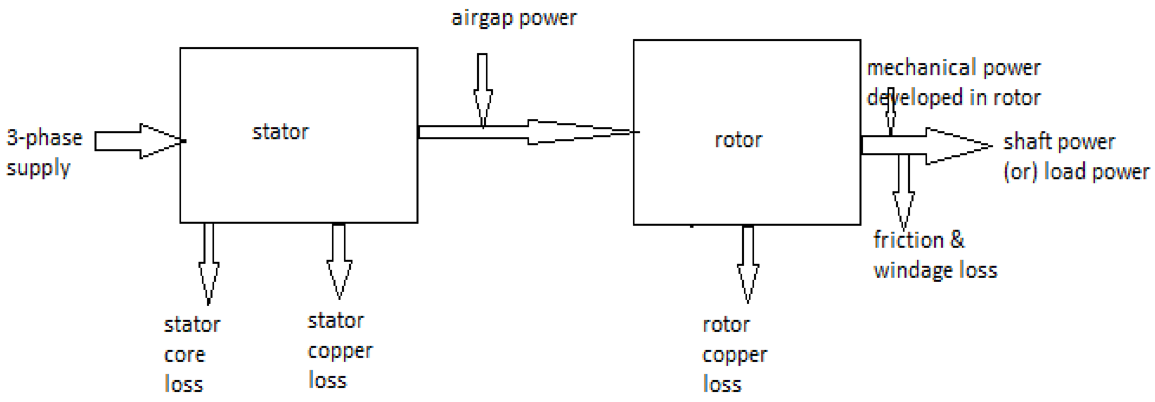


Fig.4.7 power flow diagram of induction motor

Equivalent circuit of induction motor:

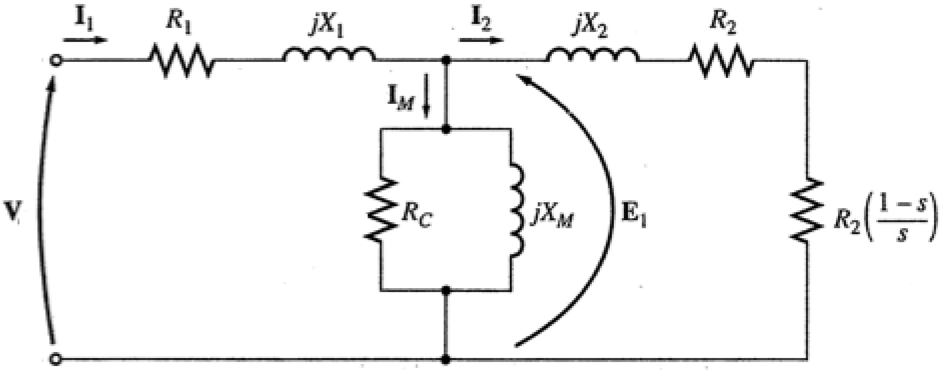


Fig.4.8 Equivalent circuit of induction motor

Where,

R_c = core loss

x_m = magnetizing component

R_1, X_1 = Resistance and inductive reactance of primary winding.

R_2, X_2 = Resistance and inductive reactance of secondary winding.

R_2^1, X_2^1 = Resistance and inductive reactance of secondary winding with referred to stator.

$R_2^1\frac{(1-S)}{S}$ = mechanical power developed in machine.

$R_2^1\frac{(1-S)}{S}$ is positive when $0<S<1$ (motoring mode)

$R_2^1\frac{(1-S)}{S}$ is negative when $S<0$ (generating mode)

Types of induction motors:

Based on the rotor construction induction motors are classified as

(i) Squirrel cage induction motor
(ii) Slip ring induction motor

Squirrel cage induction motor

- This is cylindrical in shape and it does not have rotor slots to place windings.
- Copper bars are accommodate on the rotor surface
- To provide closed path for rotor currents, rotor bars are short circuited by the end rings.

- End rings are made up of copper and brass.
- Since copper bars are short circuited, external resistance cannot be added
- Rotor bars are skewed to avoid magnetic locking, humming.

Slip ring induction motor

- This is cylindrical in shape and having rotor slots to accommodate rotor conductors
- Rotor winding is star connected and is connected to slip rings
- External resistance can be added through slip rings.

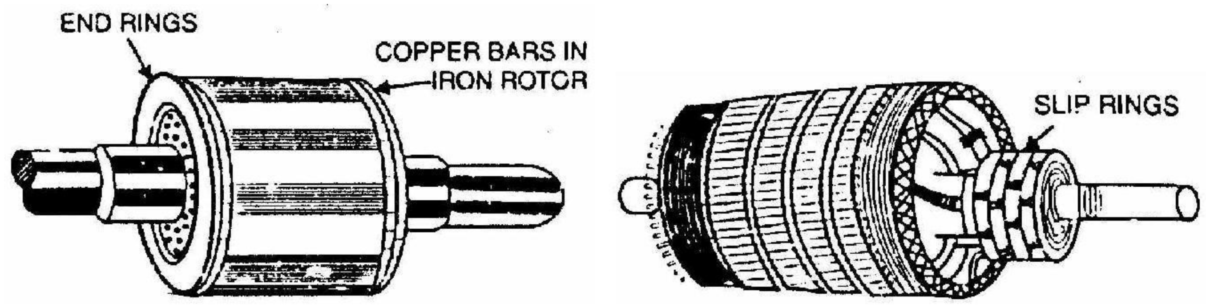


Fig.4.9 Types of rotors of motor

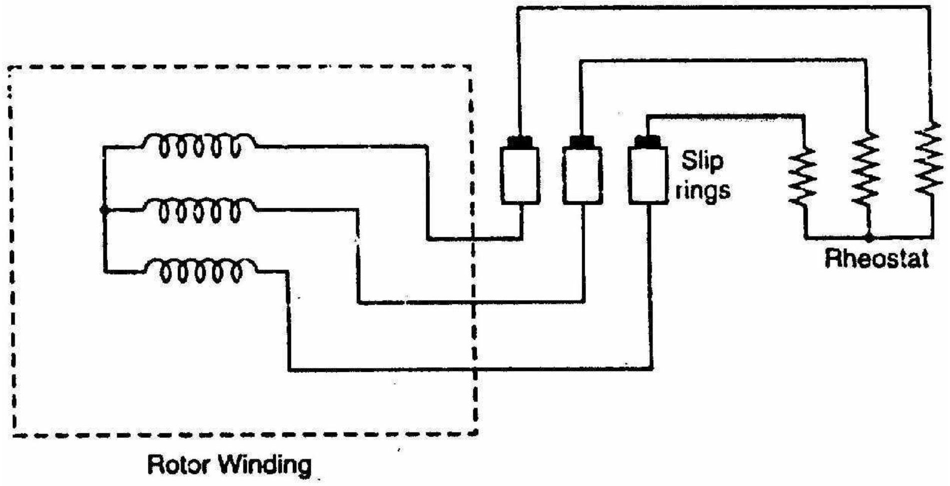


Fig.4.10 Slip ring induction motor connections

Torque Slip Characteristics of Three phase Induction Motor

The torque slip curve for an induction motor gives us the information about the variation of torque with the slip. The slip is defined as the ratio of difference of synchronous speed and actual rotor speed to the synchronous speed of the machine. The variation of slip can be obtained with the variation of speed that is when speed varies the slip will also vary and the torque corresponding to that speed will also vary. The curve can be described in three modes of operation-

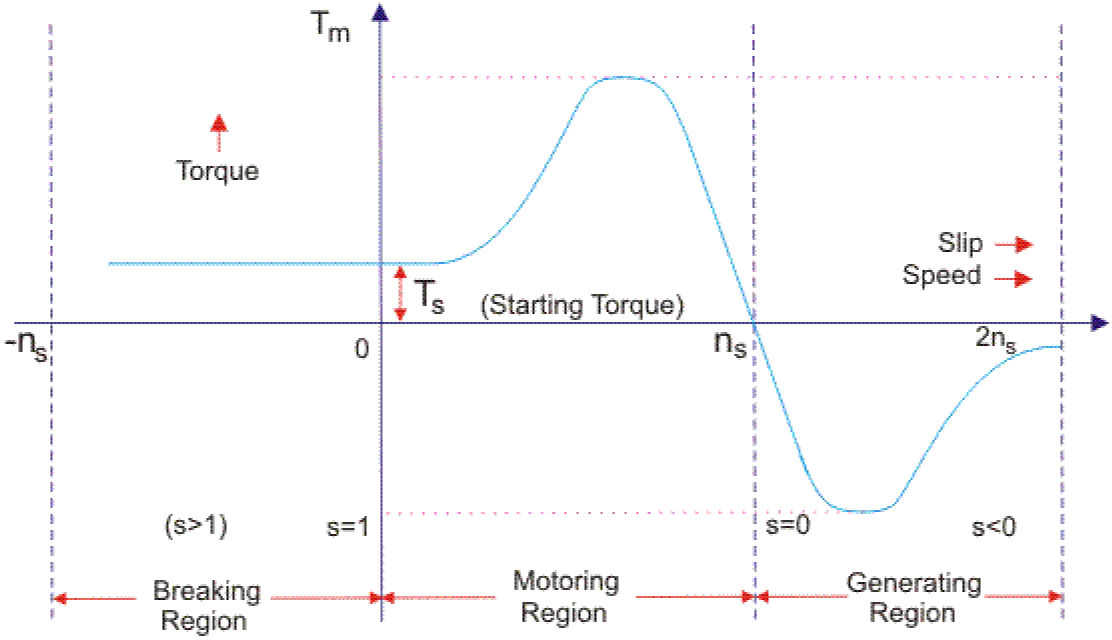


Fig.4.11 Torque slip characteristics of Three phase induction motor

Motoring Mode: In this mode of operation, supply is given to the stator sides and the motor always rotates below the synchronous speed. The induction motor torque varies from zero to full load torque as the slip varies. The slip varies from zero to one. It is zero at no load and one at standstill. From the curve it is seen that the torque is directly proportional to the slip. That is, more is the slip, more will be the torque produced and vice-versa. The linear relationship simplifies the calculation of motor parameter to great extent.

Generating Mode: In this mode of operation induction motor runs above the synchronous speed and it should be driven by a prime mover. The stator winding is connected to a three phase supply in which it supplies electrical energy. Actually, in this case, the torque and slip both are negative so the motor receives mechanical energy and delivers electrical energy. Induction motor is not much used as generator because it requires reactive power for its operation. That is, reactive power should be supplied from outside and if it runs below the synchronous speed by any means, it consumes electrical energy rather than giving it at the output. So, as far as possible, induction generators are generally avoided.

Braking Mode: In the Braking mode, the two leads or the polarity of the supply voltage is changed so that the motor starts to rotate in the reverse direction and as a result the motor stops. This method of braking is known as plugging. This method is used when it is required to stop the motor within a very short period of time. The kinetic energy stored in the revolving load is dissipated as heat. Also, motor is still receiving power from the stator which is also dissipated as heat. So as a result of which motor develops enormous heat energy. For this stator is disconnected from the supply before motor enters the braking mode.

If load which the motor drives accelerates the motor in the same direction as the motor is rotating, the speed of the motor may increase more than synchronous speed. In this case, it acts as an induction generator which supplies electrical energy to the mains which tends to slow down the motor to its synchronous speed, in this case the motor stops. This type of breaking principle is called dynamic or regenerative breaking.

Torque Slip Characteristics of Single Phase Induction Motor

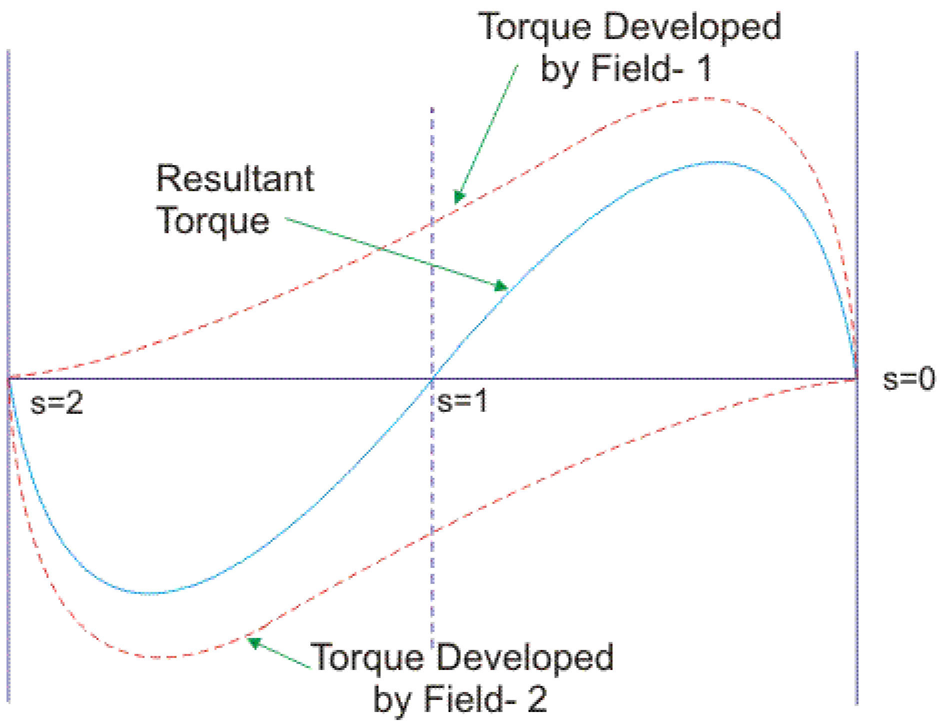


Fig.4.12 Torque slip characteristics of Single phase Induction motor

From the figure, we see that at a slip of unity, both forward and backward field develops equal torque but the direction of which are opposite to each other so the net torque produced is zero hence the motor fails to start. From here we can say that these motors are not self starting unlike the case of three phase induction motor. There must be some means to provide the starting torque. If by some means, we can increase the forward speed of the machine due to which the forward slip decreases the forward torque will increase and the reverse torque will decrease as a result of which motor will start.

From here we can conclude that for starting of single phase induction motor, there should be a production of difference of torque between the forward and backward field. If the forward field torque is larger than the backward field than the motor rotates in forward or anti clockwise direction. If the torque due to backward field is larger compared to other, then the motor rotates in backward or clockwise direction.

Applications of Induction motors

single phase motors are well suited for most applications: machine-tools (drills, lathes, mills); smaller fans and blowers; material handling (pumps, screw conveyors, short belt conveyors, etc.).

Three-phase motors are widely used in industrial drives because they are rugged, reliable and economical. Three phase induction motors are used widely for robotics, printing machines, grinding machines, varying load applications, rolling mills, section straightening etc..

UNIT-5

SPECIAL PURPOSE MACHINES

Stepper motor

A stepper motor is a pulse-driven motor that changes the angular position of the rotor in steps. Due to this nature of a stepper motor, it is widely used in low cost, open loop position control systems.

Types of stepper motors:

- Permanent Magnet stepper motors
- Variable Reluctance motor
- Hybrid stepper motor

1.1 Variable Reluctance Motor

Figure 1.1 shows the construction of Variable Reluctance motor. The cylindrical rotor is made of soft steel and has four poles as shown in Fig.1.1. It has four rotor teeth, 90^0 apart and six stator poles, 60^0 apart. Electromagnetic field is produced by activating the stator coils in sequence. It attracts the metal rotor. When the windings are energized in a reoccurring sequence of 2, 3, 1, and so on, the motor will rotate in a 30^0 step angle. In the non-energized condition, there is no magnetic flux in the air gap, as the stator is an electromagnet and the rotor is a piece of soft iron; hence, there is no detent torque. This type of stepper motor is called a variable reluctance stepper.

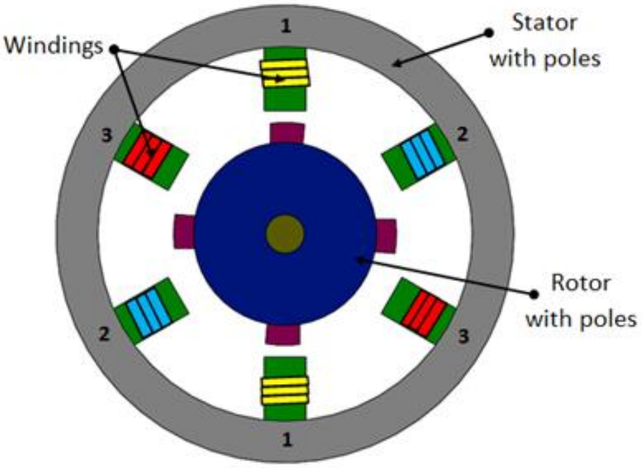


Fig.1.1 Variable reluctance stepper motor

1.2 Permanent magnet (PM) stepper motor

In this type of motor, the rotor is a permanent magnet. Unlike the other stepping motors, the PM motor rotor has no teeth and is designed to be magnetized at a right angle to its axis. Figure 4.18 shows a simple, 90^0 PM motor with four phases (A-D). Applying current to each phase in sequence will cause the rotor to rotate by adjusting to the changing magnetic fields. Although it operates at fairly low speed, the PM motor has a relatively high torque characteristic. These are low cost motors with typical step angle ranging between 7.5^0 to 15^0 .

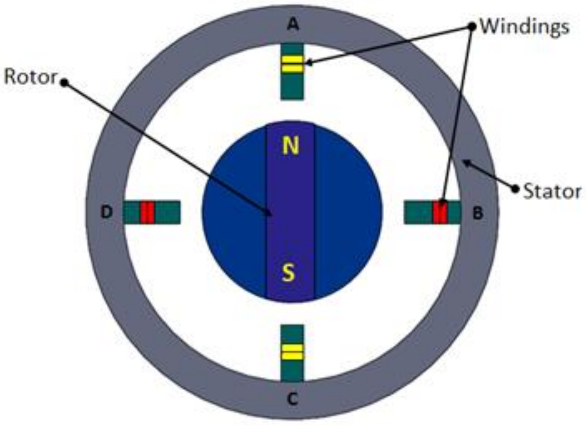


Fig.1.2 Permanent magnet stepper

1.3 Hybrid stepper motor

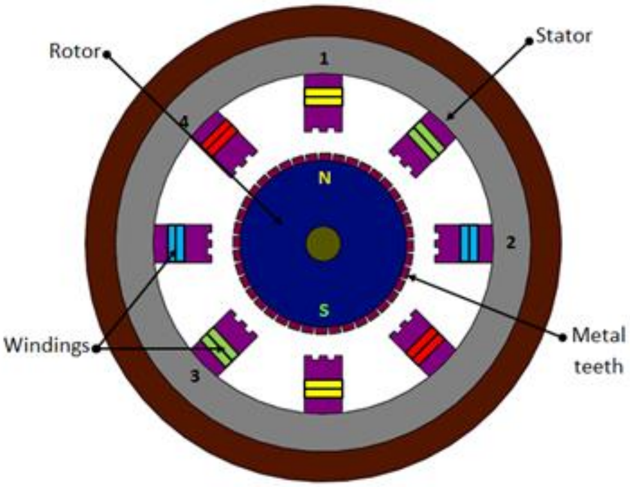


Fig. 1.3 Hybrid stepper

Hybrid stepping motors combine a permanent magnet and a rotor with metal teeth to provide features of the variable reluctance and permanent magnet motors together. The number of rotor pole pairs is equal to the number of teeth on one of the rotor's parts. The hybrid motor stator has teeth creating more poles than the main poles windings (Fig. 1.3).

Rotation of a hybrid stepping motor is produced in the similar fashion as a permanent magnet stepping motor, by energizing individual windings in a positive or negative direction. When a winding is energized, north and south poles are created, depending on the polarity of the current flowing. These generated poles attract the permanent poles of the rotor and also the finer metal teeth present on rotor. The rotor moves one step to align the offset magnetized rotor teeth to the corresponding energized windings. Hybrid motors are more expensive than motors with permanent magnets, but they use smaller steps, have greater torque and maximum speed.

Step angle of a stepper motor is given by,

$$\text{Step angle} = \frac{360^\circ}{\text{Number of poles}}$$

Advantages of stepper motors :

- Low cost
- Ruggedness

- Simplicity of construction
- Low maintenance
- Less likely to stall or slip
- Will work in any environment
- Excellent start-stop and reversing responses

Disadvantages of stepper motors :

- Low torque capacity compared to DC motors
- Limited speed
- During overloading, the synchronization will be broken. Vibration and noise occur when running at high speed.

A.C. SERIES MOTOR (or) UNIVERSAL MOTOR

A d.c. series motor will rotate in the same direction regardless of the polarity of the supply. One can expect that a d.c. series motor would also operate on a single-phase supply. It is then called an a.c. series motor. However, some changes must be made in a d.c. motor that is to operate satisfactorily on a.c. supply. The changes effected are:

(i) The entire magnetic circuit is laminated in order to reduce the eddy current loss. Hence an a.c. series motor requires a more expensive construction than a d.c. series motor.

(ii) The series field winding uses as few turns as possible to reduce the reactance of the field winding to a minimum. This reduces the voltage drop across the field winding.

(iii) A high field flux is obtained by using a low-reluctance magnetic circuit.

(iv) There is considerable sparking between the brushes and the commutator when the motor is used on a.c. supply. It is because the alternating flux establishes high currents in the coils short-circuited by the brushes. When the short-circuited coils break contact from the commutator, excessive sparking is produced. This can be eliminated by using high-resistance leads to connect the coils to the commutator segments.

Construction:

The construction of en a.c. series motor is very similar to a d.c. series motor except that above modifications are incorporated [See figure:2.1]. such a motor can be operated either on a.c. or d.c. supply and the resulting torque-speed curve is about the same in each case. For this reason, it is sometimes called a universal motor.

Operation

When the motor is connected to an a.c. supply, the same alternating current flows through the field and armature windings. The field winding produces an alternating flux that reacts with the current flowing in the armature to produce a torque. Since both armature current and flux reverse simultaneously, the torque always acts in the same direction. It may be noted that no rotating flux is produced in this type of machines; the principle of operation is the same as that of a d.c. series motor.

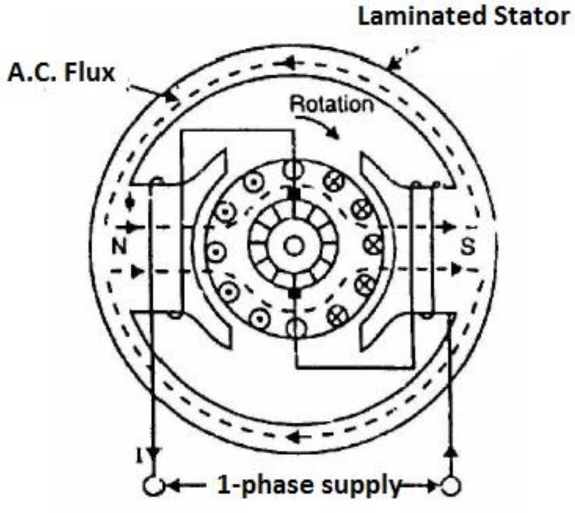


Fig: 2.1

Characteristics

The operating characteristics of an a.c. series motor are similar to those of a d.c. series motor.

(i) The speed increases to a high value with a decrease in load. In very small series motors, the losses are usually large enough at no load that limits the speed to a definite value (1500 - 15,000 r.p.m.).

(ii) The motor torque is high for large armature currents, thus giving a high starting torque.

(iii) At full-load, the power factor is about 90%. However, at starting or when carrying an overload, the power factor is lower.

Applications

The fractional horsepower a.c. series motors have high-speed (and corresponding small size) and large starting torque. They can, therefore, be used to drive:

a) high-speed vacuum cleaners
b) sewing machines
c) electric shavers
d) drills
e) Machine tools etc.

SERVO MOTOR

A servo motor is a linear or rotary actuator that provides fast precision position control for closed-loop position control applications. Unlike large industrial motors, a servo motor is not used for continuous energy conversion. Servo motors have a high speed response due to low inertia and are designed with small diameter and long rotor length.

Servo motors work on servo mechanism that uses position feedback to control the speed and final position of the motor. Internally, a servo motor combines a motor, feedback circuit, controller and other electronic circuit.

A servo motor is one of the widely used variable speed drives in industrial production and process automation and building technology worldwide.

Although servo motors are not a specific class of motor, they are intended and designed to use in motion control applications which require high accuracy positioning, quick reversing and exceptional performance.

It uses encoder or speed sensor to provide speed feedback and position. This feedback signal is compared with input command position (desired position of the motor corresponding to a load), and produces the error signal (if there exist a difference between them).

The error signal available at the output of error detector is not enough to drive the motor. So the error detector followed by a servo amplifier raises the voltage and power level of the error signal and then turns the shaft of the motor to desired position.

Types of Servo Motors

Basically, servo motors are classified into AC and DC servo motors depending upon the nature of supply used for its operation. Brushed permanent magnet DC servo motors are used for simple applications owing to their cost, efficiency and simplicity.

These are best suited for smaller applications. With the advancement of microprocessor and power transistor, AC servo motors are used more often due to their high accuracy control.

DC Servo Motors

A DC servo motor consists of a small DC motor, feedback potentiometer, gearbox, motor drive electronic circuit and electronic feedback control loop. It is more or less similar to the normal DC motor.

The stator of the motor consists of a cylindrical frame and the magnet is attached to the inside of the frame.

The rotor consists of brush and shaft. A commutator and a rotor metal supporting frame are attached to the outside of the shaft and the armature winding is coiled in the rotor metal supporting frame.

A brush is built with an armature coil that supplies the current to the commutator. At the back of the shaft, a detector is built into the rotor in order to detect the rotation speed.

With this construction, it is simple to design a controller using simple circuitry because the torque is proportional to the amount of current flow through the armature.

And also the instantaneous polarity of the control voltage decides the direction of torque developed by the motor. Types of DC servo motors include series motors, shunt control motor, split series motor, and permanent magnet shunt motor.

Working Principle of DC Servo Motor

A DC servo motor is an assembly of four major components, namely a DC motor, a position sensing device, a gear assembly, and a control circuit. The below figure shows the parts that consisting in RC servo motors in which small DC motor is employed for driving the loads at precise speed and position.

A DC reference voltage is set to the value corresponding to the desired output. This voltage can be applied by using another potentiometer, control pulse width to voltage converter, or through timers depending on the control circuitry.

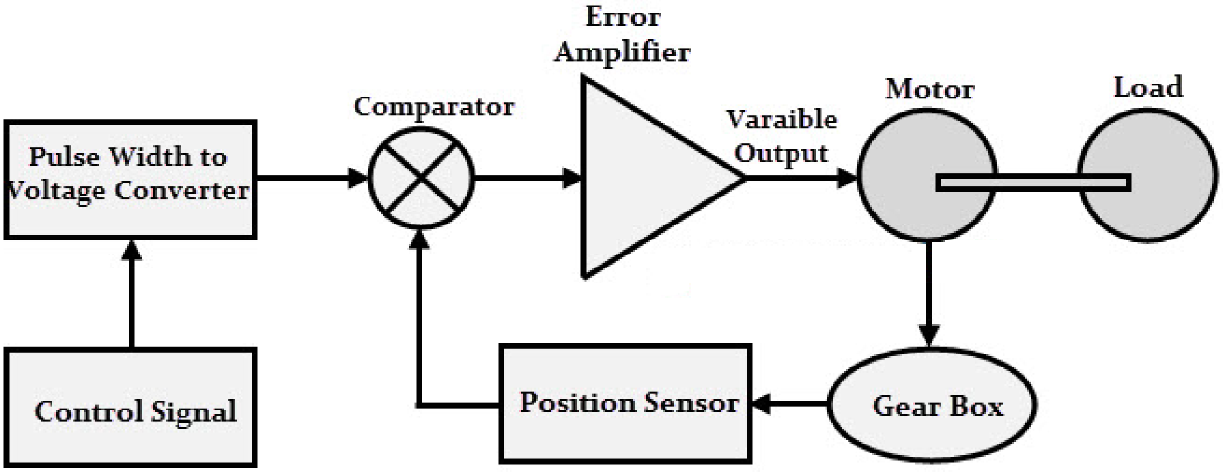
The dial on the potentiometer produces a corresponding voltage which is then applied as one of the inputs to error amplifier.

In some circuits, a control pulse is used to produce DC reference voltage corresponding to desired position or speed of the motor and it is applied to a pulse width to voltage converter.

In this converter, the capacitor starts charging at a constant rate when the pulse high. Then the charge on the capacitor is fed to the buffer amplifier when the pulse is low and this charge is further applied to the error amplifier.

So the length of the pulse decides the voltage applied at the error amplifier as a desired voltage to produce the desired speed or position.

In digital control, microprocessor or microcontroller are used for generating the PWM pluses in terms of duty cycles to produce more accurate control signals.



The feedback signal corresponding to the present position of the load is obtained by using a position sensor. This sensor is normally a potentiometer that produces the voltage corresponding to the absolute angle of the motor shaft through gear mechanism. Then the feedback voltage value is applied at the input of error amplifier (comparator).

The error amplifier is a negative feedback amplifier and it reduces the difference between its inputs. It compares the voltage related to current position of the motor (obtained by potentiometer) with desired voltage related to desired position of the motor (obtained by pulse width to voltage converter), and produces the error either a positive or negative voltage.

This error voltage is applied to the armature of the motor. If the error is more, the more output is applied to the motor armature.

As long as error exists, the amplifier amplifies the error voltage and correspondingly powers the armature. The motor rotates till the error becomes zero. If the error is negative, the armature voltage reverses and hence the armature rotates in the opposite direction.

AC Servo Motors

AC servo motors are basically two-phase squirrel cage induction motors and are used for low power applications. Nowadays, three phase squirrel cage induction motors have been modified such that they can be used in high power servo systems.

The main difference between a standard split-phase induction motor and AC motor is that the squirrel cage rotor of a servo motor has made with thinner conducting bars, so that the motor resistance is higher.

Based on the construction there are two distinct types of AC servo motors, they are synchronous type AC servo motor and induction type AC servo motor.

Synchronous-type AC servo motor consist of stator and rotor. The stator consists of a cylindrical frame and stator core. The armature coil wound around the stator core and the coil end is connected to with a lead wire through which current is provided to the motor.

The rotor consists of a permanent magnet and hence they do not rely on AC induction type rotor that has current induced into it. And hence these are also called as brushless servo motors because of structural characteristics.

When the stator field is excited, the rotor follows the rotating magnetic field of the stator at the synchronous speed. If the stator field stops, the rotor also stops. With this permanent magnet rotor, no rotor current is needed and hence less heat is produced.

Also, these motors have high efficiency due to the absence of rotor current. In order to know the position of rotor with respect to stator, an encoder is placed on the rotor and it acts as a feedback to the motor controller.

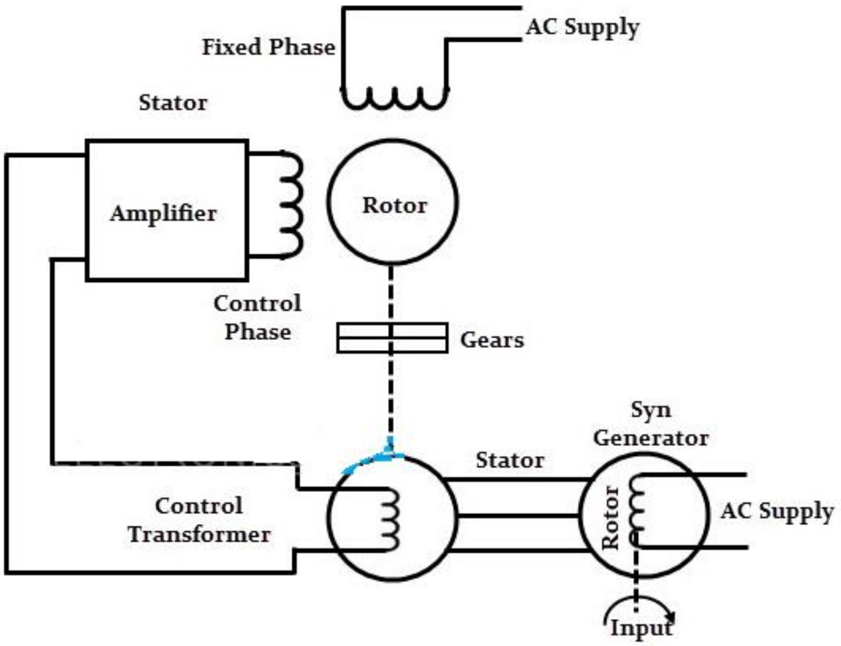
The **induction-type AC servo moto**r structure is identical with that of general motor. In this motor, stator consists of stator core, armature winding and lead wire, while rotor consists of shaft and the rotor core that built with a conductor as similar to squirrel cage rotor.

The working principle of this servo motor is similar to the normal induction motor. Again the controller must know the exact position of the rotor using encoder for precise speed and position control.

Working Principle of AC Servo Motor

The schematic diagram of servo system for AC two-phase induction motor is shown in the figure below. In this, the reference input at which the motor shaft has to maintain at a certain position is given to the rotor of synchro generator as mechanical input theta. This rotor is connected to the electrical input at rated voltage at a fixed frequency.

The three stator terminals of a synchro generator are connected correspondingly to the terminals of control transformer. The angular position of the two-phase motor is transmitted to the rotor of control transformer through gear train arrangement and it represents the control condition alpha.

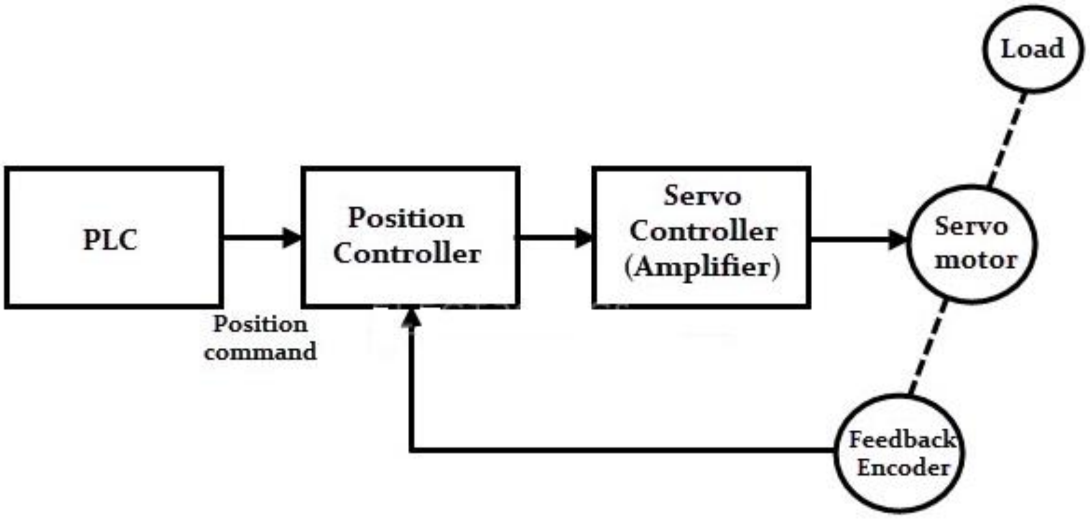


Initially, there exist a difference between the synchro generator shaft position and control transformer shaft position. This error is reflected as the voltage across the control transformer. This error voltage is applied to the servo amplifier and then to the control phase of the motor.

With the control voltage, the rotor of the motor rotates in required direction till the error becomes zero. This is how the desired shaft position is ensured in AC servo motors.

Alternatively, modern AC servo drives are embedded controllers like PLCs, microprocessors and microcontrollers to achieve variable frequency and variable voltage in order to drive the motor.

Mostly, pulse width modulation and Proportional-Integral-Derivative (PID) techniques are used to control the desired frequency and voltage. The block diagram of AC servo motor system using programmable logic controllers, position and servo controllers is given below.



Difference between the DC and AC Servo Motors

DC Servo Motor	AC Servo Motor
It delivers high power output	Delivers low output of about 0.5 W to 100 W
It has more stability problems	It has less stable problems
It requires frequent maintenance due to the presence of commutator	It requires less maintenance due to the absence of commutator
It provides high efficiency	The efficiency of AC servo motor is less and is about 5 to 20%
The life of DC servo motor depends on the life on brush life	The life of AC servo motor depends on bearing life
It includes permanent magnet in its construction	The synchronous type AC servo motor uses permanent magnet while induction type doesn’t require it.
These motors are used for high power applications	These motors are used for low power applications

Unit-VI

DIODE AND TRANSISTOR

Objectives

- To familiarize the students with the schematic details, working and characteristics of P-N junction diode
- To familiarize the students with operation of different types of rectifiers
- To familiarize the students about different types and working of transistors
- To familiarize the students with the characteristics of SCR

Syllabus :

P-N junction diode- symbol- V-I characteristics – diode applications- rectifiers – half wave and full wave and bridge rectifiers (simple problems)- pnp and npn transistors- transistor as amplifier- SCR characteristics and applications

Learning outcomes :

Student will be able to

- Describe the operational characteristics of p-n junction diode
- Understand the working principle of different types of rectifiers
- Describe the characteristics of transistor and SCR
- Select an appropriate device to meet specified performance requirements for a particular application

Unit-VI

Definition of Semiconductor

The materials that are neither conductor or insulator with energy gap of about 1 eV (electro volt) are called semiconductors.Most common type of materials that are commercially used as semiconductors are germanium (Ge) and silicon (Si) because of their property to withstand high temperature.

What is a Diode?

A diode is a device which only allows unidirectional flow of current if operated within a rated specified voltage level. A diode only blocks current in the reverse direction while the reverse voltage is within a limited range otherwise reverse barrier breaks and the voltage at which this breakdown occurs is called reverse breakdown voltage. A P-N junction is the simplest form of the diode which behaves as ideally short circuit when it is in forward biased and behaves as ideally open circuit when it is in the reverse biased. So a particular arrangement of diodes can convert AC to pulsating DC, and hence, it is sometimes also called as a rectifier. The name diode is derived from "di-ode" which means a device having two electrodes.

Symbol of Diode

The symbol of a diode is shown below, the arrowhead points in the direction of conventional current flow.

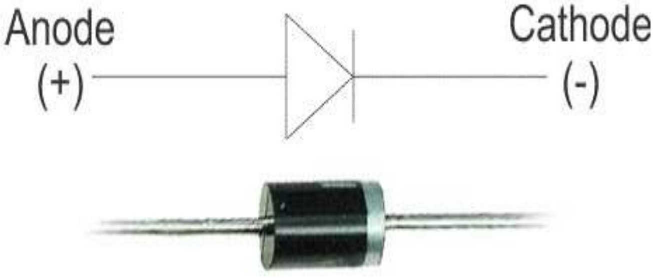


Fig 6.1: symbol of diode

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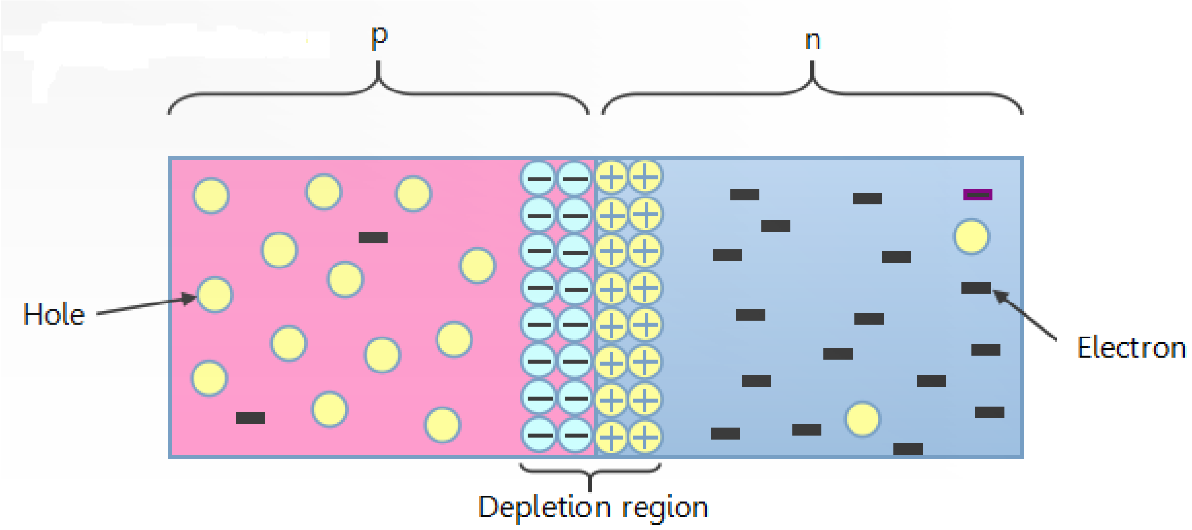
A simple PN junction diode can be created by doping donor impurity in one portion and acceptor impurity in other portion of a silicon or germanium crystal block. These make a p n junction at the middle portion of the block beside which one portion is p type (which is doped by trivalent or acceptor impurity) and other portion is n type (which is doped by pentavalent or donor impurity). Hence, it is a device with two elements, the p-type forms anode and the n-type forms the cathode. These terminals are brought out to make the external connections.

Working Principle of Diode

How depletion layer is formed:

The n side will have a large number of electrons and very few holes (due to thermal excitation) whereas the p side will have a high concentration of holes and very few electrons. Due to this, a process called diffusion takes place. In this process free electrons from the n side will diffuse (spread) into the p side and combine with holes present there, leaving a positive immobile (not moveable) ion in the n side. Hence, few atoms on the p side are converted into negative ions. Similarly, few atoms on the n-side will get converted to positive ions. Due to this large number of positive ions and negative ions will accumulate on the n-side and p-side respectively. This region so formed is called as depletion region. Due to the presence of these positive and negative ions a static electric field called as "barrier potential" is created across the p-n junction of the diode. It is called as "barrier potential" because it acts as a barrier and opposes the further migration of holes and electrons across the junction.



Situation after the formation of depletion region

When forward biased:

In a PN junction diode when the forward voltage is applied i.e. positive terminal of a source is connected to the p-type side, and the negative terminal of the source is connected to the n-type side, the diode is said to be in forward biased condition. We know that there is a barrier potential across the junction. This barrier potential is directed in the opposite of the forward applied voltage. So a diode can only allow current to flow in the forward direction when forward applied voltage is more than barrier potential of the junction. This voltage is called forward biased voltage. For silicon diode, it is 0.7 volts. For germanium diode, it is 0.3 volts. When forward applied voltage is more than this forward biased voltage, there will be forward current in the diode, and the diode will become short circuited.

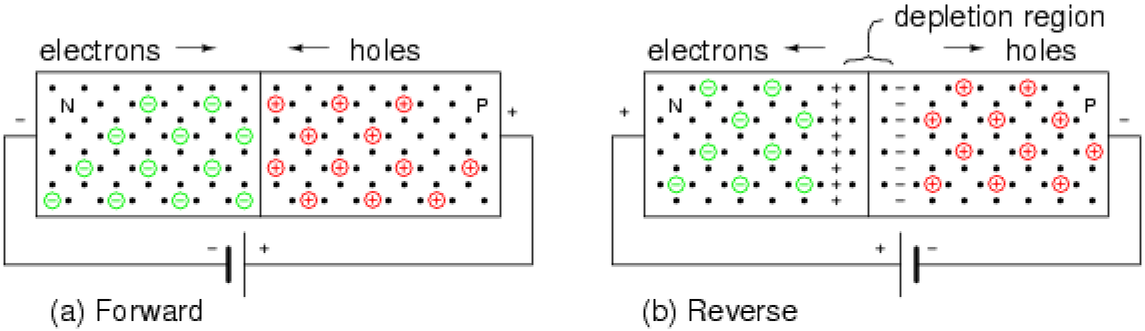


Fig 6.2: diode in forward and reverse biased conditions

When reverse biased:

Now if the diode is reverse biased i.e. positive terminal of the source is connected to the n-type end, and the negative terminal of the source is connected to the p-type end of the diode, there will be no current through the diode except reverse saturation current. This is because at the reverse biased condition the depletion layer of the junction becomes wider with increasing reverse biased voltage. Now if reverse applied voltage across the diode is continually increased, then after certain applied voltage the depletion layer will destroy which will cause a huge reverse current to flow through the diode. If this current is not externally limited and it reaches beyond the safe value, the diode may be permanently destroyed. This is because, as the magnitude of the reverse voltage increases, the kinetic energy of the minority charge carriers also increase. These fast moving electrons collide with the other atoms in the device to knock-off some more electrons from them. The electrons so released further release much more electrons from the atoms by breaking the covalent bonds. This process is termed as carrier multiplication and leads to a considerable increase in the flow of current through the p-n junction. The associated phenomenon is called Avalanche Breakdown.

P-N Junction Diode and Characteristics of p-n Junction

P-N junction diode is the most fundamental and the simplest electronics device. If, we apply forwards bias to the p-n junction diode. That means if positive side of the battery is connected to the p – side, then the depletion regions width decreases and carriers flow across the junction. If the bias is reversed the depletion width increases and no charge can flow across the junction.

Let's a voltage V is applied across a p-n junction and total current I, flows through the junction. It is given as $\mathbf{I = I_S[\exp(eV/\eta K_B T) - 1]}$

Here, I_S = reverse saturation current

e = charge of electron

η = emission co-efficient

K_B = Boltzmann constant

T = temperature

The current voltage characteristics plot is given below. The current voltage characteristics.

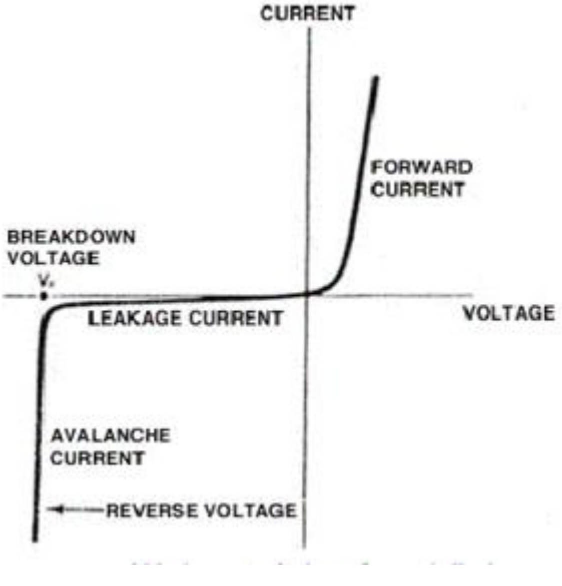


Fig 6.3 : V-I characteristics of diode

When, V is positive the junction is forward biased and when V is negative, the junction is reversing biased. When V is negative and less than V_{TH}, the current is very small. But when V exceeds V_{TH}, the current suddenly becomes very high. The voltage V_{TH} is known as threshold or cut in voltage. For Silicon diode V_{TH} = 0.6 V. At a reverse voltage corresponding to the point P, there is abrupt increment in reverse current. The PQ portion of the characteristics is known as breakdown region.

Applications of diode:

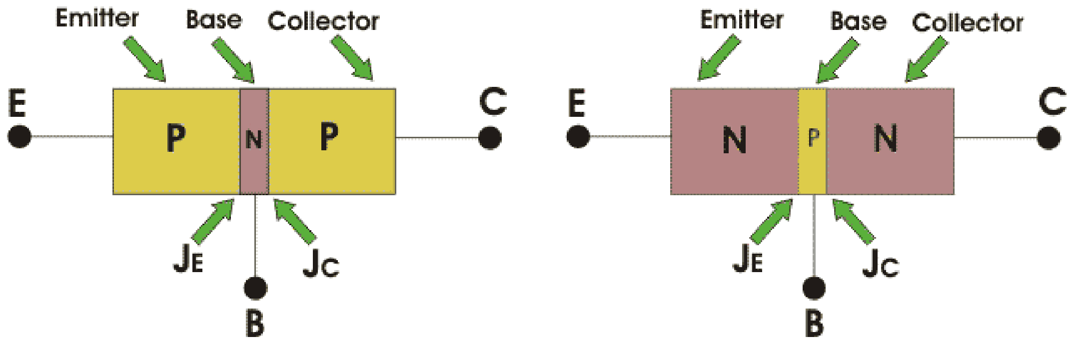
(1)Rectification (power conversion)
(2)Logic gates
(3)Radio demodulation
(4)Over voltage protection
(5)Clipper and clamper
(6)Temperature sensing device

Transistor:

The word “transistor” is derived from the words “Transfer” and “Resistor” it describes the operation of a BJT i.e. the transfer of an input signal from a low resistance circuit to a high resistance circuit. This type of transistor is made up of semiconductors. We know that silicon (Si) and Germanium (Ge) are the examples of semiconductors. Now, why this is called junction transistor? The answer lies behind the construction. We already know what is p-type and n-type semiconductors.

Definition of BJT

A bipolar junction transistor is a three terminal semiconductor device consisting of two p-n junctions which is able to amplify or “magnify” a signal. It is a current controlled device. The three terminals of the BJT are the base, the collector and the emitter. A signal of small amplitude if applied to the base is available in the amplified form at the collector of the transistor. This is the amplification provided by the BJT. Note that it does require an external source of DC power supply to carry out the amplification process. The basic diagrams of the two types of bipolar junction transistors mentioned above are given below.



PNP and NPN transistors

From the above figure, we can see that every BJT has three parts named emitter, base and collector. J_E and J_C represent junction of emitter and junction of collector respectively.In an npn transistor n - type can be sandwiched between two p-type semiconductors or similarly in a pnp transistor one p-type can be sandwiched between two n-type semiconductors. These are called p-n-p and n-p-n transistors respectively. As there are two junctions of different types of semiconductors, this is called junction transistor. It’s called “bipolar” because the conduction takes place due to both electrons as well as holes.

Transistor as a Switch

A normal switch offers open circuit when it is put in 'OFF' position and offers short circuit when it is put in 'ON' position. In other words, it can be said that a switch provides infinite resistance during its OFF condition and provides zero resistance during its ON condition. Hence a switch can be imagined as an ON-OFF controlled resistor, which provides either zero or infinite resistance to the circuit without any intermediate value.

A transistor on the other hand can be considered as a controlled resistor, as the resistance between emitter and collector is controlled by current in the base-emitter junction. By controlling base-emitter current, the emitter-collector resistance can be made either nearly infinite or nearly zero. A transistor gives quite a large resistance to the circuit but it is not ideally infinite also gives very small resistance but it is also not ideally zero.

Whenever we give small supply between emitter and base junction, large current will develop through base collector junction and so transistor will be in ON position.

In a transistor characteristic, there are three regions. They are –

- Cutoff Region
- Linear Region
- Saturation Region

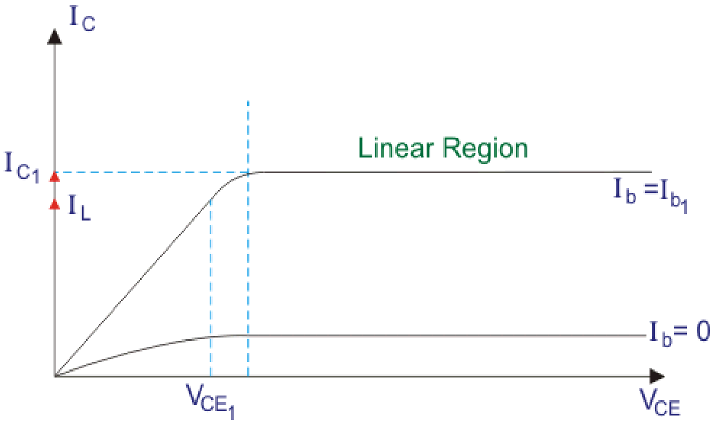


Fig 6.4: Output characteristic of transistor

The transistor behaves as a switch only when it is operated in cut - off and saturation region of its characteristic.

So for using the transistor as a switch we should make sure that the applied base-emitter current must be sufficiently high to keep the transistor in the saturation region, for given load current.Each transistor takes a finite time to go from OFF to ON condition and vice versa.

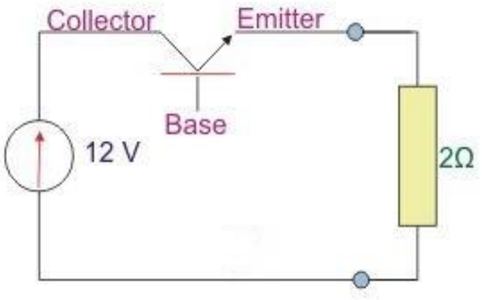


Fig 6.5: Transistor as a switch in circuit

Transistor as an Amplifier

Transistor is a semiconductor device with three terminals viz., Emitter (E), Base (B) and Collector (C) and thus has two junctions viz., Base-Emitter (BE) junction and Base-Collector (BC) junction as shown by Figure 1a. Such a device can operate in three different regions viz., cutoff, active and saturation. Transistors are fully-off in cut-off region while fully-on when operating in saturation region. However, while they operate in active region, they act as amplifiers i.e. they can be used to increase the strength of the input signal without altering it significantly.The reason behind such a behavior can be understood by analyzing the working of transistor in terms of charge carriers. For this, let us consider anpn bipolar junction transistor (BJT) biased to operate in active region (BE junction is forward biased while BC junction is reverse biased) as shown by Figure 1b.

Here, in general, the emitter will be heavily doped, the base will be lightly doped and the collector will be moderately doped. Further the base will be narrow, the emitter will be broader and the collector will be much broader.

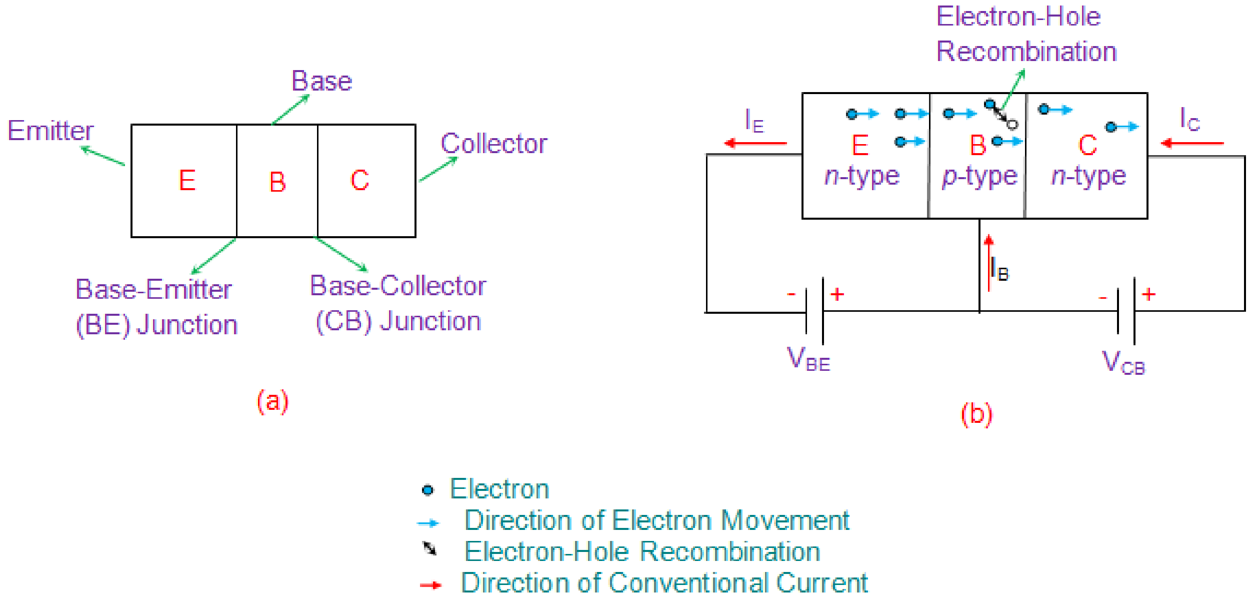


Figure 1 (a) Bipolar Junction Transistor (BJT) Showing Regions and Junctions
(b) *npn* BJT Biased to Operate in Active Region

The forward bias applied between the base and the emitter terminals of the transistor causes the flow of base current, I_B into the base region. However its magnitude is less (usually in terms of µA as V_{BE} is just around 0.6 V, in general). This can be considered as the movement ofelectrons out of the base region or the injection of holes into the base region, in equivalent sense. Further, these injected holes attract the electrons in the emitter region towards them, resulting in the recombination of holes and electrons. However due to the less doping of base in comparison with the emitter, there will be more number of electrons when compared to holes. Thus even after recombination effect, much more electrons will be left free. These electrons now cross the narrow base region and move towards the collector terminal influenced by the bias applied between the collector and the base regions. This constitutes collector current I_C moving into the collector. From this it can be noticed that by varying the current flowing into the base region (I_B), one can obtain a very large variation in collector current, I_C. This is the current amplification, which leads to the conclusion that the npnBJT operating in its active region acts as a current amplifier. The associated current gain can be mathematically expressed as-

$$\beta = \frac{I_C}{I_B}$$

Now consider the npn transistor with the input signal applied between its base and emitter terminals, while the output being collected across the load resistor RC, connected across the collector and the base terminals, as shown by Figure 2. Now consider the npn transistor with the input signal applied between its base and emitter terminals, while the output being collected across the load resistor RC, connected across the collector and the base terminals, as shown by figure 2.

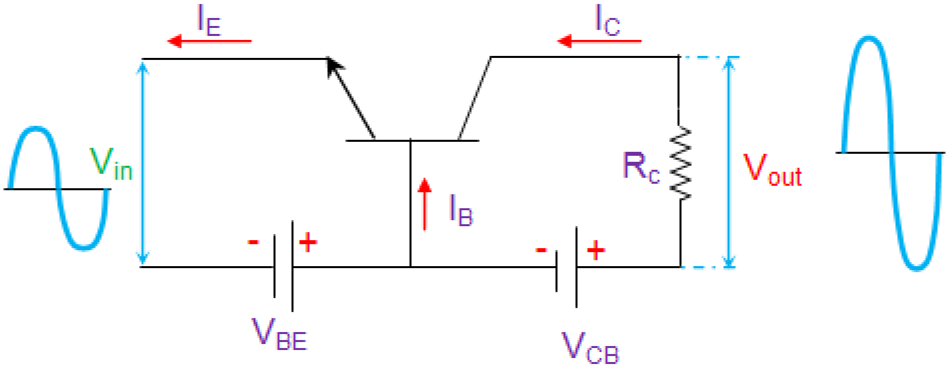


Fig 6.6: BJT as a voltage amplifier

Further note that the transistor is always ensured to operate in its active region by using appropriate voltage supplies, V_{EE} and V_{BC}. Here a small change in the input voltage V_{in} is seen to change the emitter current I_E appreciably as the resistance of the input circuit is low (due to the forward bias condition). This in turn changes the collector current almost in the same range due to the fact that the magnitude of the base current is quite less for the case under consideration. This large change in I_C causes a large voltage drop across the load resistor R_C which is nothing but the output voltage. Hence one gets the amplified version of the input voltage across the output terminals of the device which leads to the conclusion that the circuit acts like a voltage amplifier. Mathematical expression for the voltage gain associated with this phenomenon is given by

$$A_v = \frac{\Delta V_C}{\Delta V_B} = \frac{V_{out}}{V_{in}}$$

Transistor Characteristics

Transistor Characteristics are the plots which represent the relationships between the current and the voltages of a transistor

1. **Input Characteristics:** These describe the changes in input current with the variation in the values of input voltage keeping the output voltage constant.
2. **Output Characteristics:** This is a plot of output current versus output voltage with constant input current.
3. **Current Transfer Characteristics:** This characteristic curve shows the variation of output current in accordance with the input current, keeping output voltage constant.

Common Emitter (CE) Configuration of Transistor

In this configuration, the emitter terminal is common between the input and the output terminals as shown by Figure 9. This configuration offers medium input impedance, medium output impedance, medium current gain and voltage gain.

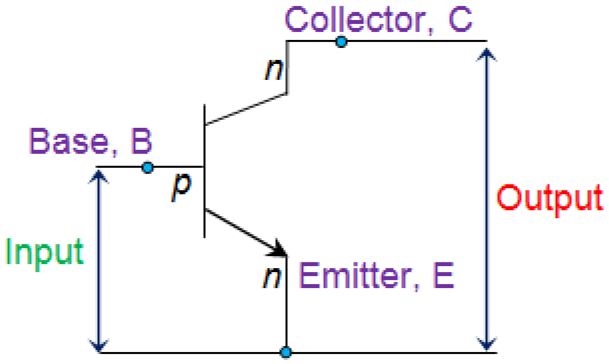


Fig 6.7: Common Emitter configuration

Input Characteristics for CE Configuration of Transistor

Figure 10 shows the input characteristics for the CE configuration of transistor which illustrates the variation in I_B in accordance with V_{BE} when V_{CE} is kept constant.

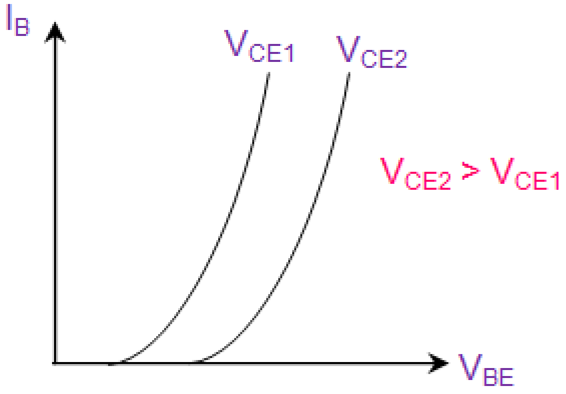


Fig 6.8 Input characteristics of CE configuration

From the graph shown, the input resistance of the transistor can be obtained as

$$R_{in} = \frac{\Delta V_{BE}}{\Delta I_B}\bigg|_{V_{CE}=constant}$$

Output Characteristics for CE Configuration of Transistor

The output characteristics of CE configuration (Figure 11) are also referred to as collector characteristics. This plot shows the variation in I_C with the changes in V_{CE} when I_B is held constant. From the graph shown, the output resistance can be obtained as

$$R_{out} = \left.\frac{\Delta V_{CE}}{\Delta I_C}\right|_{I_B=constant}$$

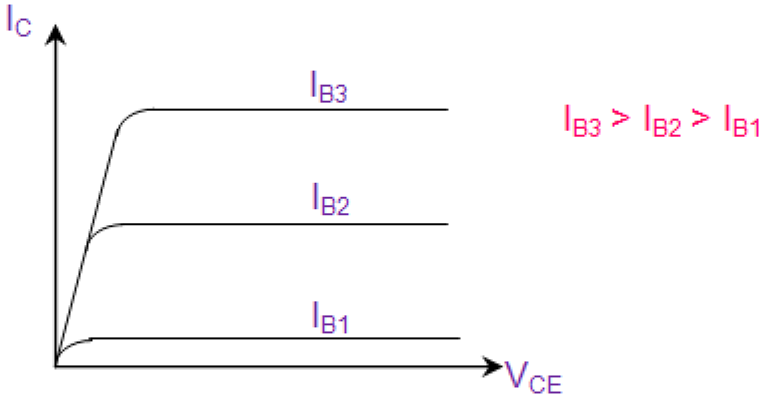


Fig 6.9: Output characteristics for CE configuration

Current Transfer Characteristics for CE Configuration of Transistor

This characteristic of CE configuration shows the variation of I_C with I_B keeping V_{CE} as aconstant. This can be mathematically given by

$$\beta = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{V_{CE}=constant}$$

This ratio is referred to as common-emitter current gain and is always greater than 1.

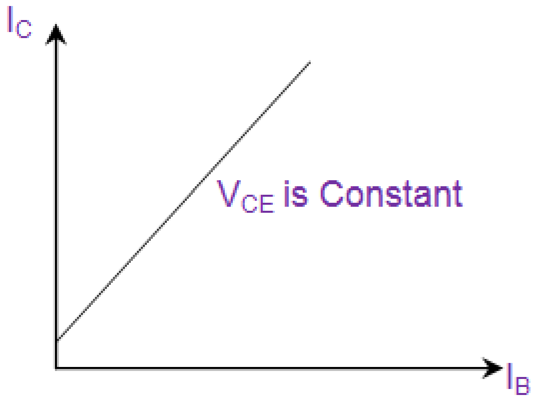


Fig 6.10: Current characteristics for CE configuration

Rectifier:

A rectifier is an electrical device that converts alternating current (AC), which periodically reverses direction, to direct current (DC), which flows in only one direction. The process is known as rectification.

Applications

The primary application of rectifiers is to derive DC power from an AC supply (AC to DC converter). Virtually all electronic devices require DC, so rectifiers are used inside the power supplies of virtually all electronic equipment.

Half Wave Diode Rectifier

Electric current flows through a p - n junction diode when it is forward biased and we get output current through the load. Let, we supply a sinusoidal voltage $V_{in} = V\sin\omega t$ as a source voltage. Now, if the input voltage is positive, the diode is forward biased and when that is negative, the diode is in reverse bias condition. When the input voltage is positive, i.e, for the positive cycle of the input voltage, the current flows through the diode.

So, the current will flow through the load also and we obtain output voltage across the load. But for the negative half cycle of the input, the p-n junction get reverse biased and no current flows through the diode as a result we obtain zero current and zero voltage across the load.

Circuit Diagram of Half Wave Rectifier

The basic diagram of half wave diode rectifier is given below,

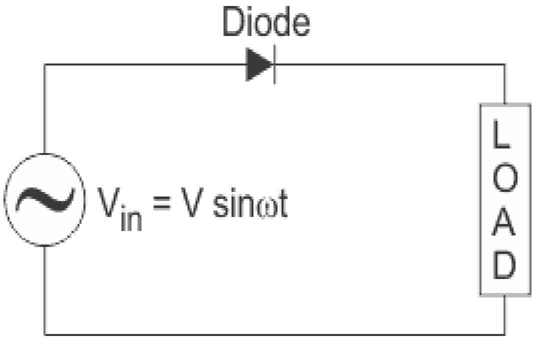
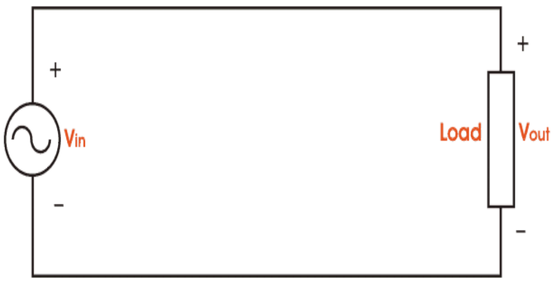
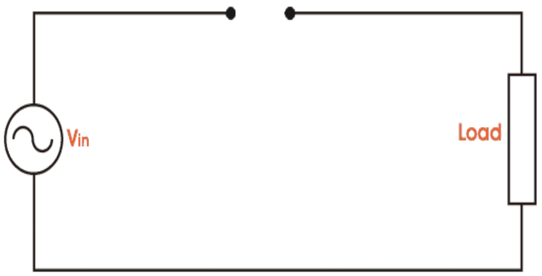


Fig 6.11: Half wave rectifier circuit

For positive half cycle



For negative half cycle



Input voltage and Output Voltage Waveforms

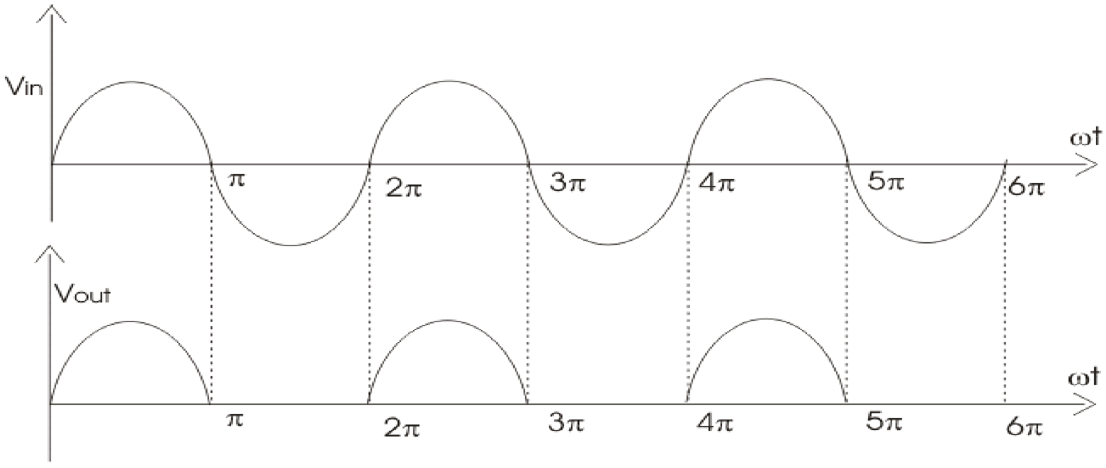


Fig 6.12: Half wave rectifier output waveforms

Now, different parameters for half wave rectifier is given below

The average of load current (I_{DC}) : Let, the load current be $i_L = I_m sinwt$,

$$I_{dc} = \frac{1}{2\pi}\int_0^{\pi} I_m \, sin\omega t = \frac{I_m}{\pi}$$

Ripple factor of half wave rectifier,

$$Ripple\ factor(r) = \frac{(I_{rms}^2 - I_{dc}^2)}{I_{dc}} = 1.21$$

The rms value of the load current (I_{rms})

$$I_{rms} = \frac{I_m}{4}$$

,

Full Wave Diode Rectifier

A rectifier circuit which gives continuous unidirectional current flow in the load circuit for both the input half cycles (for positive and negative half cycles) is known as full wave rectifier.

According to the diagram, given below is a center tapped transformer, D_1 and D_2 are two p-n junction diodes with similar characteristics. D_1 conducts for positive half of the output voltage and D_2conducts for the negative half of the output voltage. Thus we get full output voltage and the output current for the entire input cycle.

Circuit Diagram of Full Wave Diode Rectifier

The circuit diagram of the full wave diode rectifier given below,

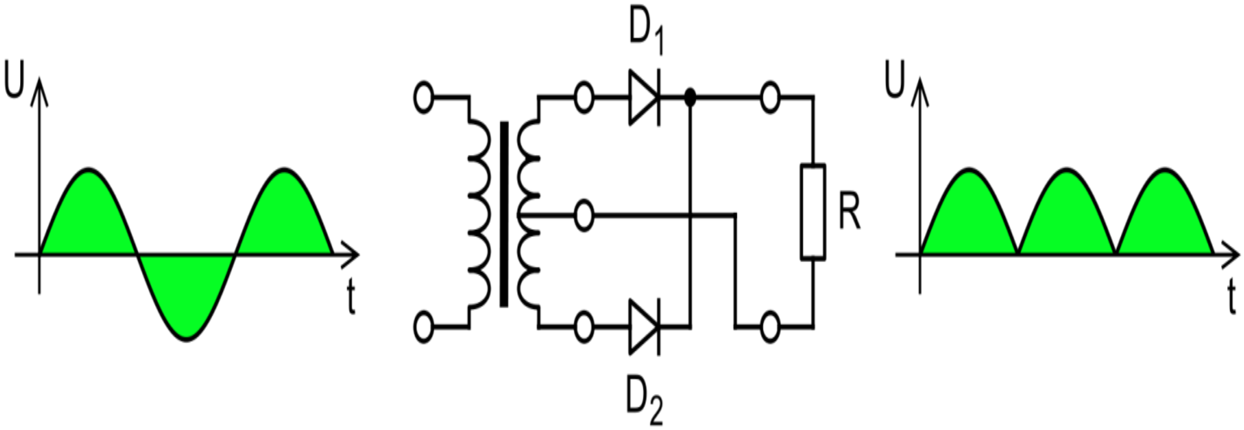
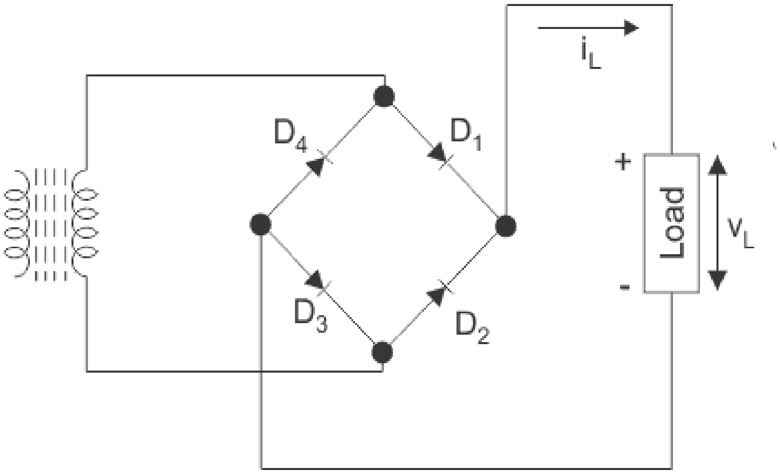


Fig 6.13: Full wavecentre tapped rectifier circuit

Full wave rectification can also be achieved using a bridge rectifier which is made of four diodes. It can be connected in two ways.



(or)

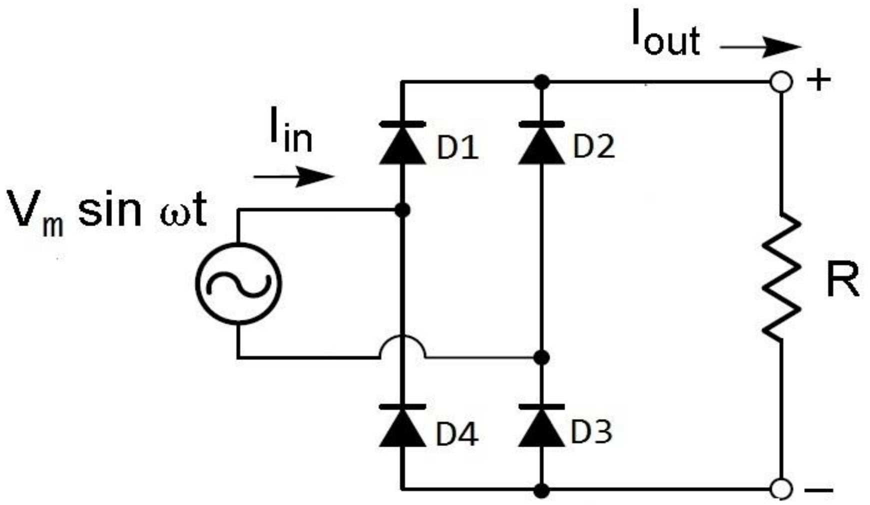


Fig 6.14: Full wave bridge rectifier circuit

According to the figure, when supply is positive D_1 and D_3 are forward biased, they conduct but D_2 and $D_4$are reverse biased.When supply is negative D_2 and $D_4$are forward biased, D_1 and D_3 are reverse biased. In both cases load current is in the same direction. Bridge rectifier has several advantages over simple full wave rectifier. It performance and efficiency is better than that of the simple full time rectifier.

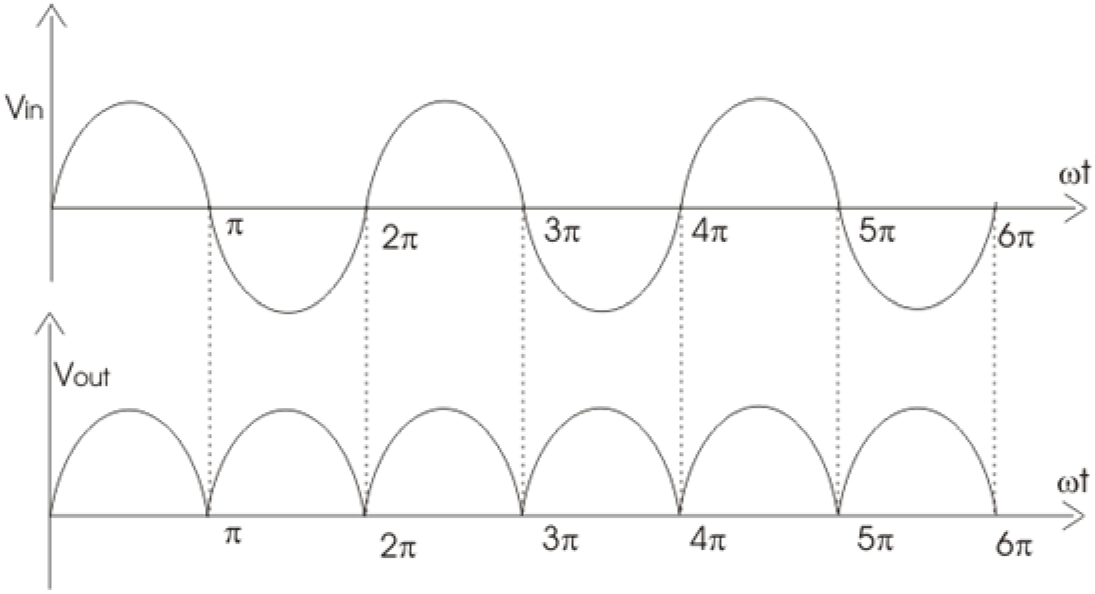


Fig 6.15: Full wave rectifier waveforms

Now, different parameters for full wave rectifier is given below.Let, the load current be $i_L = I_m \sin\omega t$

The average of load current (I_{dc}) :

$$I_{dc} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\omega t = \frac{2I_m}{\pi}$$

Ripple factor of full wave rectifier,

$$Ripple\ factor(r) = \frac{(I_{rms}^2 - I_{dc}^2)^{\frac{1}{2}}}{I_{dc}} = 0.482$$

$$Here,\ I_{rms} = \frac{I_m}{\sqrt{2}}$$

Rectifier with capacitor filter :

Here, Capacitor is used as a filter which filters out ripple frequencies and provides a DC voltage with less ripple frequency. To get a regulated DC voltage at the output we have to use a voltage regulator after filtering operation.

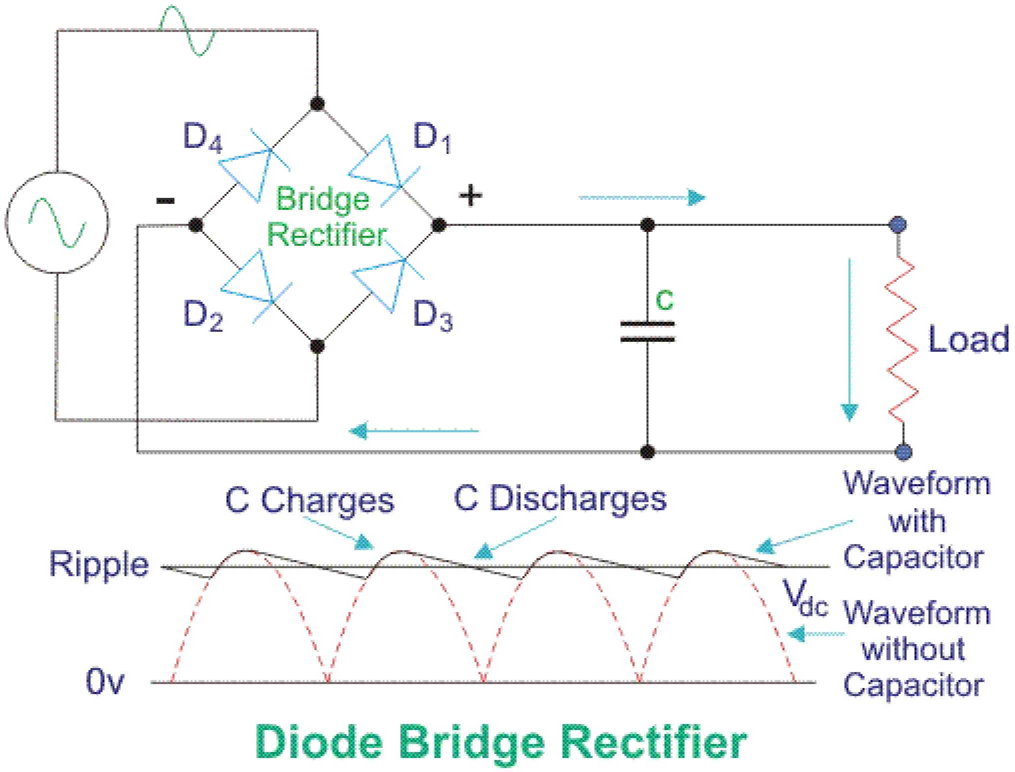


Fig 6.16: Full waverectifier circuit with capacitor filter

THYRISTOR (or) SCR

A thyristor is normally four layer three-terminal device. Four layers are formed by alternating n – type and p – type semiconductor materials. Consequently there are three p – n junctions formed inthe device. It is a bistable device. The three terminals of this device are called anode (A), cathode (K)and gate (G) respectively. The gate (G) terminal is control terminal of the device. That means, the current flowing through the device is controlled by electrical signal applied to the gate (G) terminal. A thyristor is on – off switch which is used to control output power of an electrical circuit by switching on and off the load circuit periodically in a preset interval. The main difference of thyristors with other digital and electronics switches is that, a thyristor can handle large current and can withstandlarge voltage, whereas other digital and electronic switches handle only tiny current and tiny voltage.

SILICON-CONTROLLED RECTIFIER

A silicon-controlled rectifier (or semiconductor-controlled rectifier) is a four-layer solid state current controlling device, Some sources define silicon controlled rectifiers and thyristors as synonymous

CONSTRUCTION

The Silicon Control Rectifier (SCR) consists of four layers of semiconductors, whichform NPNP or PNPN structures. It has three junctions, labeled J1, J2, and J3 and three terminals.The anode terminal of an SCR is connected to the P-Type material of a PNPN structure, and thecathode terminal is connected to the N-Type layer, while the gate of the Silicon Control RectifierSCR is connected to the P-Type material nearest to the cathode as shown in figure.An SCR consists of four layers of alternating P and N type semiconductor materials. Silicon is used as the intrinsic semiconductor, to which the proper dopants are added. The junctions are either diffused or alloyed. In this case, junction J2 is obtained by the diffusion method and then the outer two layers are alloyed to it, since the PNPN pellet is required to handle large currents. It is properly braced with tungsten or molybdenum platesto provide greater mechanical strength.

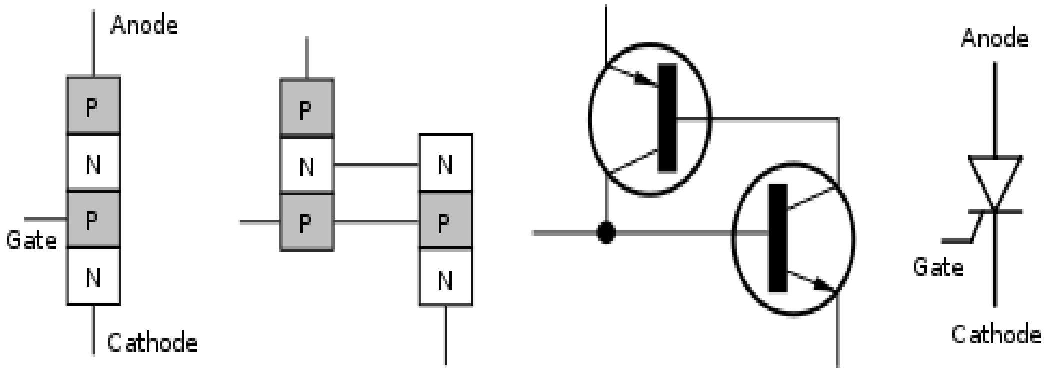


Fig 6.17: SCR Schematic and circuit symbol

MODES OF OPERATION

There are three modes of operation for an SCR depending upon the biasing given to it:

1. Forward blocking mode (off state)
2. Forward conduction mode (on state)
3. Reverse blocking mode (off state)

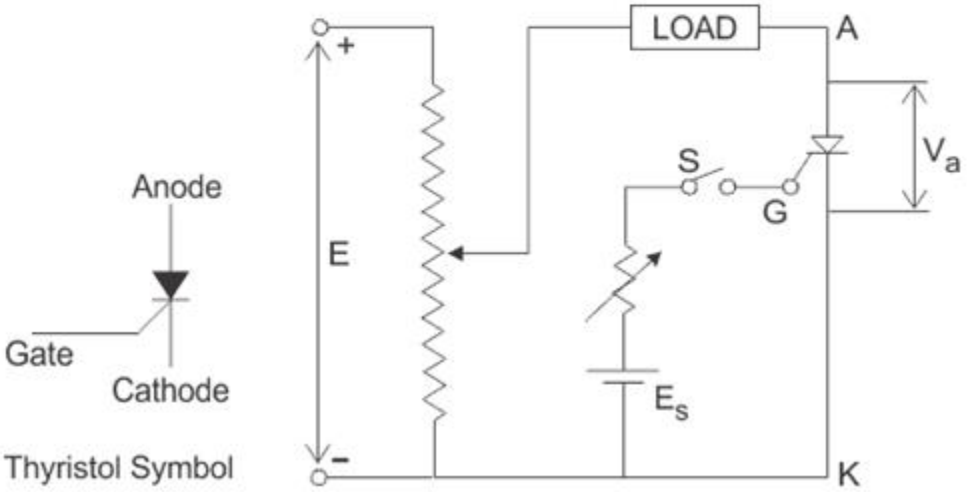
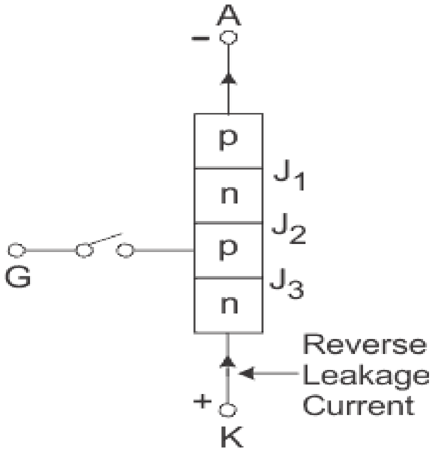


Fig 6.17: SCR or Thyristor circuit

From the circuit diagram above we can see the anode and cathode are connected to the supply voltage through the load. Another secondary supply Es is applied between the gate and the cathode terminal which supplies for the positive gate current when the switch S is closed.On giving the supply we get the requiredV-I characteristics of a thyristor shown in the figure below.

Reverse Blocking Mode of Thyristor

Initially for the reverse blocking mode of the thyristor, the cathode is made positive with respect to anode by supplying voltage E and the gate to cathode supply voltage Es is detached initially by keeping switch S open.



Reverse Blocking Mode

Here Junctions J1 and J3 are reverse biased whereas the junction J2 is forward biased. The behavior of the thyristor here is similar to that of two diodes are connected in series with reverse voltage applied across them. As a result only a small leakage current of the order of a few µAmps flows. This is the reverse blocking mode or the off-state, of the thyristor. If the reverse voltage is now increased, then at a particular voltage, known as the critical breakdown voltage VBR(Reverse breakover voltage), an avalanche occurs at J1 and J3 and the reverse current increases rapidly. A large current associated with VBR gives rise to more losses in the SCR, which results in heating. This may lead to thyristor damage as the junction temperature may exceed its permissible temperature rise. It should, therefore, be ensured that maximum working reverse voltage across a thyristor does not exceed VBR. The SCR in the reverse blocking mode may therefore be treated as open circuit.

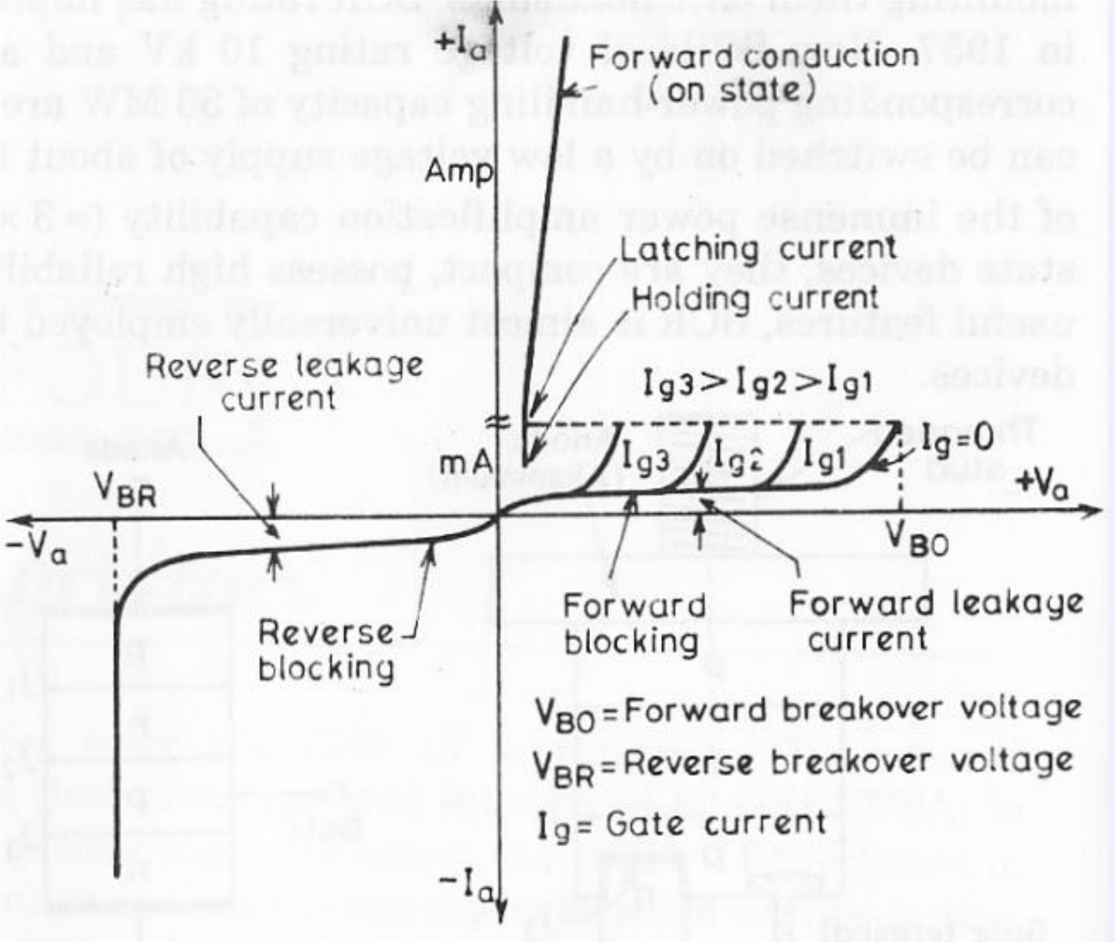
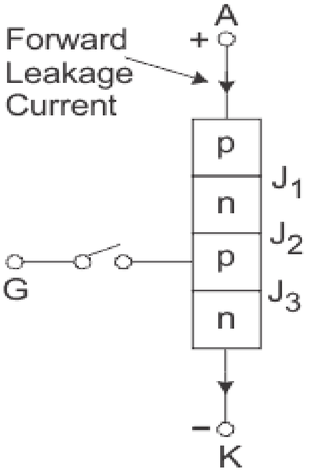


Fig 6.18: Characteristics of SCR

Forward Blocking Mode

Now considering the anode is positive with respect to the cathode, with gate kept in open condition.The thyristor is now said to be forward biased as shown in the figure below.As we can see the junctions J1 and J3 are now forward biased but junction J2 goes into reverse biased condition. In this particular mode, a small current, called forward leakage current is allowed to flow initially as shown in the diagram for characteristics of thyristor. Now, if we keep on increasing the forward biased anode to cathode voltage.



Forward Biased Condition

beyond a certain point, then the reverse biased junction J2 will have an avalanche breakdown at a voltage called forward break over voltage VB0 of the thyristor. But, if we keep the forward voltage less than VBO,we can see from the characteristics of thyristor, that the device offers a high impedance. Thus even here the thyristor operates as an open switch during the forward blocking mode.

Forward Conduction Mode

When the anode to cathode forward voltage is increased, with gate circuit open, the reverse junction J2 will have an avalanche breakdown at forward break over voltage VBO leading to thyristor turn on.Once the thyristor is turned on we can see from the diagram for characteristics of thyristor, that the point M at once shifts toward N and then anywhere between N and K. Here NK represents the forward conduction mode of the thyristor. In this mode of operation, the thyristor conducts maximum current with minimum voltage drop, this is known as the forward conduction forward conduction or the turn on mode of the thyristor.

Latching current:

It is the minimum value of anode current which it must attain during turn-on process to maintain conduction when gate pulse is removed.Latching current is associated with the turn ON process of SCR.

Holding current:

Thyristor can be turned off or returned to the forward blocking mode only if the anode current falls below a low level current called the holding current. Holding current is associated with the turn OFF process of SCR.

Commutation:

The process of turning OFF SCR is defined as “Commutation”. In all commutation techniques, a reverse voltage is applied across the thyristor during the turn OFF process. By turning OFF a thyristor we bring it from forward conducting to the forward blocking mode.

Turn-ON methods of SCR

The SCR can be switched on either by increasing the forward voltage beyond forward break over voltage VFBO or by applying a positive gate signal when the device is forward biased. Of these two methods, the latter, called the gate-control method, is used as it is more efficient and easy to implement for power control.

The following points have to be noted when designing the gate-control circuit.

1.Appropriate gate-to-cathode voltage must be applied for turn-on when the device is forward biased.

2. The gate signal must be removed after the device is turned-on.

3. No gate signal should be applied when the device is reverse-biased.

4. When the device is in the off-state, a negative voltage applied between the gate and the cathodewill improve the characteristics of the device. In such an instance, a large positive voltage will berequired to overcome this negative bias for turn-on.

The most used methods for triggering an SCR or thyristor are mentioned below:

Voltage Triggering
Thermal Triggering
Radiation Triggering
dv/dt triggering
Gate triggering

1.Voltage Triggering

The method of triggering in which the triggering of SCR or thyristor is caused by the applied voltageacross anode and cathode terminals, is known as Voltage triggering.When a thyristor is in forward biased and applied voltage across anpde and cathode is increased, thenthe depletion layer of reverse biased junction decreased. At breakdown voltage, the depletion layer istotally destroyed and as a result the thyristor triggers and comes to ON state and start conductingheavily due to increase in number of charge carriers.Because this triggering is caused by Voltage, thats why it is called Voltage triggering.

2.Thermal Triggering

The triggering method in which triggering is caused by the thermal effect i.e. by increasing the junction temperature, is called Thermal triggering.We know that conductivity of semi-conductor materials increases with increase in temperature. Herewe make use of this property. When the junction is reverse biased and the applied voltage across anode and cathode is near to breakdown voltage then we increases the temperature of the junction asa result the due to formation of electron-hole pairs, the thyristor starts conducting.In this case we should not increase the temperature to a high value, because this may cause damageto device. So, in this method the SCR or thyristor is triggered by thermal properties, therefore it is called thermal Triggering.

3.Radiation Triggering

The method in which the triggering of a SCR or thyristor is caused by radiation i.e. by thebombardment of energy particles , for example Neutrons, photons etc.In this method the triggering is done with the help of Radiation or by bombardment of energyparticles. The thyristor is bombarded by energy particles like neutrons or photons. Due to theradiation, the electron-hole pairs are generated in the device. These pairs causes the flow of current inthe device. Thus thyristor comes to ON state and hence triggered.

4.dv/dt triggering

The triggering method in which we make use of high rate of increase of voltage(or dv/dt i.e. rate ofchange of voltage with respect to time). When a thyristor is optimised for a critical rate of rise ofvoltage, then if the rate of rise of voltage increases, the thyristor will be triggered and start conducting.

5.Gate triggering

The triggering method in which triggering of SCR or thyristor is caused by applying a signal betweengate and cathode, is called Gate triggering.Gate triggering is mostly used method for triggering an SCR or thyristor. This method is used inalmost all industries and laboratories to trigger thyristor. In this method a positive signal is applied inbetween gate and cathode terminal. By using this method we can trigger the device much before itsbreakdown voltage. Hence, we can also control the firing angle(α) and the conduction angle(β=180-α) of SCR or thyristor.